Proceedings of the International Conference on Manufacturing Systems – ICMaS
Vol. 4, 2009, ISSN 1842-3183
University POLITEHNICA of Bucharest, Machine and Manufacturing Systems Department
Bucharest, Romania

MODELING THE RIGIDITY OF THE MAIN SPINDLE IN CASE OF HIGH SPEED MILLING

Daniel POPESCU, Mirela CHERCIU

Abstract: *The paper presents a mathematical model for establishing the dynamic rigidity of high speed spindles. The model is constructed based on constructive characteristics of real spindles and on rigidity simulation as function of speed and external load.*

Key words: *modeling, rigidity, main spindle, high-speed, milling.*

1. INTRODUCTION

The achievements obtained in the construction of high speed milling processing centers (Fig.1) emphasized the need to research methods for obtaining a flexible rigidity of the main spindle (Fig. 2) in case of high speed milling processes.

The goal of our research is establishing analysis models of this parameter, given its importance in increasing finished surface quality and the technological process productivity.

Fig. 1. Milling processing center.

Fig. 2. Main spindle.

The main spindle is one of the most important elements of a machine tool. The processing performance may be improved by increasing the dynamic rigidity of the spindle-bearing system. There is a large amount of research performed up to now regarding the dynamics of machine tool main spindles. Some of it concerns mathematical modeling of the movement equations that generate the transversal and longitudinal vibrations of the spindle [3] using Hamilton's variation principle.

The presented model (which is actually a finite-element model) simulates the real spindle model taking into account the interaction between the main spindle, house and bearings. Since some research indicates that the rotation inertial effect is significant when there is a large mass concentrated on the spindle, this study considers the main spindle as a Rayleigh beam.

In machining industry, the machine tools used for producing other machines are referred to as mother machines.

Machine tools are the basis for a large variety of products from household equipment to medical, automotive, aero spatial and other industries and fields.

The achievements obtained in machining and manufacturing technologies, such as high processing speeds, high precision and multi-tasking systems provide the best quality finished products.

High processing standards are ensured through latest generation machine tools and operating systems as well as new concepts such as "done in one". Computer assisted flexible manufacturing and cyber factory systems provide for organization and supervision of manufacturing processes, component supply and module assembly such that to accomplish the established delivery terms.

In order to handle increased costs with highly qualified work personnel and to reduce processing time, the manufacturing is performed in e-factories comprised of multi-tasking machines equipped with intelligent functions and hi-tech robots.

The settings are updated automatically and the manufacturing costs are significantly smaller in such e-factories as compared with conventional manufacturing plants.

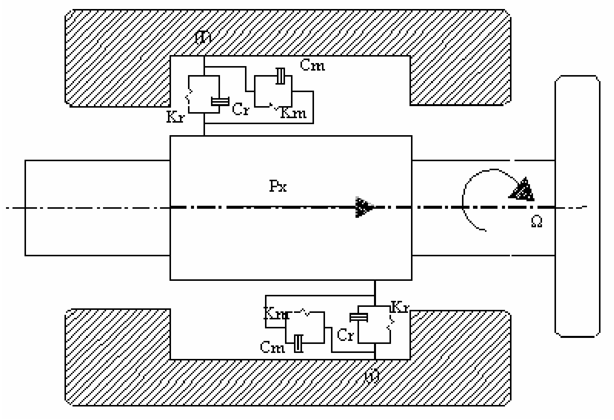

Fig. 3. Main spindle model.

2. MATHEMATICAL MODEL

The mathematical model is a continuous beam with two freedom degrees (Fig. 3).

The movement equation for the spindle and housing can be written as:

$$[M]\{\ddot{u}\}+[K]\{u\}=0\,. \qquad (1)$$

The dynamic characteristics of the two bearings are:

$$\begin{aligned} F &= \left(K_r u_i + C_r \dot{u}_i + K_b \theta_i + C_b \dot{\theta}_i\right) + \\ &+ \left(K_r u_j + C_r \dot{u}_j + K_b \theta_j + C_b \dot{\theta}_j\right), \end{aligned} \qquad (2)$$

where i, j are the nodes attached to the position of bearing elements.

