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GEAR-TYPE TOOL PROFILING – A COMPARISON BETWEEN A CAD METHOD AND THE ANALYTICAL METHOD

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Abstract: *A CAD method, developed under the CATIA soft, in order to profile a gear-type tool for generating an ordinate whirl of profiles, associated to a couple of rolling centrods, is presented. The method is based on synthesizing some "virtual mechanisms", to reproduce the enveloping profiles principle: rolling between the centrods associated to the tool and to the workpiece, by satisfying the enveloping condition. A comparison between the results given by the new method and those obtained by using the analytical "Trajectories Method" is made. The included applications are concerning embracing profiles, (case of interior tangency between the rolling centrods) as well as profiles generated through exterior tangency.*

Key words: *gear-type tool, enveloped surfaces, CAD profiling, virtual mechanism.*

1. INTRODUCTION

Methods to profile gear-type tools, which generate ordinate whirls of surfaces (profiles) through enwrapping, by rolling, were developed based on fundamental theorems of enveloped surfaces theory:

- Olivier 1st theorem, Gohman theorem, Willis theorem, [4, 5, 7, 9, and 10];
- Graphical – analytical methods [6];
- Graphical methods, by using CAD – type soft facilities [1, 2, 3].

There were also conceived complementary analytical methods like Minimum Distance Method [8] or Plain Generating Trajectories Method [8].

The solutions provided by enounced methods have the quality of giving, in all cases, numerical results compatible, from technical point of view, to the ones obtained by applying the enveloped surfaces fundamental theorems.

A new solution for profiling the gear-type cutting tool, reciprocal enwrapped to an ordinate whirl of surfaces, is suggested. Both tool and workpiece being associated to a couple of rolling centrods (usually circular), the facilities of CATIA software were used by imagining a kinematical entity, in order to reproduce the rolling motion between the centrods: C_1 – circle of R_{rp} radius, associated to the whirl of profiles to be generated and C_2 – circle of R_{rs} radius, attached to the tool.

The solution is based on *Part Environment* facilities, where the elements of a virtual mechanism, able to simulate the two centrods rolling, when the enwrapping (normal) condition is satisfied, are synthesized. The virtual mechanism elements, developed in *Part* environment are input in an *Assembly* file of the environment, then in *DMU Kinematics* environment, in order to define the preset kinematical couples. The mechanism motion is realized through *Simulation* and *Replay* commands, for some intermediary positions *Shots*.

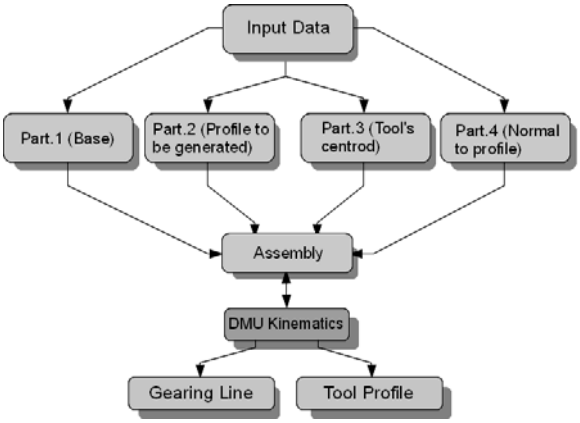

Fig. 1. Generating algorithm in CATIA graphic designing environment.

By using *Trace* command, the trajectory of any point owning to a virtual mechanism element may be drawn, relative to any other of its elements, inclusive relative to the fix element *Base*, when the gearing line results.

The trajectories drawn in CATIA soft are curves of *Spline* type, whose constitutive point co-ordinates may be output as file text, see Fig. 1.

2. GEAR-TYPE TOOL PROFILING

2.1. Virtual Mechanism to Generate in CATIA Designing Environment

Three types of profiles of workpieces to whom the generating virtual mechanisms in CATIA designing environment (G.M.C.E.) will be developed are considered (see Table 1).

