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LOCATION AND DISPERSION EFFECTS IN SINGLE-RESPONSE SYSTEM DATA FROM TAGUCHI ORTHOGONAL EXPERIMENTATION

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Abstract: *The identification of the control parameter effects on location (mean) and dispersion (variation) of the observed product quality characteristic (response) is the crucial issue in industrial experiments, for improving manufacturing process robustness. This paper discuss the use of the Taguchi's robust parameter design method for the optimisation a single-response system - automatic enamelling process in the cookware processing technology, for the product quality improvement. The analysis of experimental results was performed using location and dispersion modelling. The results were compared with the results of the commonly-used ANOVA analysis, and it showed the importance and effectiveness of the location and dispersion modelling approach in revealing significant effects of the control parameters and interactions on the response mean and variation.*

Key words: *quality improvement, Taguchi robust design, location, dispersion.*

1. INTRODUCTION

The engineering and statistical analysis of the manufacturing process automatic enamelling indicated certain problems regarding product quality characteristic – base enamel thickness and showed the necessity for process optimisation. The experiment was conducted in order to improve single-response system (automatic enamelling process) robustness, particularly to achieve the target base enamelling mean value and reduce its variation [1, 2].

The design of the experiment was performed by using orthogonal array, in order to reduce number of experimental trials. The previous analysis of experimental data was based on ANOVA method.

In this paper, the new approach for the analysis (location and dispersion modelling) was adopted to clarify the significance of control factors and interactions on the response mean and variation. Its application on the observed problem showed that this method is a useful and economical mean for the identification of all significant location and dispersion effects, but also showed more sensitivity for discovering interaction effects than the commonly-used ANOVA method.

2. DESIGN OF EXPERIMENT

In order to minimise the number of trials required in the experiment, Taguchi's method of experimental design was adopted for the process optimisation. Taguchi developed a family of fractional factorial experiments arrays, to reduce the experimental number but still obtain reasonably rich information, with certain statistical level of confidence.

In Taguchi's methodology, all factors affecting the process quality can be divided into two types: control factors and noise factors. Control factors are those set by the manufacturer and are easily adjustable.

Noise factors, on the other hand, are those undesired variables that are difficult or impossible to control, such as the ambient temperature, humidity and ageing of parts.

The major steps of implementing the Taguchi's method, followed in this experiment, are [3]:

- identify the control factors/interactions,
- identify the levels of each factor,
- select an appropriate orthogonal array (OA),
- assign the factors/interactions to columns of the OA,
- conduct the experiments,
- analyse the data and determine the optimal levels, and
- conduct the confirmation experiment.

Control parameters (factors) and interaction that were identified as critical for base enamelling process were:

- enamel parameters: *specific weight* (SW) and *deposit weight* (DW), and interaction SW·DW;
- process parameters: *pouring speed* (PS) and *automat speed* (AS), and interaction PS·AS.

These four control factors were used as design parameters in the experiment and studied at two levels.

Table 1 illustrates the list of design parameters - control factors and their levels selected for the experiment.

Design of the experiment was performed using Taguchi orthogonal technique, by orthogonal array L_{16}, giving sixteen trials in the experiment (Table 2.) to accommodate four control factors and two interactions, studied at two levels [4].

Table 1

Design parameters / control factors and levels used for the experiment

Design parameter / Control factor	Label	Unit	Level	
			–1	+1
Specific weight	SW	gram cm^{-3}	8	11
Deposit weight	DW	gram cm^{-3}	1.68	1.70
Pouring speed	PS	turns min^{-1}	0	3
Automat speed	AS	parts min^{-1}	5	9

Table 2

Plan of the experiment and measuring results [2]

trial no.	SW [gram cm^{-3}]	DW [gram cm^{-3}]	PS [turns min^{-1}]	AS [parts min^{-1}]	Enamel thickness Mean [μm]	Enamel thickness St. Deviation [μm]
1	8	1,68	0	5	73.52	4.04269
2	8	1.68	0	9	73.52	4.04269
3	11	1.70	3	5	102.20	6.60177
4	11	1.70	3	9	108.64	7.45475
5	8	1.68	3	5	81.56	4.55595
6	8	1.68	3	9	80.44	5.69415
7	11	1.70	0	5	94.60	5.43139
8	11	1.70	0	9	101.56	4.36921
9	8	1.70	3	5	89.68	5.90000
10	8	1.70	3	9	84.68	7.22680
11	11	1.68	0	5	91.16	4.93018
12	11	1.68	0	9	78.80	4.50000
13	8	1.70	0	5	89.56	5.26846
14	8	1.70	0	9	77.08	3.21351
15	11	1.68	3	5	91.72	3.88930
16	11	1.68	3	9	85.48	5.17301

For each trial, base enamel thickness was measured on five parts, and then mean and standard deviation was calculated (Table 2.). Specification limits for base enamel thickness are LSL ÷ USL = 80 ÷ 120 μm, the target base enamel thickness is 95 μm [1].

