

## ELECTRO-HYDRAULIC SERVO-SYSTEM STABILITY ANALYSIS

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**Abstract:** The present research refers to the analysis and the synthesis of an electro-hydraulic servosystem, specific to heavy machine tools. It is presented the construction and the functionality of the servosystem, the physical elements of design and its mathematical modelling, the correspondent transfer function and the results of the research, with the afferent restrictions.

Key words: servo-system, electro-hydraulic, modelling, research.

## 1. INTRODUCTION - GENERAL PROBLEMS OF DYNAMICS AND COMMAND

In general, electro-hydraulic servo-systems are made by placing the electric element (electro-hydraulic servovent-valve, the step-by-step motor and seldom the continuous current motor) directly on the hydraulic motor, thus resulting modules with small moving masses, and having small volumes in the rooms of the motor and in the coupling pipes, being characterized by high static and dynamic performances. In relation with the type of the electric command element, one can distinguish analogue and discrete electro-hydraulic servo-systems, and in relation with the used hydraulic motor, there are servo-systems with translation, revolving or oscillated movement.

The structure of the hydraulic servo-systems must consider the requirements of the tool-machine, regarding the dynamics of the movement, and of the basic characteristics of automatic systems: to be stable, rapid and precise.

The issue of increasing the performances of electrohydraulic servo-systems is usually solved by acting on the command subsystem, by synthesizing the best correction elements. The force subsystem is usually considered an invariable input data, or it is considered that the greatest reserves for increasing the dynamic performances are in this subsystem. An example of a resource is increasing the rigidity of the execution part by minimizing the liquid colon.

Choosing the servo-systems by the length of the course criteria, influences essentially the own frequencies of the advance mechanisms. Thus, for rectilinear movements, the powering can be made with linear hydraulic motor, with revolving hydraulic motor and screw-nut mechanism or with revolving hydraulic motor and pinion-rack bar. Choosing one of these solutions can be made function of result of the mass multiplied by the length.

An essential influence on the qualities in dynamic regime of the whole system of self-adjusting, is the own frequency of the hydro-motor – load assembly, because in every servo-system this assembly has the lowest own frequency, thus imposing the minimum reaction time of the whole system, and implicitly its precision level. From the point of view of the own frequency, the median position is the worst case, in which this frequency has the lowest value. This is why the dynamic study of the servosystems (for assuring the minimum accepted performances) is made considering the hypothesis of placing the piston in the median position.

# 1.1. Electro-hydraulic servo-systems with integrated command

There are four constructive alternatives of electrohydraulic servo-actuation, of which two have a high importance: the twisting moment electro-hydraulic amplifier and the linear electro-hydraulic amplifier. The latter will be referred as follows. Its functioning principle is transforming the revolving input movement in a translation output movement and amplifying the forces in the same time. These give the linear amplifier some advantages compared with the other alternatives.

The remembered servo-actuations are powered by small electric motors. Due to their force, rapidity, simplicity and precision these actuations are recommended especially where an increase of the capacity can't be made with conventional actuations. The development of electro-hydraulic revolving and translation servo-actuations, in the construction of machines and heavy machine tools, is given great attention by the technicians.

The small electric input powers will be transformed with the hydraulic energy in final big output powers, assuring between the input and output quantities well determined cinematic, static or dynamic rapports. This is obtained through one or more inverse intern or extern reaction couples.

## 2. THE CONSTRUCTION AND FUNCTIONING OF THE CONSIDERED SERVO-SYSTEM [3]

The driving system from Fig. 1 consists of the hydraulic motor, independent from the autonomous

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Fig. 1. The electro-hydraulic servo-system.



Fig. 2. The block diagram.

device for changing the reference input (DAC). The DAC consist of the electric micro motor for introducing the reference input, the summation mechanism and the hydraulic distributor.

The command pulses command the rotation of the stepping motor 1. This rotational movement is transmitted through the belt wheels 2 to the command screw 3. Subsequently, the distributor 4 fixed to the screw by the arc 5, performs an axial movement letting the oil flow to pass from the hydraulic system pipe to cylinder 6 rigidly coupled to slide 7.

When the slide moves, through pinion-rack mechanism 8–9, the table brings back the distributor in central position, after that the table movement stops.

The feedback system created in this manner implies the movement of the working part of the machine in such a way so the precision and rigidity of the driving system are considerably increased.

The linear increment  $\Delta$  of the system can be modified between large limits, by modifying the transmission ratio, as can be seen from the relation (1):

$$\Delta = \frac{1}{200} \times \frac{D_1}{D_2} \times Z_8 \times p_9 \tag{1}$$

where  $\Delta$  is in mm; 1 / 200 angular step of the stepping motor (1.8<sup>0</sup> = 1 / 200 rot.);  $D_1$ ,  $D_2$  – diameters of the belt wheels;  $p_9$  – step of the rack 9 (Fig. 1).

The block diagram is presented in the Fig. 2.