We have:

$$[F]_l = [K]_l \begin{Bmatrix} u_i \\ \theta_i \\ u_j \\ \theta_j \end{Bmatrix} + [C]_l \begin{Bmatrix} \dot{u}_i \\ \dot{\theta}_i \\ \dot{u}_j \\ \dot{\theta}_j \end{Bmatrix} \qquad (3)$$

and

$$[K]_1 = \begin{bmatrix} K_r & 0 & 0 & 0 \\ 0 & K_b & 0 & 0 \\ 0 & 0 & K_r & 0 \\ 0 & 0 & 0 & K_b \end{bmatrix};\ [\mathrm{C}]_1 = \begin{bmatrix} \mathrm{C_r} & 0 & 0 & 0 \\ 0 & \mathrm{C_b} & 0 & 0 \\ 0 & 0 & \mathrm{C_r} & 0 \\ 0 & 0 & 0 & \mathrm{C_b} \end{bmatrix}, \qquad (4)$$

where: $[M]$ – mass matrix, $[K]_1$ – bearing rigidity matrix, $[C]_1$ – damping matrix, $\{u\}$, $\{v\}$ – generalized nodal displacements.

The movement equations for node (i) (Fig. 4) are:

$$\begin{aligned} &\begin{bmatrix} m_a & 0 \\ 0 & J_a \end{bmatrix}_i \begin{Bmatrix} \ddot{u}_a \\ \ddot{\theta}_a \end{Bmatrix}_i + \begin{bmatrix} C_r & 0 \\ 0 & C_m \end{bmatrix} \begin{Bmatrix} \dot{u}_a \\ \dot{\theta}_a \end{Bmatrix}_i - \begin{bmatrix} C_r & 0 \\ 0 & C_m \end{bmatrix} \begin{Bmatrix} \dot{u}_c \\ \dot{\theta}_c \end{Bmatrix}_i + \\ &+ \left(\begin{bmatrix} K_r & 0 \\ 0 & K_m \end{bmatrix} + \begin{bmatrix} K_{1a} & 0 \\ 0 & K_{2a} \end{bmatrix}\right) \begin{Bmatrix} u_a \\ \theta_a \end{Bmatrix}_i - \begin{bmatrix} K_r & 0 \\ 0 & K_m \end{bmatrix} \begin{Bmatrix} u_c \\ \theta_c \end{Bmatrix}_i = \begin{Bmatrix} F_{ci} \\ M_{ci} \end{Bmatrix}, \end{aligned} \qquad (5)$$

$$\begin{aligned} &\begin{bmatrix} m_c & 0 \\ 0 & J_c \end{bmatrix}_i \begin{Bmatrix} \ddot{u}_c \\ \ddot{\theta}_c \end{Bmatrix}_i + \begin{bmatrix} C_r & 0 \\ 0 & C_m \end{bmatrix} \begin{Bmatrix} \dot{u}_c \\ \dot{\theta}_c \end{Bmatrix}_i - \begin{bmatrix} C_r & 0 \\ 0 & C_m \end{bmatrix} \begin{Bmatrix} \dot{u}_a \\ \dot{\theta}_a \end{Bmatrix}_i + \\ &+ \left(\begin{bmatrix} K_r & 0 \\ 0 & K_m \end{bmatrix} + \begin{bmatrix} K_{1c} & 0 \\ 0 & K_{2c} \end{bmatrix}\right) \begin{Bmatrix} u_c \\ \theta_c \end{Bmatrix}_i - \begin{bmatrix} C_r & 0 \\ 0 & C_m \end{bmatrix} \begin{Bmatrix} u_a \\ \theta_a \end{Bmatrix}_i = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}. \end{aligned} \qquad (6)$$