3. EXPERIMENTAL ANALYSIS

Unlike most other experimental design methods, Taguchi's technique allows us to study the variation of process and ultimately to optimise the process variability, as well as target, using Signal-to-Noise ratio (SNR). SNR evaluates both mean and variation of process together. It presents ratio between response mean (control factors effect) and variation (noise or uncontrollable factors effect). It shows the process robustness against noise factors [5].

Noise factors were not included in the design of the observed experiment; they were considered as unknown. However, considering the sensitivity of the automatic enamelling process to the environmental variation, it is expected that humidity, temperature and dust level in the manufacturing environment can affect the final product quality characteristic – enamel thickness. Since in current circumstances it is impossible to control the mentioned noise factors, it has been decided to employ parameter design methodology to make the process and product less sensitive to the variation.

The desired response is nominal (target) enamel thickness, so SNR value for each trial, was calculated according to the formula:

$$SNR = 10 \cdot \ln\left(\frac{\mu^2}{\sigma^2}\right). \quad (1)$$

where μ is response mean value and σ is response standard deviation.

To analyse the data resulting from conducting a robust parameter design, Taguchi suggested using different signal-to-noise ratios (SNR) for different types of data. Since the measurement of SNR takes both mean and variation into consideration, the analysis of SN ratio seems better than the traditional analysis of variance. However, many statisticians criticized its usefulness and correctness and proposed some alternative methods [6].

In order to avoid possible contradictions, in the following ANOVA analysis both standard deviation and SNR will be analysed, together with the mean response value.

3.1. ANOVA

Analysis of the results was first performed using Analysis of Variance (ANOVA), considering following response model [2]:

$$\begin{aligned} Y_i = a_{0,i} + a_{1,i}SW + a_{2,i}DW + a_{1,2,i}SW \cdot DW + \\ a_{3,i}PS + a_{4,1}AS + a_{3,4,i}PS \cdot AS \end{aligned}. \quad (2)$$

where $a_{0,i}$, $a_{1,i}$, $a_{2,i}$, $a_{1,2,i}$, $a_{3,i}$, $a_{4,i}$, $a_{3,4,i}$ are coefficients, Y_i (i = 1, ..., 3) are: Y_1=MEAN (enamel thickness mean), Y_2=STDEV (enamel thickness standard deviation), Y_3=SNR (enamel thickness Signal-to-Noise ratio).

Tables 3a, 3b and 3c present ANOVA tables for MEAN, STDEV and SNR, respectively. With the level of significance α = 0.05, significant factors for MEAN are SW, DW and PS; for STDEV significant factors are DW and PS and interaction PS·AS; factor PS and interaction PS·AS are significant for SNR.

Analysis of the ANOVA tables and belonging interaction plots of all control factors and their interactions for responses MEAN, STDEV and SNR resulted in following conclusions [1, 2]:

- in order to achieve maximal SNR value, optimal factors setting is: PS = 0; AS = 9;
- optimal factor setting for minimisation of STDEV is: DW = 1.68; PS = 0; AS = 9;
- in order to achieve target value for MEAN (95 um), the optimal factors setting is: DW = 1.70; SW = 11; PS = 0.

Considering the fact that the most important goal is to achieve the target value for MEAN (95 μm), factor DW will be set to value 1.70. Thus, the adopted optimal factors setting from ANOVA analysis is:
DW = 1.70; SW = 11; PS = 0; AS = 9.