### 3. THE TRANSFER FUNCTION [1]

It is well known that in order to determine the transfer function of a servo-system, there need to be taken into account the physical relations for the functioning of the servo-system. Thus, the flow Q through the distributor is:

 $Q = C_d \cdot \Pi . d . x \cdot \sqrt{\frac{2}{p}(p_0 - p_1)},$  or, by linearising:

$$Q = A_Q \cdot x \,, \tag{2}$$

where: *d* being the diameter of the distributor drawer and  $A_Q = C_d \cdot \pi \cdot d \cdot \sqrt{\frac{2}{\rho}} \cdot p_0 -$  flow amplification.

The open loop transfer function can be expressed as:

$$T_D(s) = T_d(s)T_r(s), \ T_d(s) = T_R(s)T_{EE}(s), \ T_r(s) = \frac{1}{z_s \cdot p_g}$$

and the transfer function of the closed circuit is:

$$T_{0}(s) = \frac{T_{d}(s)}{1 + T_{d}(s) \cdot T_{r}(s)},$$
(3)

where  $T_d(s)$  is the transfer function of the direct loop,  $T_r(s)$  the transfer function of the feedback loop,  $T_R(s) = p_s$  is the transfer function of the controller *R* and  $T_{EE}(s)$  the transfer function of the actuation system "distributor-motor" (Fig. 2).

The structural equations for the "distributor-motor" assembly are:

$$\begin{cases} Q = A_Q \cdot x \\ Q = A \frac{dz}{dt} + k_1 \cdot \Delta p + \frac{V_M}{2E} \frac{d\Delta p}{dt}, \\ \Delta p \cdot A = M \frac{d^2 z}{dt^2} + k_2 \frac{dz}{dt} \end{cases}$$
(4)

where:

$$\Delta p = \frac{M}{A} \frac{d^2 z}{dt^2} + \frac{k_2}{A} \frac{dz}{dt}$$
(5)

and:

$$\frac{d\Delta p}{dt} = \frac{M}{A}s^3 + \frac{k_2}{A}s^2 \tag{6}$$

The transfer function of the actuating element EE (Fig. 2) is:

$$T_{EE}(s) = \frac{A_{Q}}{\frac{V_{M}M}{2EA}s^{3} + \left(\frac{k_{1}M}{A} + \frac{V_{M}k_{2}}{2EA}\right)s^{2} + \left(A + \frac{k_{1}k_{2}}{A}\right)s}$$
(7)

The transfer function of the open loop  $T_D(s)$  is:

$$T_{D(s)} = \frac{A_{\varrho} \cdot \frac{P_s}{z_8 \cdot p_9}}{\frac{V_M M}{2EA} s^3 + \left(\frac{k_1 \cdot M}{A} + \frac{V_M k_2}{2EA}\right) s^2 + \left(A + \frac{k_1 k_2}{A}\right) s}.$$
 (8)

The transfer function of the closed circuit:

$$T_{0}(s) = \frac{z(s)}{\theta(s)} = \frac{A_{\varrho} \frac{D_{1}}{D_{2}} \cdot p_{s}}{B_{3} \cdot s^{3} + B_{2} \cdot s^{2} + B_{1} \cdot s + A_{\varrho}}, \qquad (9)$$

n

where:

 $C_d = 0.62 - \text{distributor constant};$ 

 $d = 0.6 \dots 2.5$  [cm] – distributor drawer diameter;  $p_0 = 50$  [daN/cm<sup>2</sup>] – pressure of the agent provided by the pump;

 $p_s = 1$  [cm] – command screw pitch;

 $\rho = 0.86 \times 10^{-6}$  [daN s<sup>3</sup>/cm] – density of the hydraulic agent;

 $A = 32 \text{ [cm}^2\text{]} - \text{piston area;}$ 

 $K_1 = 0.47 \text{ [daN·s/cm]} - \text{viscous friction coefficient;}$   $K_2 = 1.4 \text{ [cm^3/daN·s^2]} - \text{volume losses coefficient;}$   $M = 0.07 \text{ [daN·s^2/cm]} - \text{inertial mass to be moved;}$   $VM = A \text{ [cm^2]} \times \text{hub } 34\text{[cm]} - \text{linear hydraulic}$ motor volume = 1 088 [cm<sup>3</sup>];

E = 1 500 [daN/cm<sup>2</sup>] – elasticity modulus of the hydraulic agent;

P9 = 0.08 cm;

Z9 = 24.

In the following part, various criteria are applied to verify the stability of the studied system, by comparing the four methods applied.

## 4. DETERMINING THE STABILITY OF THE SERVO-SYSTEM CONSIDERING THE DRAWER DIAMETER AS VARIABLE.

The three values of the drawer diameter in are: d = 0.6 cm; d = 2 cm; d = 2.5 cm.

#### 4.1. Bode characteristics

It is known that in the analysis and design of automated systems, the Bode characteristics have a large utilisation, the most important for the open system being:

- a) amplitude-frequency characteristic
- b) phase-frequency characteristic, which are known also as frequency characteristics.

