Proceedings in Manufacturing Systems

Proceedings in Manufacturing Systems, Vol. 5 (2010), No. 1 ISSN 2067-9238

ELASTIC DEFLECTION OF THE CYLINDRICAL WORKPIECES DURING THE MACHINING

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Abstract: *During the machining process, under the action of the cutting forces, the workpiece is usually elastically deflected and, as a consequence, the current depth of cut is different in comparison with the desired one, therefore the machined surface could present dimensional and shape errors. The deflection size is influenced by the rigidity of the machine tool, cutting tool, workpiece, working parameters (depth of cut, work feed and speed), geometry of the tool active part, workpiece material and dimensions, etc. For certain work conditions, the elastic deflection could be estimated by taking into consideration theoretical relations, but appropriate results could be obtained by experimental research. The paper presents some authors' considerations concerning the modeling of the elastic deflection of the workkpiece held in the universal scroll chuck and tailstock live center. Such models could be used to establish the tool point path on CNC lathes, so that the possible machining errors to be avoided.*

Key words: *elastic deflection, rigidity, turning, universal scroll chuck, tailstock live center.*

1. INTRODUCTION

The machining accuracy characterizes the correspondence among the dimensions, shape and mutual positions specific to the obtained part and the same elements inscribed in the technical drawing of the part. There are different factors able to affect the machining accuracy. They could be classified by taking into consideration distinct aspects.

Thus, if the phenomena capable to generate the machining errors are considered, there are errors generated by the tool wear, by the thermal deformation of certain components of the system machine – tool – workpiece, by the elastic deformation of certain components of the same system, by the positioning of the tool corner, etc. [2, 4, and 5].

The cutting is a machining process based on the material removal of the tooling allowance by chips detaching process, as a result of the workpiece material pressing by the tool; as this definition shows, the cutting always supposes the presence of the forces specific to the process of chips generation and detaching. These forces act on the components of the system machine-tool-workpiece, determining their deformation. Because the system components are rigid, their deformations are low, but for several machining processes, the deformations may affect the machining accuracy.

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As known, the machining process implies the setup of the tool corner position so that the established depth of cut is materialized; due to the deformation of the machining system components, the effective depth of cut is usually lower than the desired or designed one. For this reason, it is important to know the influence exerted by the elastic deformation of the machining system on the variation of the depth of cut and, finally, on the machining accuracy.

The capacity of the machining system to oppose to the deformation generated by the machining process may be important also when the workpiece has initial shape errors.

Evidently, in order to obtain the final part, the tooling allowance has to be removed; if this allowance is not uniform, due to the workpiece shape errors, the use of only a rectilinear motion of the tool corner (as in the case of the classical machine tools) generates a non controlled variation of the depth of cut and implicitly of the machined surface dimensions.

To diminish these errors to acceptable values (this means that the machining errors are lower than the accepted deviations for the dimensions, shape and mutual positions of the machined surfaces), usually many passes with reduced depths of cut are used and this fact diminishes the material removal rate and the cutting process efficiency.

On the CNC machine tools, if the shape errors have known values and by taking into consideration the elastic deformations, the path of the tool corner could be thus designed, that finally the part accuracy corresponds with the desired one.

Once again, also in the case of the CNC machine tools, information concerning the rigidity of the machining system components (which could be able to affect the machining accuracy) has to be known.

A radial basis function neural network was used by Li in order to model the errors associated with the three cutting components [1]; the author appreciated that is very complicated to estimate the cutting-force-induced error through the calculation of the cutting force and the machine-workpiece-tool system deflection.

Topal and Çoğun proposed a compensation method based on computer aided generation of the tool path, taking into consideration an empirical model for estimation of diametral error based on cutting forces [6]; the model is valid for cantilever bar turning.

Oprean and Predincea [4] studied some ways to evaluate and characterize the linear and angular static rigidity of the machine tools; they considered that the static rigidity of a machine tool offers significant information concerning the behavior of the manufacturing system during the machining process.

The aim of this paper is to highlight the influence exerted by the elastic deflection of the workpiece held in the universal scroll chuck and tailstock live center on the turning accuracy.

2. THEORETICAL CONSIDERATIONS

One can take into consideration the case of the turning a workpiece held in the universal scroll chuck and the tailstock live center (Fig. 1), from the right end to the left one.

