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PROFILING AND RUNNING CHECK OF THE OPERATING PART OF WORM CUTTERS FOR CUTTING OF STRAIGHT-LINE MULTIPLE SPLINE SHAFTS

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Abstract: *The present paper offers an innovative method of profiling and running check of worm cutters for cutting of multiple spline shafts with straight form line of teeth. The method determines the worm cutter tooth shape, which guarantees the accuracy of the multiple spline shafts at design stage.*

Keywords**:** *profiling method, multiple spline shaft, centroidal generation.*

1. INTRODUCTION

The accuracy of profiling of the operating part of the worm cutter teeth for cutting of cylindrical gear with straight form line depends primarily on the methods used. Those methods share a common deficiency, namely that *the solution accuracy depends not on the method itself but on the production experience gained* [1, 2, and 3].

The fast pace with which computers are introduced into machine-building production provides a real chance for revising and re-thinking of some methods for design of machine tools in the direction of improving the accuracy in the most crucial areas – the profiling of their operating parts and the conducting of running checks, which guarantee the solution accuracy. Prerequisites have been created for optimizing of some basic parameters, arriving at multiple solutions, etc.

The analytical methods used for profiling of teeth in a normal section and described in specialized reference materials are developed based on the method of *centroidal generation*. The underlying idea in it is that the initial cylinders of the tooth pair "tool-gear" roll one over the other without skidding, and in the contact point of the combined teeth they have a common tangent and common normal, which passes through the gearing pole.

One of the main tasks solved through any analytical method of profiling of the operating part of worm cutters for cutting of cylindrical gear with straight form line is to change the complex theoretical transcendental form line of the starting tool surface with one that can be easily realized with the needed accuracy of the existing technological equipment used. *For the profiling case under study the theoretical profile of the cutting edge of the tooth is replaced by a circumference arc.*

When defining the parameters of the replacing circumference, we should make sure there is a minimal difference between that and the theoretical line. The solution of this task is achieved through mathematical methods which lead to approximated solutions [4, 5].

The accuracy of the solution is evaluated by comparing the error from replacing the theoretical cutting edge profile with the circumference arc and the gear teeth width allowance.

Analyzing the analytical methods for profiling of worm cutters for cutting of cylindrical gear with straight form line, we can make the following basic conclusion: *the profiling of worm cutter teeth under these methods does not provide a complex answer to the question whether the spline shaft teeth come with the required shape and size.*

2. EXPOSITION

2.1. Choice of analytical method for profiling of the operating part teeth

For the profiling of the worm cutter operating part the method of generation is used in its plane version, which can be realized in several ways. Two of them are most commonly used. They are described in the reference materials as:

- *the method of enveloping curves and surfaces;*
- *the method of common normals to the twisted profiles, using the profiling line (method of Relo).*

The final mathematical equations for the forming line of the starting tool surface are the same for both methods [6].

Both methods share the deficiency of not defining the side profile of the spline shaft teeth. This profile can be determined by the deviation of the real form line of the tool surface from the theoretical form line of the starting tool surface of the cutter.

The mathematical formulae of the second method are used for defining the theoretical profile of the cutting edges of the worm spline cutter operating part.

The profile of the spline shaft side surfaces is defined through the method of enveloping curves and surfaces. This method is used more rarely for profiling of worm cutters. But with the help of computer equipment it is used for *obtaining the theoretical profile of side surfaces, which hasn't been described in reference materials up till now.* It allows looking for optimal parameters of the replacing circumference arc of the cutting edges with minimum deviation of the theoretical straight line profile from the real one. Besides, the radius of the end of the transition curve can be determined precisely.

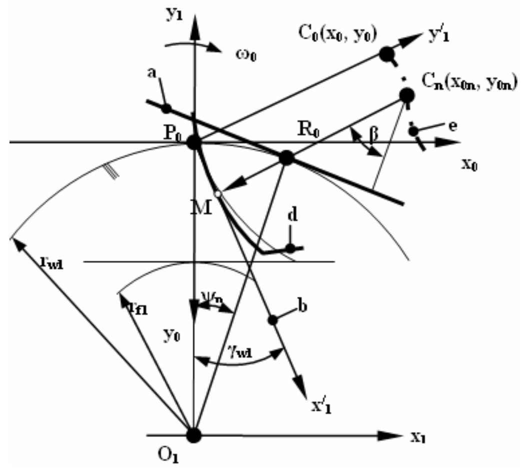

Fig. 1. Computational scheme for determining the form line of the starting tool surface.