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Table 1

Types of profiles

G.M.C.E. Type	Workpiece Profile Type
G.M.C.E. for rectilinear segments	Profiles composed from rectilinear segments
G.M.C.E. for arcs of circle	Profiles composed from arcs of circle, tangent or not in the contact points
G.M.C.E. for Spline – type of curve	Profiles composed from curves, given through points or known by equations (involute, cycloid, etc.)

2.2. Analytical Method

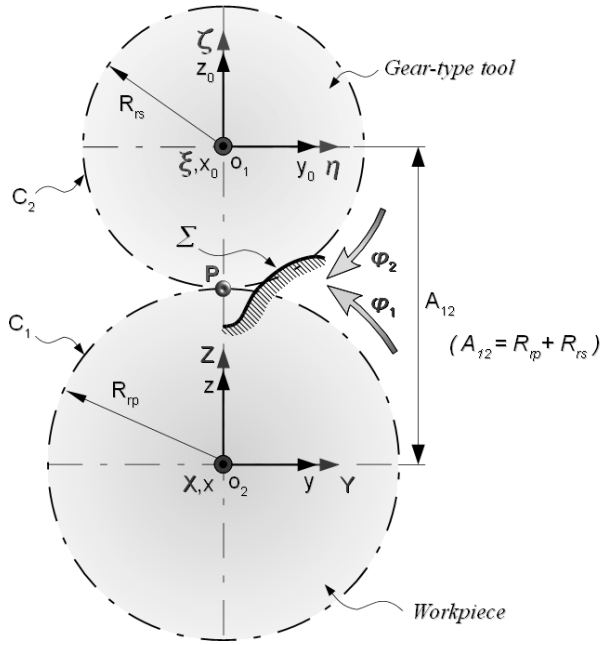

Fig. 2. Generation kinematics (exterior tangent centrods).

The following co-ordinates systems are defined (see also Fig. 2):

- *xyz*, meaning a fix system, having as *x* axis the rotation axis of the C_1 centrod, of R_{rp} radius;
- $x_0y_0z_0$ – fix system, having x_0 axis common with the gear-type tool rotation axis (the C_2 centrod axis);
- A_{12} – the distance between the two fix reference systems, having parallel axis and same orientation;
- *XYZ* – mobile system, attached to the whirl of surfaces to be generated;
- ξης – mobile system, moving together to the gear-type tool (implicit to the C_2 centrod, circle of R_{rs} radius).

Rolling process kinematics, between C_1 and C_2 centrods, tangent in the gearing pole *P*, assumes that the speed of points owning to both centrods is the same. The speed vectors in the two rotation motions, around *x* and x_0 axis, of these two points, from the two centrods, momentarily found in P, are identical.

XYZ system absolute motion can be expressed as

$$x = \omega_1^T(\varphi_1)\cdot X \ , \tag{1}$$

where
$$x = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ and } X = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \tag{2}$$

mean the matrices of the current points, into *xyz* fix system, respective into *XYZ* mobile system.

The motion of C_2 centrod, rotation around x_0 axis, is given through the equation

$$x_0 = \omega_1^T(-\varphi_2)\cdot \xi \ , \tag{3}$$

where
$$x_0 = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} \text{ and } \xi = \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} \tag{4}$$

are the current points matrices into the spaces $x_0y_0z_0$, respective ξης.

The motion (3) – C_2 centrod rotation motion – can be referred to *xyz* system

$$x_0 = x - A \ , \tag{5}$$

where
$$A = \begin{pmatrix} 0 \\ 0 \\ A_{12} \end{pmatrix} \text{ and } A_{12} = R_{rp} + R_{rs} \ , \tag{6}$$

in the case of exterior tangency between the centrods.

Thus, C_2 centrod and same time ξης system, attached to the gear-type tool absolute motion, referred to *xyz* fix system, is

$$x = \omega_1^T(-\varphi_2)\cdot \xi + A \ . \tag{7}$$

Between the angular parameters φ_1 and φ_2, of the two rotation motions, there is always satisfied the relation

$$i = \frac{R_{rp}}{R_{rs}} = \frac{\varphi_2}{\varphi_1} \ , \tag{8}$$

which, in most of the practical situations is a constant (the transmission ratio).