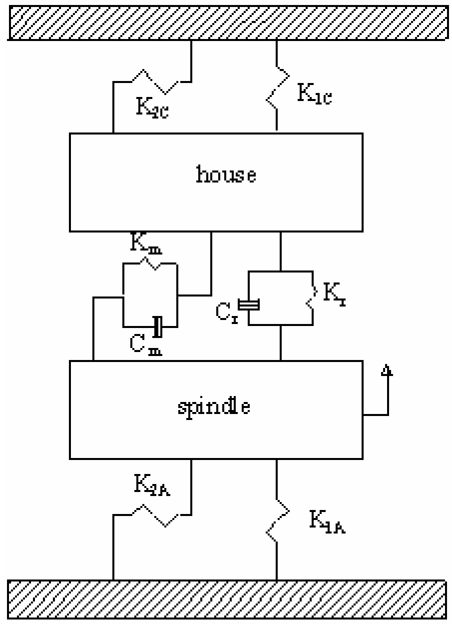

Fig. 4. Model for housing-spindle connections in node (i).

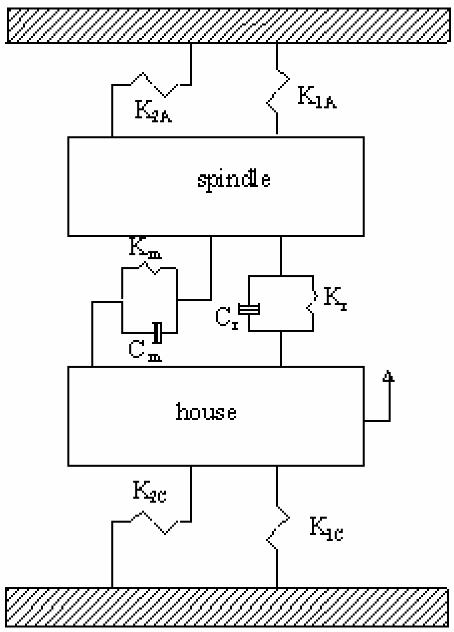

Fig. 5. Model for housing-spindle connections in node (j).

For the node (j) (Fig. 5) the movement equations become:

$$\begin{aligned} &\begin{bmatrix} m_a & 0 \\ 0 & J_a \end{bmatrix}_j \begin{Bmatrix} \ddot{u}_a \\ \ddot{\theta}_a \end{Bmatrix}_j + \begin{bmatrix} C_r & 0 \\ 0 & C_m \end{bmatrix} \begin{Bmatrix} \dot{u}_a \\ \dot{\theta}_a \end{Bmatrix}_j - \begin{bmatrix} C_r & 0 \\ 0 & C_m \end{bmatrix} \begin{Bmatrix} \dot{u}_c \\ \dot{\theta}_c \end{Bmatrix}_j + \\ &+ \left(\begin{bmatrix} K_r & 0 \\ 0 & K_m \end{bmatrix} + \begin{bmatrix} K_{1a} & 0 \\ 0 & K_{2a} \end{bmatrix}\right) \begin{Bmatrix} u_a \\ \theta_a \end{Bmatrix}_j - \begin{bmatrix} K_r & 0 \\ 0 & K_m \end{bmatrix} \begin{Bmatrix} u_c \\ \theta_c \end{Bmatrix}_j = \begin{Bmatrix} F_{cj} \\ M_{cj} \end{Bmatrix}, \end{aligned} \qquad (7)$$