2.2. Formulation of task

Appropriate computational schemes have been developed for creating the mathematical model to be used for the achievement of the goals set. The following considerations have been taken into account when developing them: in the tooth pair "tool-gear" "no windage" gearing is reproduced; the main stages of the method of centroidal generation are reproduced too; coordinates connected with the part and the tool are introduced for determining the parameters of form lines.

The theoretical straight line (*b*) and the real form line (c) of the spline shaft and those of the worm cutter – the theoretical form line of the starting tool surface (*g*) and the real form line of the starting tool surface (*d*) are determined through the computational scheme given on Fig. 1.

The line (*b*) is given in the coordinates x_1Oy_1, fixed to the part. The parameters of (*c*) are determined in $x_1^{/}P_0y_1^{/}$, fixed to the theoretical form line (b) of the part. The parameters of the theoretical (*g*) and the real (*d*) lines of the tool are determined in the coordinates, fixed to the centroidal straight line of the cutter. The parameters of the transition curve are determined in the coordinates $x_1O_1y_1$.

To facilitate the presentation of the newly developed method for determining the tooth profile of the cut spline shaft the following symbols have been adopted:

1 – parameters referring to spline shaft,

0 – parameters referring to worm cutter,

γ_{w1} – profile angle,

ψ_n – generation angle for the current position of profiling,

r_{w1} – radius of spline shaft centroidal circumference,

r_{f1} – radius of the bottom curve between spline shaft teeth,

R_0 – radius of the circumference replacing the theoretical profile of the form line of the tool cutting edge,

O_1 – spline shaft axis,

P_0 – gearing pole in "starting" position of profiling,

P_n – current position of the gearing pole,

C_0 – center of replacing circumference in "starting" position of profiling,

C_n – current position of the replacing circumference center,

$x_0P_0y_0$ – co-ordinates firmly fixed to the cutter's centroidal straight line,

$x_1^{/}P_0y_1^{/}$ – co-ordinates firmly fixed to the straight form line of the part,

$x_1O_1y_1$ – co-ordinates fixed to the part's axis,

a – tool centroidal straight line,

b – theoretical straight form line of the part's profile,

c – real form line of the part's profile,

g – theoretical form line of the starting tool surface,

d – real form line of the starting tool surface ,

e – trajectory of the replacing circumference center.

2.3. Determining the form line of the cutter tool surface

In order to determine the form line of the starting cutter tool surface the basic parameters of the tooth profile of the spline shaft should be known. The radius of its centroidal circumference r_{w1} should be noted specifically. It can either be determined through formulae or given in advance.

Using the main parameters of the tooth profile the following equation for the form line (*b*) is written down:

$$y_1 = x_1.\tan\left(90^0 - \gamma_{w1}^0\right) + R_{w1}.. \tag{1}$$

Knowing the equation of the line (*b*) we can write the mathematical equations given in different methodologies [4, 5, 6, 7, and 8], for the theoretical line (*g*) of the starting tool surface:

$$\begin{aligned} x_{01} &= r_{w1}\left[\left(\psi_n - \gamma_{w1}\right) - \left(\sin\psi_n - \sin\gamma_{w1}\right)\cos\psi_n\right] \\ y_{01} &= r_{w1}\left[\left(\sin\psi_n - \sin\gamma_{w1}\right)\sin\psi_n\right]. \end{aligned} \tag{2}$$

The equations of the line (*g*) are transcendental and its shape is quite complex. It is common from a technological aspect to replace this line within the borders of the tool tooth height by an arc from one or two circumferences [5, 7, and 8].

It is known that a circumference is defined solely by three points. It is recommended that the first point should be with coordinates (0, 0) in the system $x_0P_0y_0$, i.e. the circumference should always pass through the gearing pole P_0. It does not discount the possibility for other coordinates, different from zero. The coordinates of the other two points are chosen within the borders of the tooth height (Fig. 2).

The parameters of the replacing circumference R_0, x_0 and y_0 are determined through familiar formulae, given it the specialized reference materials [6, 7].