If now Σ profile is defined as owning to the ordinate whirl of profiles to be generated,

$$\Sigma = \begin{pmatrix} 0 \\ Y(u) \\ Z(u) \end{pmatrix} \ , \tag{9}$$

with *u* – variable parameter, the profiles family Σ can be found referred to the gear-type tool reference system:

$$\xi = \omega_1(-\varphi_2)\cdot\left[\omega_1^T(\varphi_1)\cdot X - A\right]. \tag{10}$$

After calculus and by also considering (8), we have:

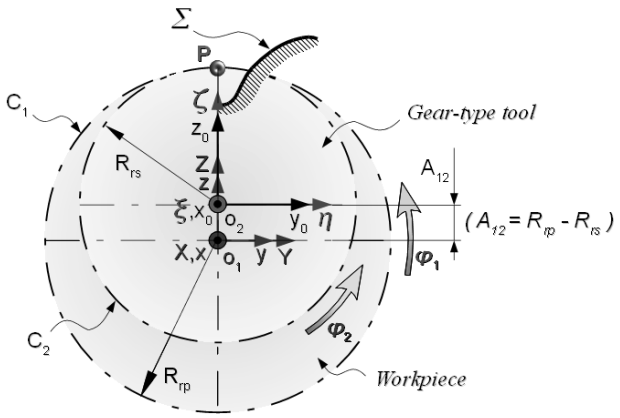

Fig. 3. Interior tangent centrods.

$$(\Sigma)_{\varphi_1}\left|\begin{array}{l}\xi = 0;\\ \eta = Y(u)\cos[(1+i_1)\varphi_1] - Z(u)\sin[(1+i_1)\varphi_1] + \\ \qquad + A_{12}\sin(i\varphi_1);\\ \zeta = Y(u)\sin[(1+i_1)\varphi_1] + Z(u)\cos[(1+i_1)\varphi_1] - \\ \qquad - A_{12}\cos(i\varphi_1).\end{array}\right. \quad (11)$$

The envelope of profiles family (11) means the gear-type tool into a section normal to ξ axis. An enveloping condition must be associated to equations (11) in order to find the shape of the wrapping gear-type tool. One can choose among enveloping basic theorems: Olivier, Gohman, Willis [4] or a specific complementary method: "minimum distance","substitution circles family" [8], "plain generation trajectories" [8].

The case of generating inner surfaces, when the two centrods are interior tangent, can be treated similarly (see Fig. 3). Thus, the profiles family specific to this generating solution becomes

$$(\Sigma)^*_{\varphi_1}\left|\begin{array}{l}\xi = 0;\\ \eta = Y(u)\cos[(1-i_1)\varphi_1] - Z(u)\sin[(1-i_1)\varphi_1] - \\ \qquad - A_{12}\sin(i\varphi_1);\\ \zeta = Y(u)\sin[(1-i_1)\varphi_1] + Z(u)\cos[(1-i_1)\varphi_1] - \\ \qquad - A_{12}\cos(i\varphi_1).\end{array}\right. \quad (12)$$

The enveloping condition, specific to "plain generation trajectories" method,

$$\frac{\eta'_u}{\eta'_{\varphi_1}} = \frac{\zeta'_u}{\zeta'_{\varphi_1}} \quad (13)$$

determines, into the fix reference system, the gearing line between the two profiles (tool profile and workpiece profile). The partial derivatives from (13) should be calculated by starting from relations (11) or (12). The ensemble of equations (11) and (13) – respective (12) and (13) – finally gives the gear-type tool profile.