$$\begin{aligned} &\begin{bmatrix} m_c & 0 \\ 0 & J_c \end{bmatrix}_j \begin{Bmatrix} \ddot{u}_c \\ \ddot{\theta}_c \end{Bmatrix}_j + \begin{bmatrix} C_r & 0 \\ 0 & C_m \end{bmatrix} \begin{Bmatrix} \dot{u}_c \\ \dot{\theta}_c \end{Bmatrix}_j - \begin{bmatrix} C_r & 0 \\ 0 & C_m \end{bmatrix} \begin{Bmatrix} \dot{u}_a \\ \dot{\theta}_a \end{Bmatrix}_j + \\ &+ \left(\begin{bmatrix} K_r & 0 \\ 0 & K_m \end{bmatrix} + \begin{bmatrix} K_{1c} & 0 \\ 0 & K_{2c} \end{bmatrix}\right) \begin{Bmatrix} u_c \\ \theta_c \end{Bmatrix}_j - \begin{bmatrix} C_r & 0 \\ 0 & C_m \end{bmatrix} \begin{Bmatrix} u_a \\ \theta_a \end{Bmatrix}_j = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}. \end{aligned} \qquad (8)$$

The movement equations for the spindle are:

$$\begin{bmatrix} m_{ai} & 0 & 0 & 0 \\ 0 & J_{ai} & 0 & 0 \\ 0 & 0 & m_{aj} & 0 \\ 0 & 0 & 0 & J_{aj} \end{bmatrix} \begin{Bmatrix} \begin{Bmatrix} \ddot{u}_a \\ \ddot{\theta}_a \end{Bmatrix}_i \\ \begin{Bmatrix} \ddot{u}_a \\ \ddot{\theta}_a \end{Bmatrix}_j \end{Bmatrix} +$$

$$+ \begin{bmatrix} C_r & 0 & 0 & 0 \\ 0 & C_m & 0 & 0 \\ 0 & 0 & C_r & 0 \\ 0 & 0 & 0 & C_m \end{bmatrix} \begin{Bmatrix} \begin{Bmatrix} \dot{u}_a \\ \dot{\theta}_a \end{Bmatrix}_i \\ \begin{Bmatrix} \dot{u}_a \\ \dot{\theta}_a \end{Bmatrix}_j \end{Bmatrix} - \begin{bmatrix} C_r & 0 & 0 & 0 \\ 0 & C_m & 0 & 0 \\ 0 & 0 & C_r & 0 \\ 0 & 0 & 0 & C_m \end{bmatrix} \begin{Bmatrix} \begin{Bmatrix} \dot{u}_c \\ \dot{\theta}_c \end{Bmatrix}_i \\ \begin{Bmatrix} \dot{u}_c \\ \dot{\theta}_c \end{Bmatrix}_j \end{Bmatrix} +$$

$$+ \left(\begin{bmatrix} K_r & 0 & 0 & 0 \\ 0 & K_m & 0 & 0 \\ 0 & 0 & K_r & 0 \\ 0 & 0 & 0 & K_m \end{bmatrix} + \begin{bmatrix} K_{1ai} & 0 & 0 & 0 \\ 0 & K_{2ai} & 0 & 0 \\ 0 & 0 & K_{1aj} & 0 \\ 0 & 0 & 0 & K_{2aj} \end{bmatrix} \right) \begin{Bmatrix} \begin{Bmatrix} u_a \\ \theta_a \end{Bmatrix}_i \\ \begin{Bmatrix} u_a \\ \theta_a \end{Bmatrix}_j \end{Bmatrix} -$$

$$- \begin{bmatrix} K_r & 0 & 0 & 0 \\ 0 & K_m & 0 & 0 \\ 0 & 0 & K_r & 0 \\ 0 & 0 & 0 & K_m \end{bmatrix} \begin{Bmatrix} \begin{Bmatrix} u_c \\ \theta_c \end{Bmatrix}_i \\ \begin{Bmatrix} u_c \\ \theta_c \end{Bmatrix}_j \end{Bmatrix} = \begin{Bmatrix} F_{ei} \\ M_{ei} \\ F_{ej} \\ M_{ej} \end{Bmatrix} \tag{9}$$