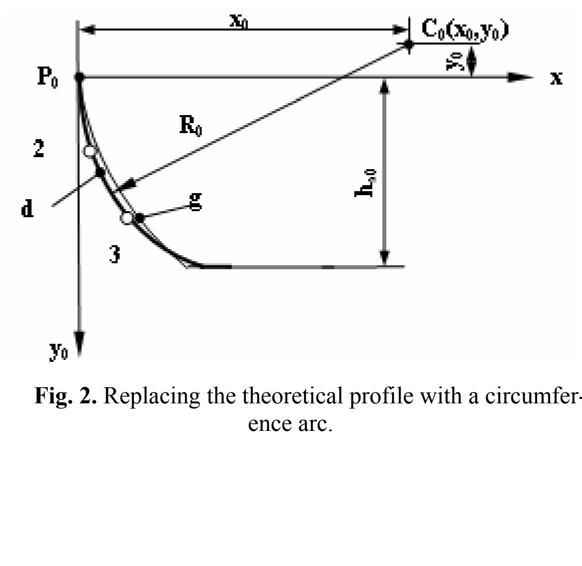

Fig. 2. Replacing the theoretical profile with a circumference arc.

The classical approach for evaluating the solution acquired is to compare the maximum deviation between the replacing circumference arc (*d*), within the borders of the tooth height, and the theoretical form line (*g*). If this deviation is smaller than the experimentally determined values, it can be concluded that the solution acquired is good and hence, *it can be implied that the part surfaces possess the needed accuracy and shape*. This ends the entire profiling process. Unfortunately, these methods do not provide an answer to the issue discussed in this paper – what the true shape and size of the treated teeth is, which depends on the profile of the worm cutter cutting teeth.

2.4. Defining the real form line of the side surface of the spline shaft teeth

Defining the equations of the form line (*c*) of the spline shaft is realized through the computational pattern given on Fig.1. It is assumed that the part is immobile. The centroidal straight line (*a*) of the worm cutter performs rotating generation movement (it rolls without skidding) along the centroidal circumference with a radius r_{w1} of the spline shaft.

The real form line (*c*) encircles a family of circumference arcs with a radius R_0 and a center C_0, characterizing the parameters x_0 and y_0.

The center C_0 of the replacing circumference and its arc, which describes the real form line (*d*) of the tool surface of the worm cutter, are fixed to its centroidal straight line (*a*). While performing its generating movement the center follows a shortened involute [9]. The equations of this transcendental curve are given in x_1Oy_1:

$$\begin{aligned} x_c &= (r_{w1}+y_0)\sin\psi_n + (x_0 - r_{w1}\cdot\psi_n)\cos\psi_n \\ y_c &= (r_{w1}+y_o)\cos\psi_n - (x_0 - r_{w1}\cdot\psi_n)\sin\psi_n \end{aligned} \qquad (3)$$

In the equations (3) the ordinate $\mathbf{y_0}$ is replaced by its absolute value.

The profiling (contact) points will stand at an equal distance R_0 from the trajectory of the center C_0 of the replacing circumference. Then the equations of the enveloping line of these contact points will be:

$$\begin{aligned} x_M &= x_c - R_0\sin(\beta+\psi_n) \\ y_M &= y_c - R_0\cos(\beta+\psi_n) \end{aligned} \qquad (4)$$

The angle β, characterizing the point on the cutting edge at the moment of profiling, is defined through the equation:

$$tg\beta = \frac{x_0 - r_{w1}\cdot\psi_n}{y_0}. \qquad (5)$$

The equations of the enveloping line in x_1Oy_1 will be:

$$\begin{aligned} x_M &= (r_{w1}+y_0)\sin\psi_n + (x_0 - r_{w1}\cdot\psi_n)\cos\psi_n - R_0\sin(\beta+\psi_n) \\ y_M &= (r_{w1}+y_0)\cos\psi_n - (x_0 - r_{w1}\cdot\psi_n)\sin\psi_n - R_0\cos(\beta+\psi_n) \end{aligned} \qquad (6)$$

For the sake of convenience, the equations of the real form line (**c**) of the spline shaft tooth profile are defined in the coordinates $x_1^{/}P_0y_1^{/}$ (Fig. 3) using the equations in (6) and written down with the equations (7).

