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# MATHEMATICAL MODEL OF THE SPRAY DRY

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***Abstract:*** *The article describes the activities of spray dry, for which was designed a mathematical model of the drying. This model allows to identify and define the process throughout drying. The output of this model is the equation of balance.*

***Key words:*** *spray dry, mathematical model, combustion, humidity, granulate.*

## 1. INTRODUCTION

Ceramic industry gets recently forefront of interest and finds great use in engineering production as well as in many other areas. Production of granulate lies in preparation of emulsion, drying and separation of granulate from drying gas. Especially the drying process presents the main part, what leads to paying attention to keep this process stable, and to permanent checking of its parameters. Main parameter presents the moisture that influences the quality of produces material. On the base of acquired information and measurements done, the mathematical model of the kiln was created. This model describes and defines the balance equation, from which wanted dependence was expressed [1].

## 2. PRODUCTION PROCESS OF CERAMIC GRANULATE

Production of ceramic granulate consists of preparation of emulsion, drying and separation of granulate from drying gas.

### 2.1. Preparation of emulsion

To main part of drying process belongs preparation of emulsion that consists out of following parts:

- loading of raw materials,
- overflowing,
- watering,
- milling,
- stirring.

Necessary amount of raw materials for preparation of emulsion is brought according to the requirements. Emulsion is created as a mixture of solid and water. Main part of the solid is created by ceramic powders of different granularities, rest is created by soot, starch and plastification reagents [3].

### 2.2. Drying

Drying is one of basic actions in chemical and food production technologies. Though it has significant participation not only in these technologies, but also in branch of consumable industries and power engineering. Under drying we understand physical action, where by the influence of the heat, the volume of fluids in materials is lowered, while preserving their chemical composition. Moisture is eliminated with evaporation.

In our case, the drying is the part of production of ceramic products. Granulate, as final product of drying, results from drying the mixture of water and solid (according to the requirements). Then it is used in further production phases.

### 2.3. Separation of granulate

Drying gas flowing from the kiln includes the scraps of the dry granulate. This "mixture" flows through the systems of cyclones, where residual granulate is separated and gas is flowing into the chimney system.

### 2.4. Dispense kiln

For production of ceramic granulate there is mostly used the system of emulsion dispense. Drying with dispense can be defined as drying of slightly dispensed dilution of emulsion, or suspension in the atmosphere of hot gas, overheated steam, internal gas or mixture of combustion products with the air, in the condition that allow regaining the dried product in the form of fine powder.

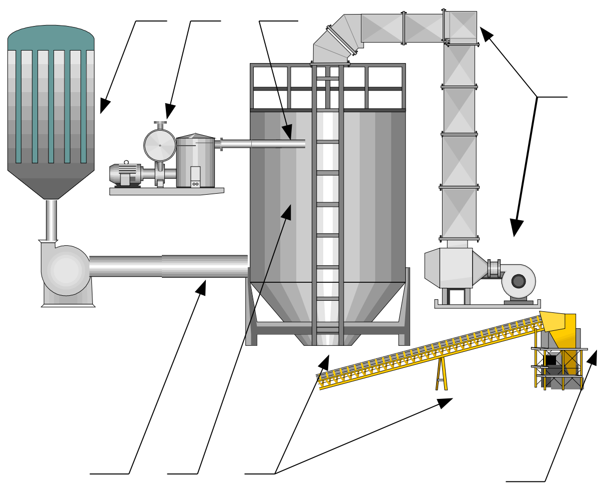

**Fig. 1.** Scheme of sprinkle drying kiln [1]:
1 – gas input, 2 – granulate injection spot, 3 – drying chamber, 5 – hot air input, 6 – pump for emulsion transport, 7 – cyclone, 8 – chimney system.

### 2.5. Products of silicon carbide drying

**Sealing rings**

Materials SiSiC and SSiC thanks to their parameters, high strength, abrasion resistance and chemical resistance are used in production of sliding rings. They have longer life than the materials based on graphite and they resist to chemical media. Disadvantage presents lower last with deficiency of lubrication. Parts up to the diameter 1 000 mm can be produced in one piece [5].

**Nozzles**

They have high abrasion resistance, erosion and chemical resistance, what leads to their usage in the areas where other materials have short life. They are used in chemical industry – spraying nozzles.

**Burners**

High thermal resistance (1 350 to 1 500 °C) allows to use the ceramics as completion of burners.

**Sliding bearings**

In the area of sliding bearing are these materials used thanks to their tribological properties.

**Sorting and grinding devices**

They are used in production of sorting wheels.

**Armatures**

Usage in hard conditions of chemical production, power engineering, paper industry – ball valves, pipeline parts, pipeline fillings.

**Work parts of pumps**

They are used in production of pump parts – pump vanes.