3. RECTILINEAR PROFILES GENERATION

3.1. CAD Method

A specific generation mechanism (G.M.C.E.) to enable, as consequence of rolling between two centrods

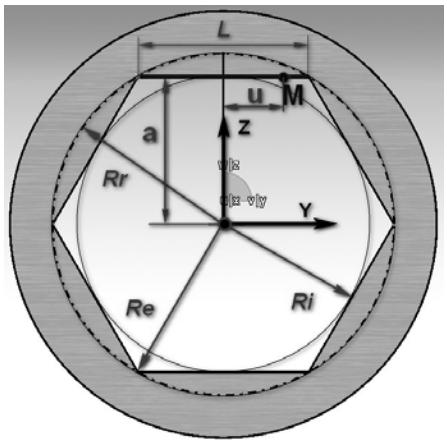

Fig. 4. Regular hexagonal hole.

defined in the beginning, to obtain in both graphical and numerical form, the gear-type tool transversal profile and, same time, the gearing line. More than that, it will become possible to study the shape of singular points trajectories, as starting point for further studying the interference processes.

In Fig. 4 a regular polygonal hole is depicted, further considered as example for finding the profile of the gear-type tool to generate it.

The input parameters values are given in Table 2.

In Fig. 5 the generation mechanism (G.M.C.E.) for generating a straight line profile is presented. It has the following component elements: *Base*, *Workpiece*, *Gear-type tool*, *Tappet*.

In Fig. 6 one can see a specific G.M.C.E., applied in the case of a hexagonal bush and a gear-type tool, having

Table 2

Input values in the case of a regular hexagonal hole

Crt. No.	Input Parameter	Value
1	Sides number, NrL	6
2	Circumscribed circle radius, Re	50 mm
3	Inscribed circle radius, Ri	43.3 mm
4	Side length, L	50 mm
5	Rolling radius, Rr	50 mm
6	Tool teeth ratio, NrDS	6/3

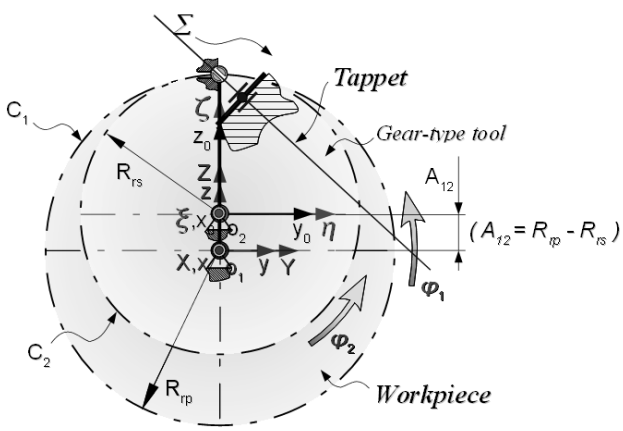

Fig. 5. Virtual mechanism to be used for a rectilinear segment generation, by using a gear-type tool.

the transmission ratio of 6/3, which if receives the *Simulation* command, realizes the rolling between the two interior tangent centrods.

In Fig. 7 and Table 3 the shape and the co-ordinates of a gear-type tool for some among the 101 considered points of its profile are presented.

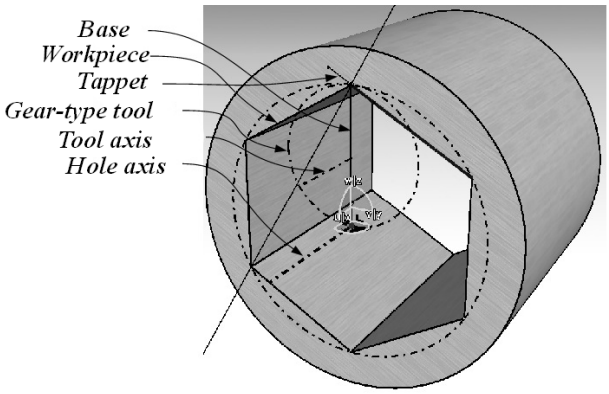

Fig. 6. G.M.C.E. for a regular hexagonal hole.