Even this modality of workpiece holding does not ensure a high precision from the point of view of coaxiality of the cylindrical zones, it is relatively frequently preferred, especially for rough machining.

During the machining process developing, cutting forces appear and they act on the workpiece and the cutting tool.

Three components of the cutting force are usually taken into consideration. In order to obtain a simplified mathematical model, only the component F_x, directed along the axis Ox is here considered. One has to mention that the elastic deformation generated by this component directly affects the diameter of the turned surface.

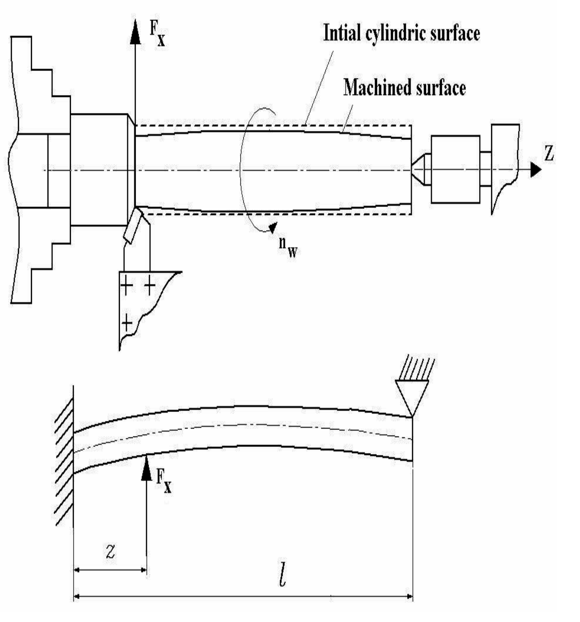

Fig. 1. Longitudinal turning of the workpiece fixed in the universal chuck and in live centre.

If the cutting speed is high enough, it is possible to appreciate that this cutting parameter (cutting speed) does not affect the size of the component F_x; it is known that for low cutting speed, the cutting force has a maximum size and after exceeding the size corresponding to this maximum, the cutting force diminishes slowly, due to the better flowing of the workpiece material in contact with the surfaces of the cutting tool.

According to the above mentioned aspects, the cutting force F_x can be written as a function of the actual depth of cut a_{pa}, the cutting feed f and the hardness HB of the workpiece material:

$$F_z = C_{Fx} \cdot a_{pa}^{x_{Fx}} \cdot f^{y_{Fx}} \cdot HB^{n_{Fx}}, \quad (1)$$

where C_{Fx} is a coefficient whose size depends on various other cutting conditions, x_{Fx}, y_{Fx}, n_{Fx} are exponents experimentally determined. Under the action of the cutting force F_x, the components of the machining system are elastically deformed.

Various mathematical relations could be used to characterize the elastic deformation of the machining system components; if only the workpiece elastic deformation is considered and one supposes that this deformation corresponds to a bar rigid fixed at one of its ends and simply supported to the other end, the elastic deformation δ_F developed at the distance a from the left end of the bar is given [3] by the expression:

$$\delta_F = \frac{F_x \cdot a^3 \cdot b^2 \cdot (3 \cdot a + 4 \cdot b)}{12 \cdot E \cdot I_z \cdot l^3}, \quad (2)$$

where b is the distance from the plane where the force F_x is applied to the right end of the bar, l – the length of the bar from the universal chuck to its right end (where the bar is held in the tailstock live center), E - the workpiece material elasticity modulus and I_z – the second moment area of the cross section corresponding to the bar workpiece.

For the circular cross section of the workpiece bar, the second moment area corresponds to the relation $I_z = \pi d^4 / 64$, d being the bar diameter. In fact, there are two distinct diameters, one - corresponding to the cylindrical surface of the workpiece and the second corresponding to the turned surface.

In order to obtain simplied mathematical model, one may take into consideration [5] an average diameter d, determined as arithmetic average of the two diameters above mentioned.