These materials can be used in all areas of engineering production. They prolong the lifespan of devices, they can fully substitute materials working in hard conditions in combination with heavy load (heat, corrosion, abrasion, erosion etc.).

## 3. MATHEMATICAL MODEL OF THE DISPENSE KILN

Function of mathematical model of kiln is to theoretically describe inputs and outputs, to find the relations between them and to derivate searched dependence of granulate moisture on moisture of drying gas on kiln output. At the same time this mathematical model allows theoretical description of dependences of output values and their impact to input parameters.

Kiln is a device, where the material is on purpose dried after leading in and out the drying gas. Drying gas is prepared by `heating of the air sucked from environment. The heating is often obtained from incineration of combustibles. Drying gas then consist of air and combustion products. To main parameters of drying gas belong temperature t, relative moisture φ, absolute moisture *Y* and enthalpy i. Air form the environment with parameters $t_0$, $\varphi_0$, $Y_0$, $i_0$ is sucked into the combustion chamber.

After heating the mixture of drying gas (air + combustion products) enters the kiln with parameters $t_1$, $\varphi_1$, $Y_1$, $i_1$. After entering the kiln the gas comes to contact with dried material and removes the moisture out of it. That changes on the output its parameters to $t_2$, $\varphi_2$, $Y_2$, $i_2$.

Gas used as drying environment is the mixture of dry air and water steam, so called moist air. Physical quantities of moist air are graphically presented in moist air diagram (annexes No. 1). Volume of water steam Y (absolute moisture) in the air can not be variable. Water steam are in overheated state, at the line of fullness (full state) the steam is rich [2, 3].

Enthalpy is combined form enthalpy of dry air and enthalpy of water steam.

## 4. BASIC PROCESS COMPONENT

### 4.1. Drying gas coming into the kiln

Drying gas, in this case the mixture of combustion products and air sucked from environment, is brought into the kiln with required temperature. Heat that is necessary for drying comes from heat source. To main parameters of drying gas belong: [2]

- temperature ($t_1$) [°C],
- mass flow ( $\dot{m}_{sp}$ ) [kg s$^{-1}$],
- absolute moisture ($Y_1$) [kg kg$^{-1}$],
- relative moisture($\varphi_1$) [%],
- enthalpy ($i_1$) [J kg$^{-1}$].

### 4.2. Emulsion

Emulsion is the mixture of water and solid, that is brought into the kiln where it is sprayed over by nozzle. Because of flowing of drying gas, the water is vaporized from the mixture, that is taken out of the kiln together with drying gas. To main parameters of emulsion belong [2]:

**Water**

- water temperature ($t_{v1}$) [°C],
- water mass flow ( $\dot{m}_{v1}$ ) [kg.s$^{-1}$],
- mensural moisture ($u_1$) [kg.kg$^{-1}$],
- specific thermal capacity of water ($c_v$) [J.kg$^{-1}$.K$^{-1}$]/

**Solid**

- solid temperature ($t_{s1}$) [°C],
- solid mass flow ( $\dot{m}_s$ ) [kg s$^{-1}$],
- mensural moisture ($u_1$) [kg kg$^{-1}$],
- specific thermal capacity of solid ($c_{s1}$) [J kg$^{-1}$ K$^{-1}$].

### 4.3. Drying gas coming out of the kiln

Drying gas flows from the kiln together with certain volume of granulate. This granulate falls after going through the cyclone into the collecting container and is added again into prepared emulsion. Combustion products flow to smokestack [2].

Main parameters:

- temperature ($t_2$) [°C],
- mass flow ( $\dot{m}_{sp}$ )[kg s$^{-1}$],
- absolute moisture ($Y_2$) [kg kg$^{-1}$],
- relative moisture ($\varphi_2$) [%],
- enthalpy ($i_2$) [J kg$^{-1}$].

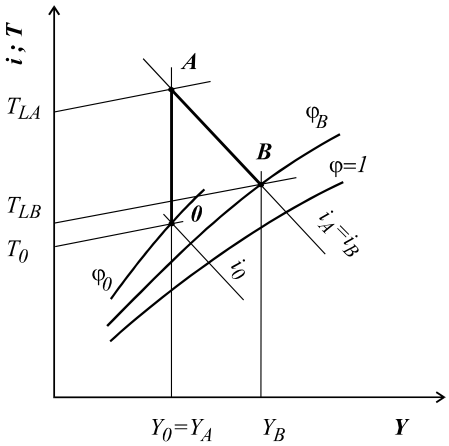


**Fig. 2.** Representation of drying process for theoretical kiln [3].

### 4.4. Granulate

Dried granulated of requested moisture falls from drying tower onto the conveyor, from where is after going through vibration screen stored in the bin. Its moisture is the main indicator of its qualitative parameters and has significant influence to further use. [2]