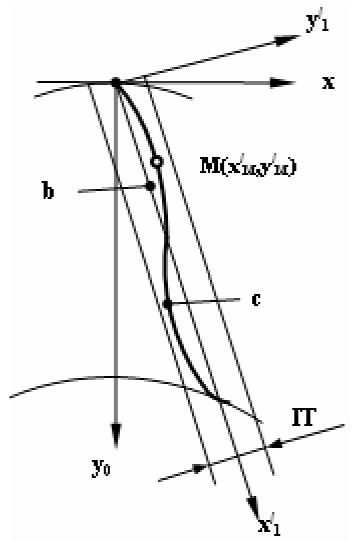

Fig. 3. Spline shaft tooth profile.

The ordinate values $y_1^{/}$ are the deviations of the real profile (*c*) of the side surfaces of the gear teeth belonging to the theoretical profile (*b*) (Fig. 3).

$$\begin{aligned} x_{M1}^{/} &= (r_{w1}+y_0)\cos\mu + (x_0 - r_{w1}\cdot\psi_n)\sin\mu + R_0\cos(\beta+\mu) + r_{w1}\cos\gamma_{w1} \\ y_{M1}^{/} &= (r_{w1}+y_0)\sin\mu + (x_0 - r_{w1}\cdot\psi_n)\cos\mu - R_0\sin(\beta+\mu) + r_{w1}\cos\gamma_{w1} \end{aligned} \qquad (7),$$

where $\mu = \psi_n + \gamma_{w1}$.

The maximum and minimum width of the spline teeth will be:

$$b_{max} = b + 2y_{M1\,max}^{/},$$
$$b_{min} = b - 2y_{M1\,min}.$$

For quantitative evaluation of the replacement of the theoretical form line of the starting tool surface with its real form line (circumference arc) a check is performed

$$b_{max} - b_{min} \le IT_в.$$

If this condition is not fulfilled, another combination of the parameters of the replacing arc (R_0, x_0, y_0) is sought. For this purpose some software has been developed, whose main pattern is shown in Table 1.

2.5. Defining the transition curve in the basis of the spline shaft teeth

The main "deficiency" of all centroidal tools, and the worm cutters, respectively, is that transition spaces (curves) appear at the basis of the teeth cut. By changing the parameters of the tool and the tooth pair "worm cutter – spline shaft" the location and shape of the transition curve can be changed, put not its presence at the basis of the gear wheel.

The transition curve in the basis of the wheel tooth is formed by the crossing point of the side and top cutting tooth of the tool B_0 (Fig. 4)

The existing methods which look into this issue make a substantial compromise while defining the starting conditions, namely, that the top point of the starting tool surface lays on the real form line of the tool (the replacing circumference arc). Theoretically and practically, such a version is unacceptable, since big errors would