Table 3a

ANOVA for MEAN vs. SW, DW, PS and AS [2]

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*P*
SW	1	677.56	677.561	677.561	30.34	0.000
DW	1	526.70	526.702	526.702	23.59	0.001
PS	1	124.32	124.323	124.323	5.57	0.043
AS	1	35.40	35.403	35.403	1.59	0.240
SW·DW	1	48.58	48.581	48.581	2.18	0.174
PS·AS	1	8.94	8.940	8.940	0.40	0.543
Res.Err.	9	200.98	200.979	22.331		
Total	15	1622.49	1622.49			

Table 3b

ANOVA for STDEV vs. SW, DW, PS and AS [2]

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*P*
SW	1	0.3616	0.3616	0.3616	0.62	0.451
DW	1	4.6634	4.6633	4.6633	8.01	0.020
PS	1	7.1524	7.1524	7.1524	12.29	0.007
AS	1	0.0695	0.0694	0.0694	0.12	0.738
SW·DW	1	0.2734	0.2733	0.2733	0.47	0.510
PS·AS	1	4.1504	4.1504	4.1504	7.13	0.026
Res.Err.	9	5.2386	5.2386	0.58207		
Total	15	21.9092	21.9092			

Table 3c

ANOVA for SNR vs. SW, DW, PS and AS [2]

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*P*
SW	1	1.9340	1.934	1.934	1.02	0.340
DW	1	1.0167	1.0167	1.0167	0.53	0.483
PS	1	10.5539	10.553	10.553	5.55	0.043
AS	1	0.5280	0.5280	0.5280	0.28	0.611
SW·DW	1	0.1559	0.1559	0.1559	0.08	0.781
PS·AS	1	10.4915	10.491	10.491	5.51	0.043
Res.Err.	9	17.1259	17.1259	1.9029		
Total	15	41.8060	41.8060			

3.2. Location and dispersion modelling

The location and dispersion modelling approach gives models for measures of location and dispersion separately, in term of control factors and interactions main effects on response. Since noise factors were considered as unknown and were not included in this experiment, here at each control factors setting y_i (the sample mean) and sample variance are used to present the location and dispersion, and calculated as [5]:

$$\gamma_i = \frac{1}{n_i}\sum_j y_{ij},$$

$$\sigma_i^2 = \frac{1}{n_i - 1}\sum_j (y_{ij} - \overline{y_i})^2 . \tag{3}$$

where n_i is number of replicates at for the i^{th} control factors setting (i = 1, ..., 16).

In order to model the relationship between the response location / dispersion and control factors, half-normal plots were generated to show the significance of the factors and their interactions effects on the response – enamel thickness mean and variation.

The half-normal probability plot is a graphical tool that uses these ordered estimated effects to help assess which factors are important and which are unimportant. A half-normal distribution is the distribution of the $|Y|$ with Y having a normal distribution.

Quantitatively, the estimated effect of a given main effect or interaction and its rank relative to other main effects and interactions is given via least squares estimation (that is, forming effect estimates that minimize the sum of the squared differences between raw data and the fitted values from such estimates). Having such estimates in hand, one could then construct a list of the main effects and interactions ordered by the effect magnitude.

Unimportant factors are those that have near-zero effects and important factors are those whose effects are considerably removed from zero. Thus, unimportant effects tend to have a normal distribution centred near zero while important effects tend to have a normal distribution centred at their respective true large (but unknown) effect values [7, 8]. For the observed experiment, Fig. 1 presents half-normal plot for base enamel thickness mean (MEAN), where location factors and interactions could be discovered. Significant effects of factors SW, DW, PS and interaction AS·SW·DW on location were noticed.

Interaction between three control factors were not considered in ANOVA analysis, so it shows that the half-normal plots are convenient manner to reveal significant effects of factors and all possible interactions in the model, on the observed response. According to half-normal plot of location (MEAN) presented at figure 1, regression analysis was performed including all mentioned significant effects.

The obtained regression equitation is:

$$\begin{aligned} MEAN &= 87.8 + 6{,}51 \cdot SW + 5.74 \cdot DW \\ &+ 3.02 \cdot SW \cdot DW \cdot AS + 2.79 \cdot PS. \end{aligned} \tag{4}$$