Main parameters:

- granulate temperature ($t_{s2}$) [°C],
- granulate mass flow ( $\dot{m}_s$ ) [kg s$^{-1}$],
- mensural moisture ($u_2$) [kg kg$^{-1}$],
- specific thermal capacity of granulate ($c_{s2}$) [J kg$^{-1}$ K$^{-1}$]

## 5. CREATION OF BALANCE EQUATION

On the base of these incomes and outcomes there were balance equation created. After comparison of thermal flows on input and output, the balance equation has the form [2]:

$$\dot{m}_{sp} \cdot i_1 + \dot{m}_{vi} \cdot c_{v1} \cdot t_{v1} + \dot{m}_s \cdot c_{s1} \cdot t_{s1} = \dot{m}_{sp} \cdot i_2 + \dot{m}_s \cdot c_{s2} \cdot t_{s2} \quad (1)$$

where:

- $\dot{m}_{sp}$ – mass flow of drying gas on kiln input [kg s$^{-1}$],
- $i_1$ – specific enthalpy of drying gas on kiln input [J kg$^{-1}$],
- $\dot{m}_{v1}$ – mass flow of water on kiln input [kg s$^{-1}$],
- $c_{v1}$ – specific thermal capacity of water [J kg$^{-1}$ K$^{-1}$],
- $t_{v1}$ – temperature of water on kiln input [°C],
- $\dot{m}_s$ – mass flow of solid on kiln input [kg s$^{-1}$],
- $c_{s1}$ – specific thermal capacity of solid on input [J kg$^{-1}$ K$^{-1}$],
- $t_{s1}$ – temperature of solid on kiln input [°C],
- $i_1$ – specific enthalpy of drying gas on kiln output [J kg$^{-1}$],
- $c$ – specific thermal capacity of granulate on output [J kg$^{-1}$ K$^{-1}$],
- $t_{s2}$ – temperature of solid on output [°C].

Product $\dot{m}_{sp} \cdot i_1$ presents thermal flow of drying gas on kiln input per time unit. Emulsion values express the parameters of water and solid. Thermal flow of water is expressed by product of mass flow of water, specific thermal capacity of water and temperature of solid $\dot{m}_s \cdot c_{s1} \cdot t_{s1}$ . Because of theoretical way of drying, the mass flow of solid $\dot{m}_s$ does not change. It only changes its specific thermal capacity and temperature $\dot{m}_s \cdot c_{s2} \cdot t_{s2}$ . For specific enthalpy of drying gas on input there is an equation:

$$i_1 = c_{pv} \cdot t_1 + Y_1 (r_0 + c_{pvp} \cdot t_1), \text{ [J kg}^{-1}\text{]}, \quad (2)$$

where:

- $c_{pv}$ – middle mensural heat of gas [J kg$^{-1}$ K$^{-1}$],
- $t_1$ – temperature of drying gas on kiln input [°C],
- $Y_1$ – moisture of drying gas on input [kg kg$^{-1}$],
- $r_0$ – evaporative heat of water for temperature 0 °C [J kg$^{-1}$],
- $c_{pvp}$ – middle mensural heat of steam [J kg$^{-1}$ K$^{-1}$].

Similarly for specific enthalpy of drying gas on input, there is equation:

$$i_2 = c_{pv} \cdot t_2 + Y_2 (r_0 + c_{pvp} \cdot t_2), \text{ [J.kg}^{-1}\text{]}, \quad (3)$$

where:

- $c_{pv}$ – middle mensural heat of gas [J kg$^{-1}$ K$^{-1}$],
- $t_2$ – temperature of drying gas on kilt output [°C],
- $Y_2$ – moisture of drying gas on output [kg kg$^{-1}$],
- $r_0$ – evaporation heat of water of temperature 0 °C [J kg$^{-1}$],
- $c_{pvp}$ – middle mensural heat of steam [J.kg$^{-1}$.K$^{-1}$].

`After substitution to the equation (1) we get following:

$$\begin{aligned} &\{\dot{m}_{sp} \cdot [c_{pv} \cdot t_1 + Y_1 (r_0 + c_{pvp} \cdot t_1)]\} + \dot{m}_{v1} \cdot c_{v1} \cdot t_{v1} + \dot{m}_S \cdot c_{S1} \cdot t_{s1} = \\ &\{\dot{m}_{sp} \cdot [c_{pv} \cdot t_2 + Y_2 (r_0 + c_{pvp} \cdot t_2)]\} \dot{m}_s \cdot c_{s2} \cdot t_{s2}. \end{aligned} \quad (4)$$

Mass flow of water $\dot{m}_{v1}$ can be expressed based on the equation

$$\dot{m}_{v1} = \dot{m}_s \cdot u_1 \text{ [kg.kg}^{-1}\text{]}, \quad (5)$$

where

- $u_1$ – mensural moisture at the beginning of drying process [kg.kg$^{-1}$],
- $\dot{m}_{v1}$ – mass flow of water on kilt input [kg.s$^{-1}$],
- $\dot{m}_s$ – mass flow of solid on kiln input [kg.s$^{-1}$].