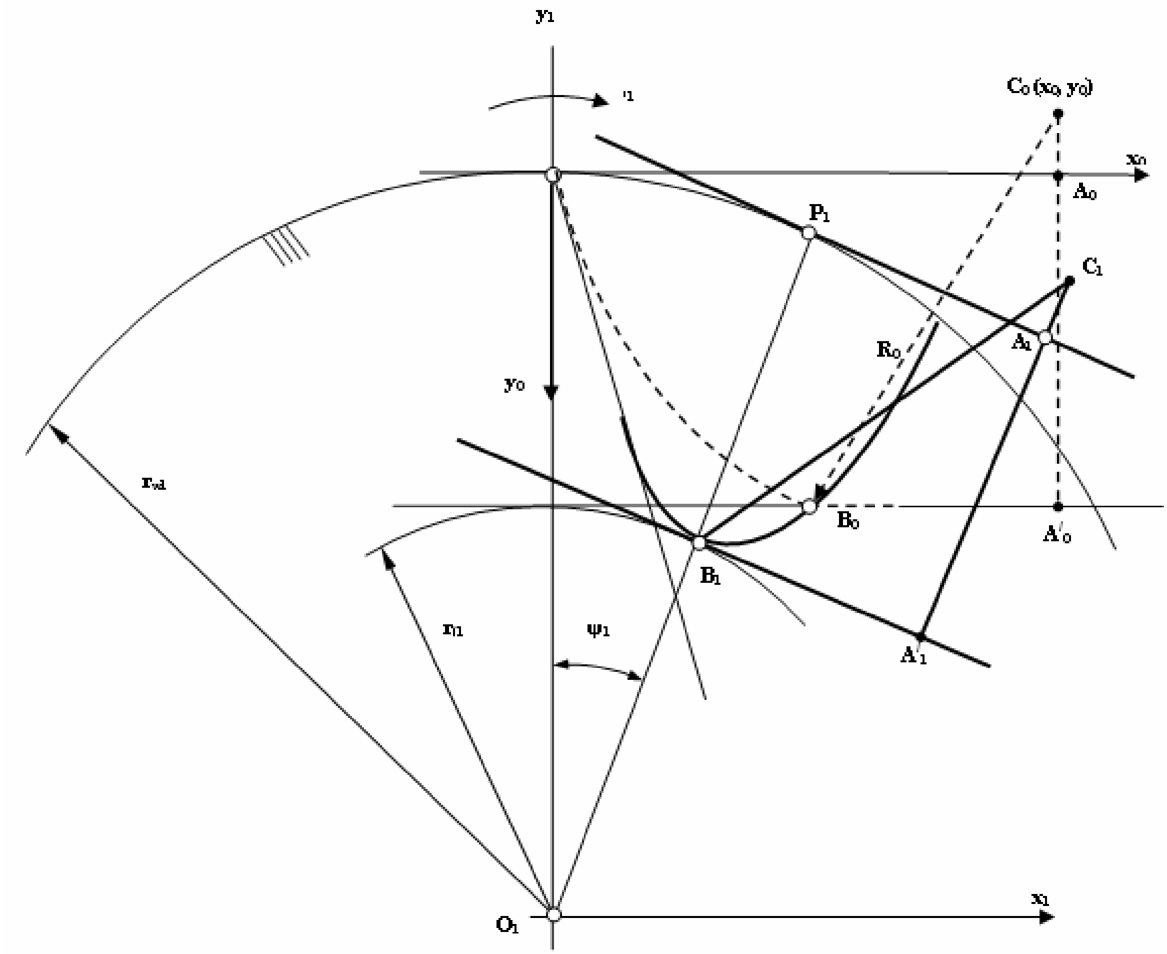

Fig.4. Computational scheme for determining the end of the transition segment in the basis of the spline shaft teeth.

occur when the theoretical profile is replaced. In the new method for profiling the worm cutter cutting teeth suggested this deficiency is avoided. All theoretical equations for determining the real profile of the spline shaft are carried out using the real cutting tooth of the worm cutter gear.

2.5.1. Defining the end of the transition curve

In order for the suggested method to be used (that of the enveloping curves and surfaces) we adopt the following condition, which is in compliance with the main requirements for the centroidal generation: the centroidal circumference of the spline shaft is immobile, and the centroidal straight line of the worm cutter rolls over it without skidding.

When the generation movement is carried out with immobile gear wheel, point B_0**,** which belongs to the two cutting edges, will touch the inner circumference of the wheel, when the generation angle is ψ_1. With the increase of the generation angle ψ_1, the transition curve will start to form, which for the case under discussion will end in B_1 (Fig. 4).

The unknown parameter for defining the end of the transition curve, i.e. when the tooth top trajectory touches the inner circumference, is the generation angle ψ. It is determined by the condition that the two centroidal lines roll one over the other without skidding (Fig. 4).

The segment P_0A_0 is equal to: $\overline{P_0A_0} = \overset{\frown}{P_0P_1} + \overline{P_1A_1}$.

The segment and its parts are determined with the equations ($\Delta B_0A'_0C_0 \cong \Delta B_1A'_1C_1$):

$$\overset{\frown}{P_0A_0} = x_0 ,\ P_0P_1 = r_{w1}.\psi_1 ,$$

$$\overline{P_1A_1} = \overline{B_1A_1'} = \sqrt{R_0^2 - (y_0 + h_{a0})^2} ,$$

$$x_0 = r_{w1} \cdot \psi_1 + \sqrt{R_0^2 - (y_0 + h_{a0})^2} .$$

Using the generation angle equations we will get:

$$\psi_1 = \frac{x_0 - \sqrt{R_0^2 - (y_0 + h_{a0})^2}}{r_{w1}} . \tag{8}$$

When we know the critical generation angle ψ_1 we can determine the coordinates of the end of the transition curve, and use them to perform a quantitative evaluation for the accurate gearing of the spline "shaft" and "bush".

The calculations for the case when the two parts are centered along the inner diameter "*d*", taking into account the height of the jutted tooth are analogous.