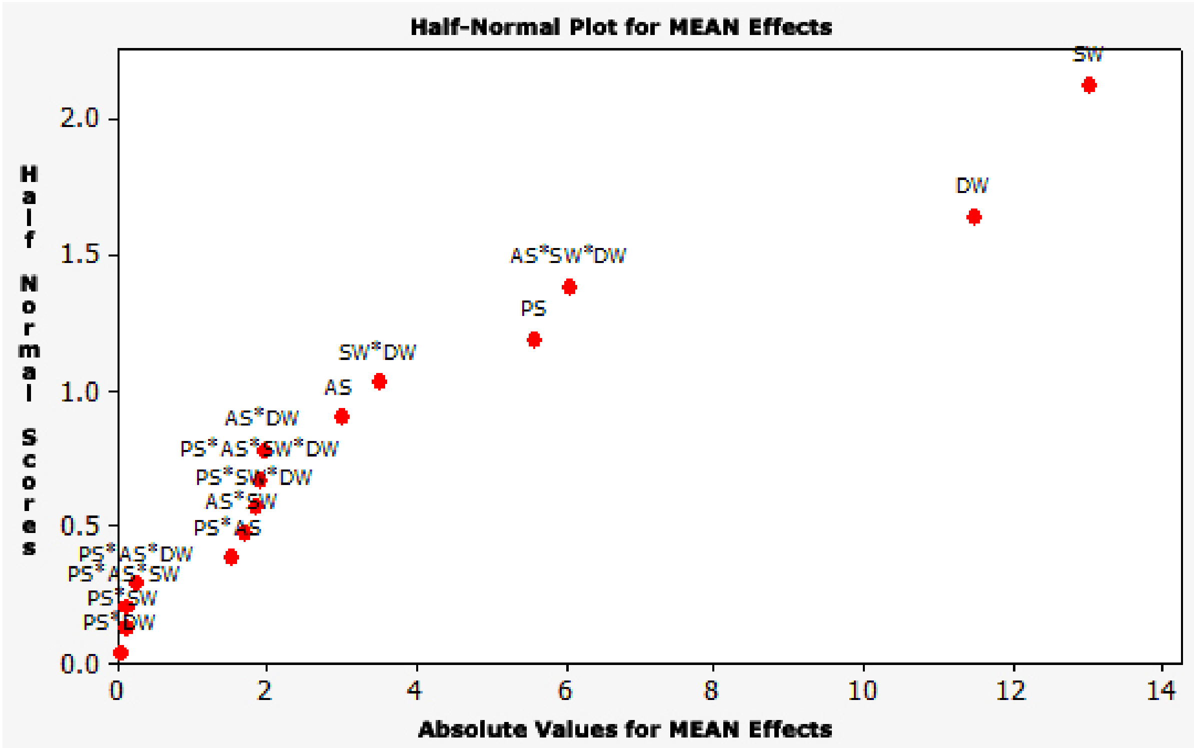

Fig. 1. Half-normal plot of location effects.