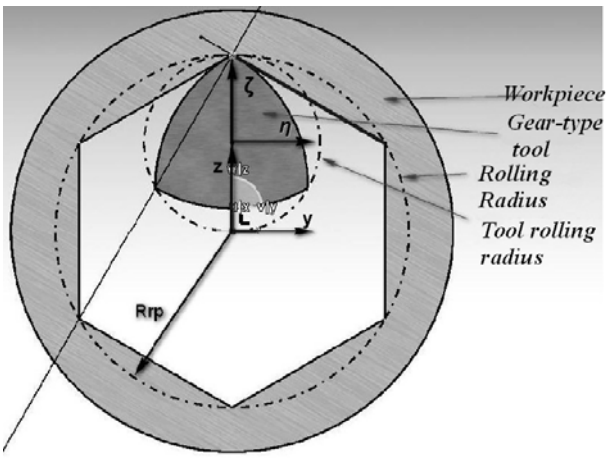

Fig. 7. Gear-type tool for hexagonal hole (R_{rp} = 50 mm, i = 6/3).

Table 3

Co-ordinates of points from tool cutting edge – hexagonal hole, CAD method

Crt. no.	η [mm]	ς [mm]	Crt. no.	η [mm]	ς [mm]
1	0.000	–25.000	46	–14.719	–10.998
4	–1.273	–24.235	49	–15.408	–9.896
7	–2.498	–23.442	52	–16.064	–8.774
10	–3.675	–22.622	55	–16.685	–7.633
13	–4.809	–21.776	58	–17.272	–6.472
16	–5.899	–20.906	61	–17.826	–5.292
19	–6.949	–20.012	64	–18.345	–4.091
22	–7.959	–19.095	67	–18.830	–2.871
25	–8.930	–18.156	70	–19.279	–1.631
28	–9.864	–17.195	73	–19.693	–0.370
31	–10.761	–16.213	76	–20.071	0.911
34	–11.622	–15.210	79	–20.411	2.215
37	–12.449	–14.187	82	–20.713	3.540
40	–13.242	–13.144	85	–20.976	4.889
43	–13.997	–12.081	101	–21.647	12.001

3.2. Analytical Method

According to Fig. 4, the parametric equations of the workpiece profile (hexagonal hole) are defined:

$$\Sigma \left| \begin{array}{l} X = 0; \\ Y = u; \\ Z = a, \end{array} \right. \tag{14}$$

where u – variable,

$$R_{rp} = L, -\frac{R_{rp}}{2} \le u_{\min} \le \frac{R_{rp}}{2}, a = \frac{\sqrt{3}}{2} L .$$

From (14), the profiles family, referred to the gear–type tool reference system, can be found:

$$(\Sigma)_{\varphi_1} \left| \begin{array}{l} \eta = u\cos[(1-i)\varphi_1] - a\sin[(1-i)\varphi_1] - A_{12}\sin(i\varphi_1); \\ \zeta = u\sin[(1-i)\varphi_1] + a\sin[(1-i)\varphi_1] - A_{12}\cos(i\varphi_1). \end{array} \right. \tag{15}$$

The specific enveloping condition is

$$\varphi_1 = \arcsin\left(\frac{u}{R_{rp}}\right). \tag{16}$$

The ensemble formed by equations (15) and (16) gives, into ξη reference system, the tool profile. In Table 4 the co–ordinates of points from gear–type tool profile, used to generate the same surface as in the example considered in 3.1 are presented.

Table 4

Co–ordinates of points from tool cutting edge – hexagonal hole, analytical method

Crt. no.	η [mm]	ς [mm]	Crt. no.	η [mm]	ς [mm]
1	0.000	–25.000	46	–14.719	–10.998
4	–1.273	–24.235	49	–15.408	–9.896
7	–2.498	–23.442	52	–16.064	–8.774
10	–3.675	–22.622	55	–16.685	–7.633
13	–4.809	–21.776	58	–17.272	–6.472
16	–5.899	–20.906	61	–17.826	–5.292
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25	–8.930	–18.156	70	–19.279	–1.631
28	–9.864	–17.195	73	–19.693	–0.370
31	–10.761	–16.213	76	–20.071	0.911
34	–11.622	–15.210	79	–20.411	2.215
37	–12.449	–14.187	82	–20.713	3.540
40	–13.242	–13.144	85	–20.976	4.889
43	–13.997	–12.081	101	–21.647	12.001