2.5.2. Defining the beginning of the transition curve

Solving this task using the method proposed in the present paper, some operating advantages can be achieved during the technological assembly of the shaft and bush, namely: such parameters of the replacing circumference (R_0, x_0, y_0) can be sought, where the transition surface formed will not impede the assembly of the spline bush, which has chamfers on the canals.

Point $B2$ is a specific profile point of the side surface of the gear wheel tooth. First, it is the last point of the form line (b) of the part, resulting from the top point B_0 of the side cutting tooth; secondly, it is the first point of the transition curve, formed from point B_0, resulting from the crossing of the two cutting edges.

With generation angle ψ_2 points B_0 and B_2 coincide in one point, which belongs to the gearing (profiling) line (Fig. 5). Hence, the common normal should be perpendicular to the tangent to the circumference with a radius R_0, i.e., it should pass through the current position of the gearing pole P_1.

The generation angle, determining the position of B_2, will be determined by the equation:

$$\overline{P_0A_0} = \overset{\frown}{P_0P_1} + \overline{P_1A_1} . \tag{9}$$

The values of the above equation are determined, considering the main positions of the centroidal generation:

$$\overset{\frown}{P_0P_1} = r_{w1}.\psi_1 \, , \overline{P_1A_1} = y_0 \cdot tg\beta \tag{10}$$

where $\cos\beta = \dfrac{y_0 + h_{a0}}{R_0}$;

h_{a0} – tooth height.

After substitution and processing of the above equations we get the following for the generation angle $\boldsymbol{\psi_2}$:

$$\psi_2 = \frac{x_0 - y_0 \cdot tg\beta}{r_{w1}} . \tag{11}$$

Using the computational scheme given on Fig. 5, we determine the coordinates of B_2, from which the transition curve starts, and its radius R_2. To determine those parameters it is necessary to calculate the parameters of the gearing pole in its current position P_1 in advance:

$$\begin{aligned} x_{P1} &= r_{w1} \cdot \sin\psi_2 \\ y_{P1} &= r_{w1} \cdot \cos\psi_2 \end{aligned} , \tag{12}$$

The coordinates for B_2 are determined:

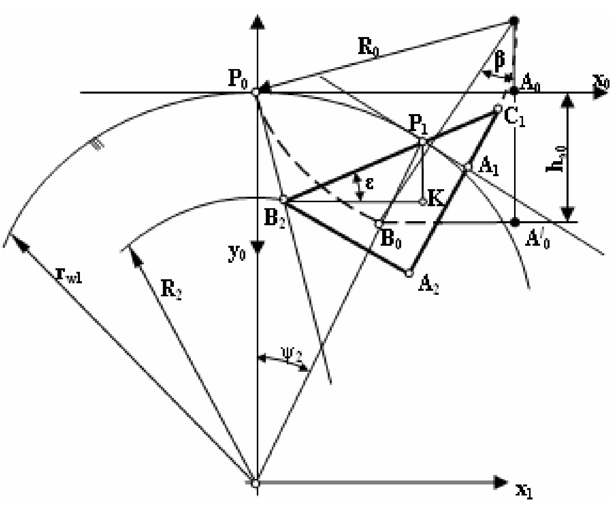

Fig. 5. Computational scheme for determining the beginning of the transition segment in the basis of the spline shaft.

$$\begin{aligned} x_{b2} &= x_{P1} - \overline{B_2P_1} \cdot \cos\varepsilon \\ y_{b2} &= y_{P2} - \overline{B_2P_1} \cdot \sin\varepsilon, \end{aligned} , \tag{13}$$

where $\varepsilon = \dfrac{\pi}{2} - (\beta + \psi_2)$; $\overline{B_2P_1} = R_0 - \dfrac{y_0}{\cos\beta}$.

After substituting equations (12) in (13) the following equations are obtained for the coordinates of the transition point B_2:

$$\begin{aligned} x_{b2} &= r_{w1} \cdot \sin\psi_2 - \left(R_0 - \frac{y_0}{\cos\beta} \right) \cos\varepsilon \\ y_{b2} &= r_{w1} \cdot \cos\psi_2 - \left(R_0 - \frac{y_0}{\cos\beta} \right) \sin\varepsilon \cdot \end{aligned} \tag{14}$$

The radius of the beginning of the transition curve R_2 is:

$$R_2 = \sqrt{x_{b2}^2 + y_{b2}^2} \, . \tag{15}$$

The radius of the beginning of the transition curve R_2, obtained by the real cutting tooth profile of the cutter, can be used for carrying out the respective checks for the different types of centering of the shaft with the bush.