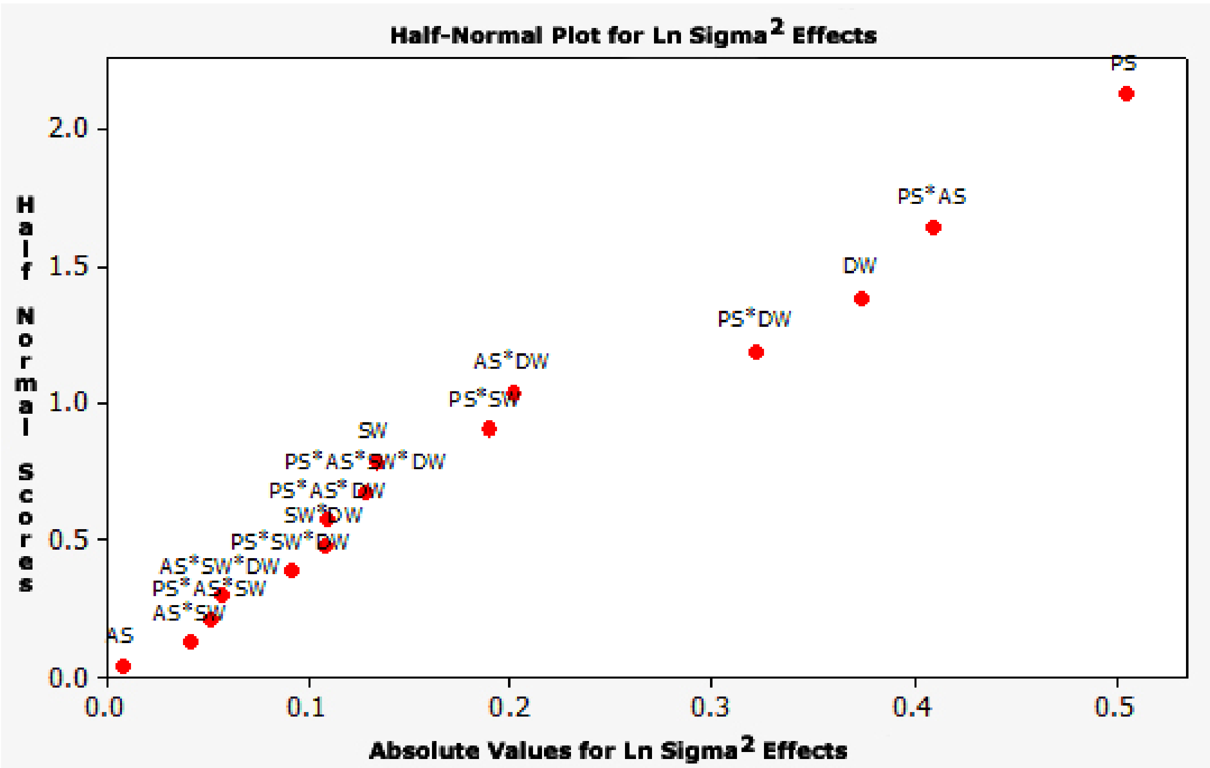

Fig. 2. Half-normal plot of dispersion effects.

Statistics calculated with respect to the equitation (4) are presented in the Table 4.

The predictors used for this multiple regression are control factors and interactions which effects were found as significant for the mean value of the observed response, from half-normal plot. The coefficients are used with the predictors to calculate the fitted value of the response. Each predictor in a regression equation has a coefficient associated with it.

In multiple regressions the estimated coefficient (*Coef*) indicates the change in the mean response per unit increase in the responding predictor when all other predictors are held constant.

If the *p*-value of a coefficient is less than the chosen α-level ($\alpha = 0.05$), there is evidence of a significant relationship between the predictor or factor level and the response.

Table 4

Statistical parameters of regression equitation for location modelling

Predictor	***Coef***	***SE Coef***	***T***	***P***
Constant	87.7625	0.9162	95.79	0.000
SW	6.5075	0.9162	7.10	0.000
DW	5.7375	0.9162	6.26	0.000
SW·DW·AS	3.0225	0.9162	3.30	0.007
PS	2.7875	0.9162	3.04	0.011
S = 3.66474 R – Sq = 90.9% R – Sq(adj) = 87.6%				

Table 5

Statistical parameters of regression equitation for dispersion modelling

Predictor	***Coef***	***SE Coef***	***T***	***P***
Constant	3.22446	0.05828	55.33	0.000
PS	0.25217	0.05828	4.33	0,001
PS·AS	0.20448	0.05828	3.51	0.005
DW	0.18675	0.05828	3.20	0.008
DW·PS	0.16110	0.05828	2.76	0.018
S = 0.233118 R – Sq = 81.6% R – Sq(adj) = 75.0%				

In the Table 4, value *SE Coef* presents standard error of the estimated coefficient; the estimated standard deviation of the coefficient. Value *T* is used for comparison with the *t*-distribution to determine if a predictor is significant. *P*-values are usual mean to decide whether the predictor is significant or not, depending of α-level.

Half-normal plot for response dispersion is shown at Fig, 2, presented over Ln Sigma^2 value. The reason to use the natural logarithm of the variance instead of the variance, is that it maps positive values to real (both positive and negative) values, and by taking its inverse transformation, any predicted value on the ″ln″ scale will be transformed back to a positive value on the original scale. Also, ″ln″ transformation converts a possible multiplicative relationship into an additive relationship, which is much easier to model statistically [5].

At the Fig. 2, effects of PS, PS·AS, DW and PS·DW can be noticed as significant for the response dispersion, thus they were used for regression analysis. According to half-normal plot of dispersion (Ln Sigma^2) presented at Fig. 2, regression analysis was performed including all mentioned significant effects on dispersion. The regression equation for dispersion effects is:

$$\begin{aligned} LnSigma^2 &= 3.22 + 0.25 \cdot PS + 0.2 \cdot PS \cdot AS \\ &+ 0.19 \cdot DW + 1.6 \cdot DW \cdot PS. \end{aligned} \tag{5}$$

Statistical parameters of regression equitation (5) for dispersion effects are presented in the Table 5.

Since the objective of the experiment is to achieve the nominal (target) response means value and according to the two-step procedure for Nominal-the-Best (NTB) problem [5], the first step is to select the levels of dispersion factors to minimise dispersion.