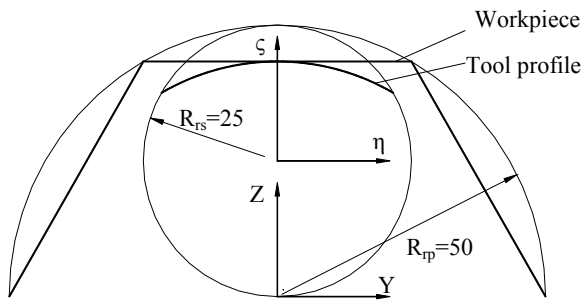

Fig. 8. Gear–type tool profile – analytical method.

As it can be observed from a comparison between the last two tables, the co–ordinates of the points having the same rank are identical if considering the first three (even four) decimals, which shows a high accuracy of the CAD method. In Fig. 8 the tool profile is drawn, which was found by both methods.

4. CIRCULAR PROFILES GENERATION

4.1. CAD Method

A virtual mechanism specific to generate profiles in arc of circle was designed (Fig. 9), which further allows, in CATIA design environment, to determine the profile of the required gear–type tool (in the considered example, a tool for interior to generate a hub with triangular slots).

In Fig. 10 and Table 5 the hub shape and specific input parameters are presented.

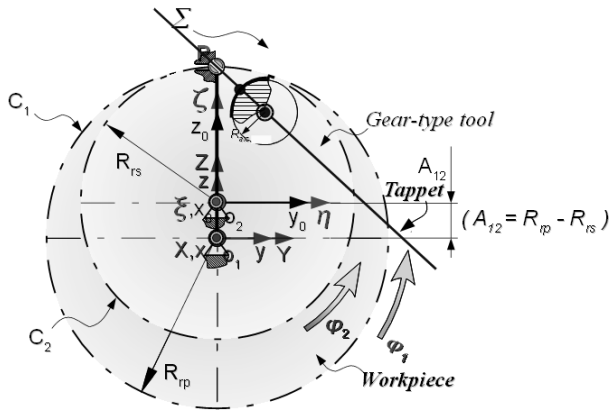

Fig. 9. G.M.C.E. to generate profiles in arc of circle.

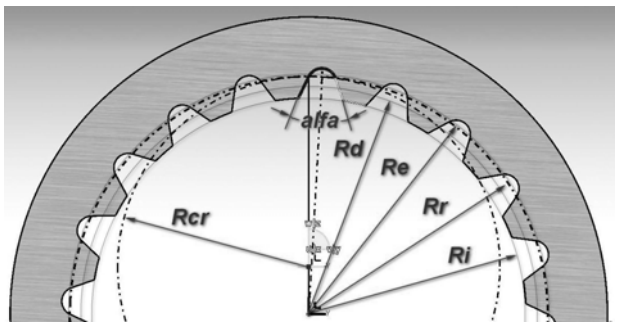

Fig.10. Hub with triangular slots.

Table 5

Input values in the case of a hub with triangular slots

Crt. No.	Input Parameter	Value
1	Slots number, NrC	20
2	Flank exterior radius, Re	55 mm
3	Flank interior radius, Ri	50 mm
4	Flank pitch circle radius, Rd	52.5 mm
5	Flank angle, α	45°
6	Tool teeth ratio, NrDS	20/16
7	Workpiece rolling radius, Rr	52.5 mm
8	Gear–type tool rolling radius Rcr	44 mm

The profiles ensemble which represents the composite profile of the generator tool for triangular slots was obtained by simulating the specific virtual mechanisms rolling. In Table 6, co–ordinates of points from the gear–type tool are presented, owning to the left flank and to the inner arc of circle, found by applying the CAD method.