The Bode characteristics give us information about the relative stability of the open system through the module reserve and the phase reserve. It can be noticed that for the drawer diameter values of 0.6and 2.00 cm, the module reserve is positive, so the system is stable. For all other values of the drawer diameter: 2.5 cm or more, the system is unstable. The phase reserve is positive for all values of the drawer diameter.

#### 4.2. Nyquist criterion

The Nyquist criterion analyses the transfer locus of the open system *TD* ( $j\omega$ ). According to this criterion, a linear automated system is stable if the transfer locus of the open circuit *TD* ( $j\omega$ ) for values from  $\omega = -\infty$  to  $\omega = +\infty$ , is inside the domain bordered by the critical point (-1, j0).

Thus, it can be noticed that for the drawer diameter values of 0.6 and 2.00 cm, the Nyquist transfer locus is located within the domain bordered by the critical point (-1, j0), so the system is stable for these values, but for all other values of the drawer diameter: 2.5 cm or more, the system is unstable.

Considering the facts shown above, it can be concluded that starting with the value 2.5 of the drawer diameter, the system becomes unstable. This means that the Nyquist transfer locus surrounds the critical point (-1, j0) for the values of the drawer diameter of 2.5 cm or more.

#### 4.3. Pole-zero map

The roots of the polynomial expression from the denominator must be calculated and placed in the complex system, for several values of the drawer diameter.

The system is completely stable if all of the transfer function's poles are placed on the left side in the complex plane (s). The complex character of the poles (with a negative real part) will determine the system's dampened oscillatory aspect  $x_e(t)$ .

If the location of the poles is on the left side of the roots diagram, the system described by means of this transfer function will be a stable one. A condition for aumatic systems ist that their transitory phaze must be as short as posible. This means that the time functions must be dampened as quick as posible. In the *S* plane this condition consists in a limiting of the left semyplane which has the polse of the transfer function.

The transitory character duration of service is not the only condition imposed for obtaining a satisfactory answer. The transitory regime has to present an acceptable over-adjustment and therefore a proper dampening behaviour. The conjugated complex poles that give the oscillatory aspect of the transitory regime have to be limited. The limitation of the imaginary part of the complex conjugated poles is done starting with the origin of the plane s under angle  $\theta$ . This limitation correponds to a limitation of the amortisation factor  $\xi$  of the oscillatory answer written as  $\cos \theta$ .

#### 4.4. The step response

The step response of driving system, reflect the output data evolution, respectively the functional stability of the driving system for different values of drawer diameter.

# 5. DIAGRAMS FOR THREE VALUES OF THE DRAWER DIAMETER [2]

In Figs. 3–6, the diagrams for SYSTEM 1 (d = 0.6 cm) are presented. The SYSTEM 3 (d = 2 cm) is characterized by diagrams in Figs. 7–10. The Figs. 11–14 show the diagrams for SYSTEM 4 (d = 2.5 cm).







**Fig. 4.** Nyquist Open Loop Sys1, d = 0.6 cm (SYSTEM 1).



**Fig. 5.** Pole-Zero Closed Loop Sys1, d = 0.6 cm (SYSTEM 1).











**Fig. 8.** Nyquist Open Loop Sys2, d = 2 cm (SYSTEM 3).



**Fig. 9.** Pole-Zero Closed Loop Sys 2, d = 2 cm (SYSTEM 3).







Fig. 11. Bode Open Loop Sys3, d = 2.5 cm (SYSTEM 4).



Fig. 12. Nyquist Open Loop Sys3, d = 2.5 cm (SYSTEM 4).



Fig. 13. Pole-Zero Closed Loop Sys3, d = 2.5 cm (SYSTEM 4).



**Fig. 14.** Step Response Closed loop Sys 3, d = 2.5 cm (SYSTEM 4).

## 6. CONCLUSIONS

- 1. The paper researched the behaviour of an electrohydraulic servo-system used in the actuation of Industrial robots.
- 2. The behaviour has been analysed from the point of view of the stability, through the presented methods.
- 3. The following steps were followed:
  - drawing the block diagram according to the constructive sketch of the system;

- linearising and obtaining the transfer function by applying the Laplace transformation;

- analysing the stability of the servo-system by modifying a parameter in the presented situation, the drawer diameter;.

- 4. From the analysis of the stability of the presented servo-system, the same stability condition was concluded; namely the drawer diameter must be less than 2.5 cm. The present analysis refers strictly to this parameter.
- 5. For drawer diameter values over 2.5 cm, the system becomes unstable.
- 6. The comparative dynamic analysis shown through the four presented methods leads us to the same conclusion mentioned at the fifth point.

- 7. These analysis methods are beneficial in the design phase. In this case it is possible to establish the variation field of the drawer diameter in order to achieve a stable functioning of this system.
- 8. Expecially in the Pole-Zero diagram, we can notice that already at a drawer diameter value of 2.5 cm, the system's functionality is at the stability limit, the transfer function poles being in the right semiplane.
- 9. We can notice that the stability depends on the elements and the conditions in which the system works, that is why this research is important even from the design phase.

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