From the relation (5) and Fig. 2, the recommended levels of dispersion factors are:

- PS "−1" level, AS "+1" level, DW can be set to both levels "−1" and "+1" (considering the interaction DW·PS).

Then factor SW can be used to bring the mean on the target depends of the level of DW. From the relation (4) and knowing target is 95 μm, by solving equitation (6):

$$\begin{aligned} 95 &= 87.8 + 6.51 \cdot SW + 5.74 \cdot DW \\ &+ 3.02 \cdot SW \cdot DW \cdot (+1) + 2.79 \cdot (-1). \end{aligned} \tag{6}$$

there are two possible solutions:

- if DW is set to the level "−1", then calculated value for SW is 16.2;
- if DW is set to the level "+1", then calculated value for SW is 10.5.

Since in the current conditions it is not possible to set factor SW to 16.2, the second solution is adopted giving final factors setting obtained form the location and dispersion modelling approach:

- DW = 1.70; SW = 11; PS = 0; AS = 9.

4. DISCUSSION

According to ANOVA analysis and location and dispersion modelling approach, and knowing the desired response values:

- MEAN = 95 μm; STDEV = Min.; SNR = Max;

following control factors setting was found optimal:

- SW = 11; DW = 1.70; PS = 0; AS = 9;

giving predicted responses:

- MEAN = 96.72 μm; STDEV = 4.85 μm; SNR = 26.24 db .

From the ANOVA analysis it could be concluded that factors DW, PS and AS are dispersion factors that effect dispersion of the measured characteristic, explained by standard deviation and signal-to-noise ratio. Factors DW, SW and PS are location factors that influence the mean value of the measured characteristic. The only location factor that is not dispersion factor is SW, so it can be considered as adjustment factor, used to directly adjust the mean value of the measured characteristic [5].

Although both methods for analysis resulted in the same optimal factors setting, it could be noticed that location (mean) and dispersion (variation) modelling technique revealed new interaction found as significant in terms of effects on mean (AS·SW·DW) and variation (PS·DW), comparing to ANOVA analysis. This is especially important when the process is controlled by many control factors and interactions and also when noise factors are included in the design of experiment.

As a process performance measure, Taguchi advocates the use of the quadratic loss function measuring quality loss. Quadratic Loss Function (also known as Quality Loss Function**)** was used to quantify the loss incurred by the user due to deviation from target performance, with initial (previous) and optimised factors setting. According to formula for Taguchi quality loss function:

$$L(Y) = K \cdot (Y - t)^2 , \tag{7}$$

where Y is the system response, K − a cost constant called the quality loss coefficient, and t − the required target.

Loss caused by previous performance is Lp(Y) = K·70.06 units, where previous mean was 103.37 μm [1], and loss that will be encountered after optimisation is Lo(Y) = K·2.99 units.

Thus, by implementation of optimal control factors setting into practice, it can be expected that loss will be reduced, approximately, twenty three times.

Using optimised parameters setting, verification run was performed. It confirmed the results of the experiment. In order to increase the robustness of the automatic enamelling process, this parameters setting was adopted for use on long-term production run. Verification of the adopted parameters values with regards to other product quality characteristics (i.e. visual appearance) are in progress. Since no problems were encountered in the actual experiment as well as in the verification, this experiment can be described as successful.

5. CONCLUSIONS

In today's modern industry, extremely short product life cycles and demands for high-quality and high-added value products require efficient and objective use of experimentation to develop the next generation of processes and products. However, with the limited amount of data provided in unreplicated experiments based on orthogonal arrays, it is quite ambitious to study both location and dispersion effects in a single experiment [8].

In the observed experiment, Taguchi's robust design technique that searches for parameters setting to make products and processes immune to noise sources, was applied. The experimental plan was based on L_{16} orthogonal array.

Analysis of experimental results was performed using two methods: ANOVA, and location and dispersion modelling. The later revealed previously unknown interactions among control factors that have significant effects on response mean and variation. Based on this analysis, optimal control factors setting was determined and verified in a confirmation run.

The outcome of this research is the optimised automatic enamelling process which improves the product quality with respect to base enamelling thickness. The use of the location and dispersion modelling approach clarified a total contribution of control factors and interactions to the variation in the process and product quality. It could be concluded that the use of the Taguchi's orthogonal technique and robust parameter design in combination with location and dispersion modelling was successful method to optimise the observed single-response system.

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