The new method of profiling of the worm cutter cutting teeth allows in some cases, the replacement of the theoretical form line of the worm cutter's starting tool surface to be carried out with just one arc, instead of two arcs from two circumferences, as it is supposed to be. This reduces substantially the technological time needed for making the tool, and respectively, its prime cost.

In the cases when the centering is carried out along the inner diameter d of the shaft, the cutter is jutted, i.e. the height of the teeth is increased. In order for the sides of the shaft from the centroidal to the inner circumference to be machined accurately, the following should be true

$$R_2 \leq R_{вт} + f,$$

where R_{in} is the inner bush opening;
f – the chamfer of the spline grooves.

If it is not true, the height of the cutter teeth is increased until the inequality condition is met.

When the spline joint is centered along the outer diameter D or the sides b of the shaft, the inner diameter d is often assumed equal to the minimum, allowed by the standard – d_1. In such cases it is expedient to define the active height of the shaft sides $\overline{P_0B_2}$ and to use that for further calculations.

2.6. Practical application of the new method for profiling of the cutter's cutting teeth

The new method developed for profiling the worm cutter cutting edges in order to cut spline shafts with a straight form line is set up at a level that allows the design of a software program, whose end product is the parameters necessary for designing the worm cutter's tooth profile.

Following the method described and the methodology developed, the optimum parameter values are determined for the replacing circumference of several spline joints which are given in Table 1.

Table 1

Main parameters of the cutter teeth

Parameters of spline joint	Parameters of replacing circumference			**1/2 IT** of the groove width	Deviation of the profile	Bush chamfer radius	Radius of the transition beginning
	R_0	x_0	y_0			R_{em}	R_2
$D-10\times 92\times 102\dfrac{H8}{e8}\times 14\dfrac{D9}{d9}$	22.254	21.879	– 4.068	0.022	0.017	46.500	45.347
$D-8\times 42\times 48\dfrac{H7}{f7}\times 8\dfrac{F8}{f8}$	11.49	11.22	– 2.49	0.011	0.009	21.400	20.602
$d-6\times 28\dfrac{H7}{f7}\times 32\dfrac{H12}{a11}\times 7\dfrac{D9}{d8}$	6.420	6.225	– 1.569	0.011	0.005	14.300	14.320

2.7. Optimizing the parameters of the real form line of the tool surface

Solving optimization problems in machine-building and in the design of machine-tools in particular, is inevitably connected with the use of computers, which allows the processing and solving of a great amount of data.

From the above-mentioned it has become clear that the form line of the real tool surface is a *shortened involute*, located equidistantly R_0 from the center of the replacing circumference C_0, to which it is fixed.

The main parameter, on which the shape of the shortened involute depends, provided that the radius of the circumference, along which the straight line rolls has already been determined, is the distance of C_0 to the centroidal straight line (a), i.e. *on the ordinate* y_0 of the replacing circumference center (Fig. 2). Consequently, the shape and dimensions of the shortened involute will depend on the ordinates of the chosen points, which determine the parameters of the replacing circumference:

The shape of the shortened involute = $\boldsymbol{f\,(y_1, y_2, y_3)}$

The optimum parameters of the replacing circumference are determined through changing the ordinates of the three points, on which the approximation of the theoretical profile is performed (Table 1). The deviation from a straight line of the gear wheel side tooth surfaces is used as a criterion for this optimization process. The deviation from the straight line of the gear teeth side surfaces is used as a criterion in this optimization process.

3. CONCLUSIONS

- A new analytical method has been developed for defining the worm cutter cutting edge profile for cutting of spline shafts with a straight tooth form line.
- Through this method the minimum errors of the spline shaft tooth profile can be determined, depending on the parameters of the replacing circumference cutting edge of the cutter.
- It is possible in some cases to replace the "two radius" replacement of the cutting edge theoretical profile with just one circumference arc.
- A more profitable method for profiling of the worm cutter cutting edge has been developed, which is competitive to all known methods in real production conditions.
- Combined with other graphic methods to form a graphic-analytical methodology, it can provide a realistic picture of the shaping of particular profile segments.

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