Mass flow of solid $\dot{m}_s$ can be expressed from equation

$$\dot{m}_s = \frac{\dot{m}_{v2}}{u_2} \text{ [kg.kg}^{-1}\text{]}. \quad (6)$$

After substitution from equations (5) and (6) into equation (4) we get following:

$$\begin{aligned} &\{\dot{m}_{sp} \cdot [c_{pv} \cdot t_1 + Y_1 (r_0 + c_{pvp} \cdot t_1)]\} + \dot{m}_s \cdot u_1 \cdot c_{v1} \cdot t_{v1} + \\ &\dot{m}_s \cdot c_{s1} \cdot t_{s1} = \{\dot{m}_{sp} \cdot [c_{pv} \cdot t_2 + Y_2 (r_0 + c_{pvp} \cdot t_2)]\} + \frac{\dot{m}_{v2}}{u_2} \cdot c_{s2} \cdot t_{s2} \end{aligned},$$

from here the mensural moisture of granulate can be expressed $u_2$

$$u_2 = \frac{\dot{m}_{v2} \cdot c_{s2} \cdot t_{s2}}{\left\{\dot{m}_{sp} \cdot \left[c_{pv} \cdot t_1 + Y_1\left(r_0 + c_{pvp} \cdot t_1\right)\right]\right\} - \left\{\dot{m}_{sp} \cdot \left[c_{pv} \cdot t_2 + Y_2\left(r_0 + c_{pvp} \cdot t_2\right)\right]\right\} + \dot{m}_s \cdot u_1 \cdot c_{v1} \cdot t_{v1} + \dot{m}_s \cdot c_{s1} \cdot t_{s1}} \text{ [kg.kg}^{-1}\text{]}. \quad (7)$$

### 5.1. Sensibility function

More parameters from equation (11) can be defined only with certain preciseness. To express the influence of this preciseness to the total value of the function, the sensibility function was used. This express change of function after influence of imprecision of parameter measuring [1].

Maximal change of parameter is expressed by partial derivation of function according to selected parameter:

$$\Delta f(x_1)_{\max} = \frac{\partial f(x_1)}{\partial x_1} \Delta x_i , \quad (8)$$

- $x_i$ – parameter,
- $\Delta x_i$ – imprecision in parameter determination.

For total change of imprecision expressed by influence of imprecision $n$ of parameter there is equation:

$$\Delta f(x_1)_{\max} = \sum_{i=1}^{n} \frac{\partial f(x_1)}{\partial x_1} \Delta x_i . \quad (9)$$

If we determine:

$$z_i = \frac{\partial f(x_1)}{\partial x_1} \quad (10)$$

Then middle change can be expressed as:

$$\Delta f(x_1) = \sqrt{\left(\sum_{i=1}^{n} z_i\right)^2 \cdot \left(\sum_{i=1}^{n} \Delta x_i\right)^2} . \quad (11)$$

Relative change of function is used more often for expressing the sensibility of the function. For calculation of middle relative change of the function there is an equation:

$$\left(\frac{\Delta f(x_1)}{|f(x_1)|}\right)_{\max} = \frac{\sum_{i=1}^{n} \frac{\partial f(x_1)}{\partial x_1} \Delta x_i}{f(x_1)} . \quad (12)$$

Middle relative change of the function can be calculated as:

$$\frac{\Delta f(x_1)}{|f(x_1)|} = \frac{\sqrt{\left(\sum_{i=1}^{n} \frac{\partial f(x_1)}{\partial x_1}\right)^2 \cdot \left(\sum_{i=1}^{n} \Delta x_i\right)^2}}{f(x_1)} . \quad (13)$$

These equations express, how the change of the function will be shown.

## 5. CONCLUSIONS

Expressed function defines the dependence of mensural granulate moisture $u_2$ on moisture of drying gas on kiln output $Y_2$. It is non linear function, its progress depends on all parameters present in equation. Parameter values depend on drying device and emulsion composition. This dependence expresses the theoretical course of drying process, next stage of research has to concern heat losses and plumbing losses.

Measurements were realized in order to point out to the impact of selected factors to quality of granulate. Experiments realized in dissertational thesis [1] were executed in laboratory conditions, in future it will be necessary to realize additional measurements with application of all operational conditions that are affecting this process.

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