For the housing they become:

$$\begin{bmatrix} m_{ci} & 0 & 0 & 0 \\ 0 & J_{ci} & 0 & 0 \\ 0 & 0 & m_{cj} & 0 \\ 0 & 0 & 0 & J_{cj} \end{bmatrix} \begin{Bmatrix} \begin{Bmatrix} \ddot{u}_c \\ \ddot{\theta}_c \end{Bmatrix}_i \\ \begin{Bmatrix} \ddot{u}_c \\ \ddot{\theta}_c \end{Bmatrix}_j \end{Bmatrix} +$$

$$+ \begin{bmatrix} C_r & 0 & 0 & 0 \\ 0 & C_m & 0 & 0 \\ 0 & 0 & C_r & 0 \\ 0 & 0 & 0 & C_m \end{bmatrix} \begin{Bmatrix} \begin{Bmatrix} \dot{u}_c \\ \dot{\theta}_c \end{Bmatrix}_i \\ \begin{Bmatrix} \dot{u}_c \\ \dot{\theta}_c \end{Bmatrix}_j \end{Bmatrix} - \begin{bmatrix} C_r & 0 & 0 & 0 \\ 0 & C_m & 0 & 0 \\ 0 & 0 & C_r & 0 \\ 0 & 0 & 0 & C_m \end{bmatrix} \begin{Bmatrix} \begin{Bmatrix} \dot{u}_a \\ \dot{\theta}_a \end{Bmatrix}_i \\ \begin{Bmatrix} \dot{u}_a \\ \dot{\theta}_a \end{Bmatrix}_j \end{Bmatrix} +$$

$$+ \left(\begin{bmatrix} K_r & 0 & 0 & 0 \\ 0 & K_m & 0 & 0 \\ 0 & 0 & K_r & 0 \\ 0 & 0 & 0 & K_m \end{bmatrix} + \begin{bmatrix} K_{1ci} & 0 & 0 & 0 \\ 0 & K_{2ci} & 0 & 0 \\ 0 & 0 & K_{1cj} & 0 \\ 0 & 0 & 0 & K_{2cj} \end{bmatrix} \right) \begin{Bmatrix} \begin{Bmatrix} u_c \\ \theta_c \end{Bmatrix}_i \\ \begin{Bmatrix} u_c \\ \theta_c \end{Bmatrix}_j \end{Bmatrix} -$$

$$- \begin{bmatrix} K_r & 0 & 0 & 0 \\ 0 & K_m & 0 & 0 \\ 0 & 0 & K_r & 0 \\ 0 & 0 & 0 & K_m \end{bmatrix} \begin{Bmatrix} \begin{Bmatrix} u_a \\ \theta_a \end{Bmatrix}_i \\ \begin{Bmatrix} u_a \\ \theta_a \end{Bmatrix}_j \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{10}$$

Hence, the movement of the entire system spindle-housing-bearings is given by:

$$\left\{ \begin{array}{c|c} [M]_a & [0] \\ \hline [0] & [M]_c \end{array} \right\} \begin{Bmatrix} \{\ddot{u}\}_a \\ \{\ddot{u}\}_c \end{Bmatrix} + \left\{ \begin{array}{c|c} [C]_l & -[C]_l \\ \hline -[C]_l & [C]_l \end{array} \right\} \begin{Bmatrix} \{\dot{u}\}_a \\ \{\dot{u}\}_c \end{Bmatrix} + \left\{ \begin{array}{c|c} [K]_a + [K]_l & -[K]_l \\ \hline -[K]_l & [K]_c + [K]_l \end{array} \right\} \begin{Bmatrix} \{u\}_a \\ \{u\}_c \end{Bmatrix} = \begin{Bmatrix} \{F_x\} \\ \{0\} \end{Bmatrix}, \tag{11}$$

where:

- m_{ai}, m_{ci}, m_{aj}, m_{cj} – mass of unit length in nodes (*i*), (*j*);
- J_{ai}, J_{ci}, J_{aj}, J_{cj} – inertial moments of respective surfaces (*i*), (*j*);
- $\{u\}_a$, $\{u\}_c$ – nodal displacement matrices for spindle and housing;
- $\{F_x\}$ – spindle axial load force.