The distance a from the origin of the coordinate system to the plane of the actual cutting process could be symbolized by z (Fig. 1); in such a case, $b = l - z$ and the expression (2) becomes:

$$\delta_F = \frac{F_x \cdot z^3 \cdot (l - z)^2 (4 \cdot l - z)}{12 \cdot E \cdot I_z \cdot l^3}. \quad (3)$$

If in the relation (2) the expression (1) for the cutting force component F_x is considered, the elastic deformation could be written as:

$$\delta_F = \frac{C_{Fx} \cdot a_{pa}^{x_{Fx}} \cdot f^{y_{Fx}} \cdot HB^{n_{Fx}} \cdot z^3 \cdot (l-z)^2 \cdot (4 \cdot l - z)}{12 \cdot E \cdot I_z \cdot l^3} . \quad (4)$$

The elastic deformation δ_F of the workpiece generates a shape error ε_s, due to the change of the desired depth of cut a_{pd} into the actual depth of cut a_{pa}:

$$\varepsilon_s = a_{pd} - a_{pa} = \delta_F . \quad (5)$$

This means that:

$$a_{pd} - a_{pa} = \frac{C_{Fx} \cdot a_{pa}^{x_{Fx}} \cdot f^{y_{Fx}} \cdot HB^{n_{Fx}} \cdot z^3 (l-z)^2 (4 \cdot l - z)}{12 \cdot E \cdot I_z \cdot l^3} . \quad (6)$$

One may consider $x_{Fx} \approx 1$ and this allows to write:

$$a_{pd} = a_{pa} \cdot \left[1 + \frac{C_{Fx} \cdot f^{y_{Fx}} \cdot HB^{n_{Fx}} \cdot z^3 \cdot (l-z)^2 (4 \cdot l - z)}{12 \cdot E \cdot I_z \cdot l^3}\right] . \quad (7)$$

By taking into consideration the last relation, the actual depth of cut a_{pa} can be written as:

$$a_{pa} = \frac{a_{pd}}{1 + \frac{C_{Fx} \cdot a_p^{x_{Fx}} \cdot f^{y_{Fx}} \cdot HB^{n_{Fx}} \cdot z^3 \cdot (l-z)^2 (4 \cdot l - z)}{12 \cdot E \cdot I_z \cdot l^3}} . \quad (8)$$

The relation (5) for the shape error becomes:

$$\varepsilon_s = a_{pd} \left(1 - \frac{1}{1 + \frac{C_{Fx} \cdot a_p^{x_{Fx}} \cdot f^{y_{Fx}} \cdot HB^{n_{Fx}} \cdot z^3 \cdot (l-z)^2 (4l - z)}{12 \cdot E \cdot I_z \cdot l^3}}\right) . \quad (9)$$

As result of the shape error presence, the actual diameter d_a of the machined surface corresponds to:

$$d_a = d_d + \varepsilon_s , \quad (10)$$

or:

$$d_a = d_d + a_{pd} (1 + \frac{1}{1 + \frac{C_{Fx} \cdot a_p^{x_{Fx}} \cdot f^{y_{Fx}} \cdot HB^{n_{Fx}} \cdot z^3 \cdot (l-z)^2 (4 \cdot l - z)}{12 \cdot E \cdot I \cdot_z l^3}}) . \quad (11)$$

3. EXPERIMENTAL RESULTS

In order to verify the validity of the mathematical model given by the relation (11), a test piece having the average diameter d_{tp} = 40.16 mm was held in the universal scroll chuck and tailstock live center on a universal lathe characterized by the workpiece maximum diameter of 500 mm and the workpiece maximum length of 1 000 mm.

The length of the test piece between the universal scroll chuck and the tailstock live center was l = 362 mm. The material of the test piece was a medium carbon steel (containing 0.45 % C).

Some cutting passes with low depth of cut (a_p = 0.5 mm) were previously made in order to diminish the possibility to transmit an eventual workpiece shape error to the test piece.

The test piece was machined by using rough cutting parameters, in order to obtain a significant elastic deflection during the turning process: depth of cut a_{pd} = 3 mm, work feed f = 0.3556 mm/rev and a spindle speed n = 800 rev/min, corresponding to a cutting speed v = 100 m/min.

A relatively high cutting speed was used in order to avoid the occurrence of the built-up edge and a non-uniform roughness of the machined surface.

A tungsten carbide (ISO P20) tipped – tool was used to allow a high cutting speed and to avoid the significant wear of the tool corner.