Relation (11) can be written in reduced form as:

$$[\overline{M}]\{\ddot{v}\} + [\overline{C}]\{\dot{v}\} + [\overline{K}]\{v\} = \{F\}, \tag{12}$$

where:

$$[\overline{M}] = \left\{ \begin{array}{c|c} [M]_a & [0] \\ \hline [0] & [M]_c \end{array} \right\},$$

$$[\overline{C}] = \left\{ \begin{array}{c|c} [C]_l & -[C]_l \\ \hline -[C]_l & [C]_l \end{array} \right\},$$

$$\{v\} = \begin{Bmatrix} \{u\}_a \\ \{u\}_c \end{Bmatrix},$$

$$[\overline{K}] = \left\{ \begin{array}{c|c} [K]_a + [K]_l & -[K]_l \\ \hline -[K]_l & [K]_c + [K]_l \end{array} \right\},$$

$$\{F\} = \begin{Bmatrix} \{F_x\} \\ \{0\} \end{Bmatrix}. \tag{13}$$

The second order differential equations (8) can be transformed into first order differential equations (16) according to:

$$[T_1]\{\dot{v}(t)\} + [T_2]\{v(t)\} = \{F(t)\}, \tag{14}$$

where: $$[T_1] = \left\{ \begin{array}{c|c} [\overline{C}] & [\overline{M}] \\ \hline [\overline{M}] & [0] \end{array} \right\};$$

$$[T_2] = \left\{ \begin{array}{c|c} [\overline{K}] & [0] \\ \hline [0] & -[\overline{M}] \end{array} \right\}; \{F(t)\} = \begin{Bmatrix} \{F\} \\ \{0\} \end{Bmatrix}. \tag{15}$$

The dynamic rigidity can be obtained projecting (14) into frequency domain:

$$[K_d]^{-1}\{v(t)\} = \{F(t)\}. \tag{16}$$

Based on data obtained in case of real spindle-housing-bearings models [4] we have the parameters presented in Tables 1 and 2.

Table 1

Constructive characteristics of the real model

d (mm)	*A*	D_R (mm)	D_{ci} (mm)	D_W (mm)	γ
25	100	8.62	31	6.75	0.159
40	100	10.16	46	7.94	0.142

d (mm)	d_m (mm)	α°	*Y*	Q_c (N)	*n*
25	41	15	1	2650.97	14
40	53.94	15	1	3298.68	16

Table 2

Rigidity and damping characteristics of the main spindle bearings

d (mm)	K_r (N/m)	K_m (N/m)	C_r (N/m)	C_m (NMs/rad)
25	1.2×10^8	6×10^6	240	150
40	1.25×10^8	6.5×10^6	200	100

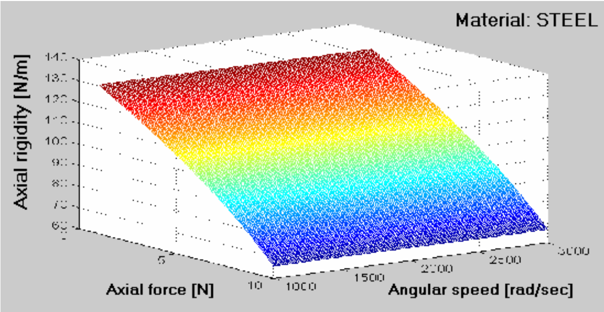

Fig. 6. Variation of dynamic rigidity as function of speed and axial load (*d* = 20 mm).