The main geometrical parameters of the cutting tool were: clearance angle α = 6°, rake angle γ = 12°, entering angle κ = 70°, end cutting edge angle κ_1 = 20°, nose radius r_ε = 0.8 mm. Both before and after the cutting test, the test piece diameter was measured using a digital micrometer (accuracy of 0.001 mm).

Three measurements were made in planes placed at 120 °; the distances z where the diameter was measured, the proper diameters and the average diameters were included in the Table 1.

In the last line of the Table 1, the diameters determined using the relation (11) were inscribed; in this relation, the following were considered: d_w = 34.16 mm, C_{Fx} = 0.027, HB = 225, E = 211 111 daN/mm^2, y_{Fx} = 0.75, n_{Fx} = 2 [5].

Using the data included in the Table 1, the diagram from the Fig. 2 was obtained. As one may see, as expected, the diameter of the test piece presents a maxi-

Table 1

Values measured and calculated for the diameter of the machined surface

Diameters, mm	Distance z, mm								
	65	100	135	170	205	240	275	310	345
Dimeter of the turned surface, mm	34.495	34.734	34.754	34.736	34.738	34.730	34.719	34.718	34.676
	34.467	34.723	34.726	34.734	34.733	34.724	34.718	34.722	34.674
	34.471	34.729	34.7320	34.734	34.742	34.734	34.713	34.719	34.677
Average diameter of the turned surface, mm	34.4836	34.7286	34.7376	37.7346	34.7376	34.7293	34.7166	34.7196	34.6756
Diameter calculated with the relation (11), mm	34.1895	34.2039	34.2117	34.2112	34.2034	34.1914	34.1794	34.1711	34.1679

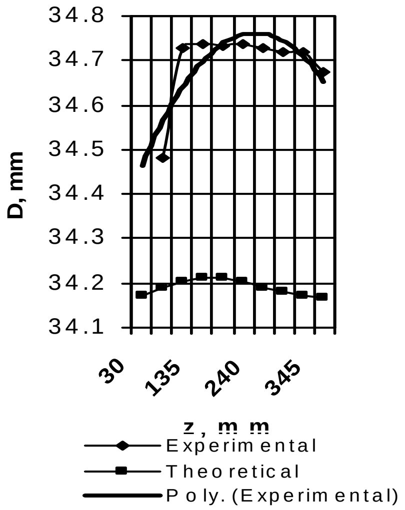

Fig. 2. Diameter of the machined surface.

mum in a zone placed nearer to the universal scroll chuck, due essentially to the elastic deflection of the workpiece; of course, the elastic deformation of the tool holder could also affect the diameter of the machined surface.

The diameters experimentally determined are generally higher than the ones determined by means of the relation (11).

Possible explanations of this result could be:

a) The elastic displacement of the universal scroll chuck and of the tailstock live center, together with the tailstock. Even these components of the machining system seem to be very rigid, if the cutting force is high enough, their elastic deformations could become significant. A higher rigidity corresponds to the universal scroll chuck and to the main spindle of the lathe; this situation is reflected both by the experimental results and by the results offered by the relation (11);

b) The obtained expression (11) takes into consideration only the deformation generated by the component F_x, while in actual cutting conditions, the test piece is affected also by the elastic deformation produced by the action of the component F_y and F_z;

c) The relation (11) does not consider the clearances existing among the machining system components.

Extending the experimental researches by considering workpieces with different diameters and lengths, a more complex empirical model corresponding to the diameter of the turned surface could be determined; such a model could be used in the case of the CNC lathe.

In this way, the path of the tool corner could be established so that, considering the elastic deflection of the components of the machining system, the dimensions of the machined surface after the machining will correspond to the desired ones.

4. CONCLUSIONS

In the case of the workpiece held in the universal scroll chuck and tailstock live center, the turning accuracy could be affected by the elastic deflections of the workpiece and of the other components of the machining system.

Considering the workpiece elastic deflection as significant, a mathematical model was elaborated, taking into consideration the action of the component F_x of the cutting force.

The experimental research confirmed the validity of the model, but the actual diametral dimensions of the machined surface are higher than those corresponding to the mathematical model and some explanations were given. A more complex mathematical model may be developed in the future, taking into consideration other factors able to affect the turning accuracy of the parts held in the universal scroll chuck and tailstock live center.

Experimental models thus determined could be used to establish adequate paths of the tool corner, inclusively by taking into consideration the elastic deflections of the machining system components.

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