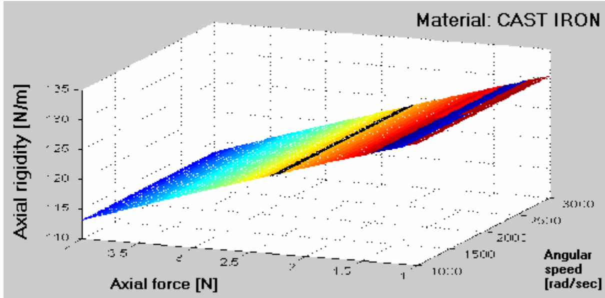

Fig. 7. Variation of dynamic rigidity as function of speed and axial load (*d* = 20 mm)

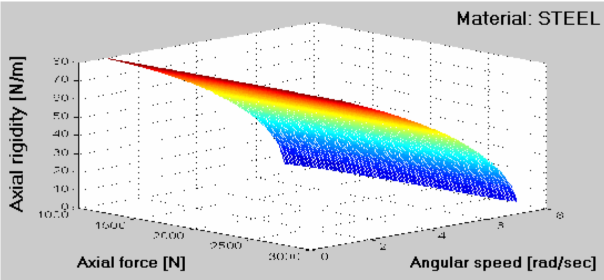

Fig. 8. Variation of dynamic rigidity as function of speed and axial load (*d* = 40mm).

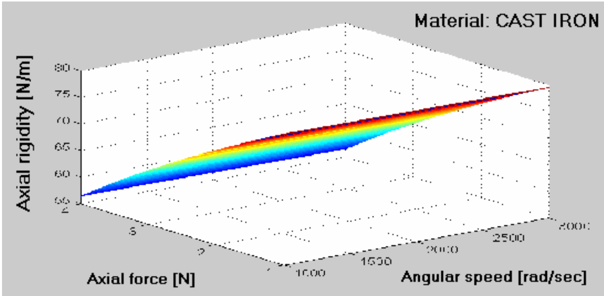

Fig. 9. Variation of dynamic rigidity as function of speed and axial load (*d* = 40 mm).

Simulations (Figs. 6, 7, 8 and 9) were obtained by varying the definition domain of two parameters $n_a = 15100 \div 19000$ rot/min, $F = 1 \div 10$ N, corresponding to direct measuring results.

3. CONCLUSIONS

Knowing the dynamic rigidity of the system allows us to establish the importance of static rigidity in establishing its value and also the parameters on which it depends [1, 2, 5].

Simulations were realized for working materials such as steel and cast iron.

In both cases it is observed a diminution of the rigidity with the increase of the final load, with a steeper slope for smaller diameters.

Also the dynamic rigidity remains constant or varies little with the variation of the main spindle rotation speed.

REFERENCES

[1] Chen C.H., Wang K.W., Shin Y.C. (1994). *An Integrated Approach Toward The Dynamic Analysis of High-Speed Spindles – System Model*, Journal of Vibration and Acoustics, pp. 506–512.

[2] Chen C.H., Wang K.W. (1994). *An Integrated Approach Toward The Dynamic Analysis of High-Speed Spindles – Dynamics Under Moving End Load*, Journal of Vibration and Acoustics, Vol. 116, pp. 514–522.

[3] Popescu D., Ispas C. (1999). *Dynamics of Grinding Processes, Modeling and Simulation Elements*, pp. 56–65, Craiova, Romania.

[4] Popescu D. (1999). *Theoretical and Experimental Contributions Regarding Improvement of The Processing Precision at Internal Grinding Machines*, PhD Thesis, University of Bucharest, Romania.

[5] Popescu D., Dumitru C. (2005). *A Model For Dynamic Of Vibrations in Milling Operations. The Dynamic Response Of The System*, The International Conference CI 2005, October 16–18 Timisoara, Romania, pp. 89–90.

Authors:

PhD, Daniel POPESCU, Assoc. Professor, University of Craiova, Faculty of Mechanical Engineering,
E-mail: daniel.popescu1119@yahoo.com
PhD, Mirela CHERCIU, Assoc. Professor, University of Craiova, Faculty of Mechanical Engineering,
E-mail: mirela_cherciu2005@yahoo.com