4.2. Analytical Method

Because triangular slots profile is symmetrical, only its left half will be further considered (Fig. 11); it consists of two parts: an arc of circle *AB* and a rectilinear segment *BC*. The equations giving the workpiece frontal profile corresponding to arc *AB* are

$$\Sigma_{AB}\left|\begin{array}{l} Y = -r\sin\theta; \\ Z = R_0 + r\cos\theta, \end{array}\right. \tag{17}$$

which generates, referred to the tool system, the family of profiles (see also (12):

$$\left(\Sigma_{AB}\right)_{\varphi_1}\left|\begin{array}{l} \eta = -r\sin\theta\cos\left[(1-i)\varphi_1\right] - \\ \quad -\left(R_0 + r\cos\theta\right)\sin\left[(1-i)\varphi_1\right] - A_{12}\sin\left(i\varphi_1\right); \\ \zeta = -r\sin\theta\sin\left[(1-i)\varphi_1\right] + \\ \quad +\left(R_0 + r\cos\theta\right)\cos\left[(1-i)\varphi_1\right] - A_{12}\cos\left(i\varphi_1\right), \end{array}\right. \tag{18}$$

Table 6

Co–ordinates of points from tool cutting edge – hub with triangular slots, CAD method

Crt. no.	η [mm]	ς [mm]	Crt. no.	η [mm]	ς [mm]
28	8.2336	39.5821	101	10.7095	42.6767
29	8.2659	39.6240	102	10.7601	42.7360
30	8.2983	39.6659	103	10.8121	42.7940
31	8.3307	39.7079	104	10.8656	42.8509
32	8.3631	39.7498	105	10.9205	42.9065
33	8.3956	39.7917	106	10.9767	42.9608
34	8.4281	39.8337	107	11.0342	43.0137
35	8.4606	39.8757	108	11.0931	43.0652
36	8.4932	39.9176	109	11.1531	43.1153
37	8.5259	39.9596	110	11.2144	43.1640
38	8.5585	40.0016	111	11.2768	43.2112
39	8.5912	40.0436	112	11.3403	43.2569
40	8.6240	40.0856	113	11.4049	43.3010
41	8.6567	40.1276	114	11.4705	43.3436
42	8.6896	40.1697	115	11.5371	43.3846

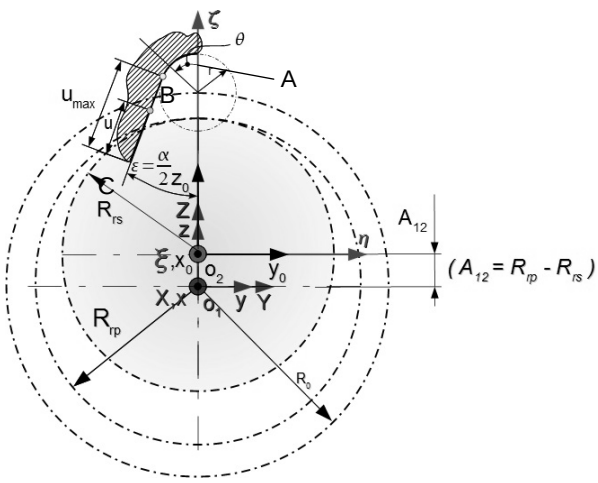

Fig. 11. Reference systems and notations.

Table 7

Co–ordinates of points from tool cutting edge – hub with triangular slots, analytical method

Crt. no.	η [mm]	ς [mm]	Crt. no.	η [mm]	ς [mm]
28	8.2344	39.5829	101	10.7095	42.6765
29	8.2667	39.6248	102	10.7600	42.7357
30	8.2990	39.6667	103	10.8121	42.7938
31	8.3314	39.7086	104	10.8656	42.8507
32	8.3638	39.7505	105	10.9204	42.9063
33	8.3963	39.7925	106	10.9767	42.9605
34	8.4288	39.8344	107	11.0342	43.0135
35	8.4613	39.8763	108	11.0931	43.0650
36	8.4939	39.9183	109	11.1531	43.1151
37	8.5265	39.9603	110	11.2144	43.1638
38	8.5592	40.0022	111	11.2768	43.2110
39	8.5919	40.0442	112	11.3403	43.2567
40	8.6246	40.0862	113	11.4049	43.3008
41	8.6574	40.1282	114	11.4705	43.3434
42	8.6902	40.1703	115	11.5371	43.3844

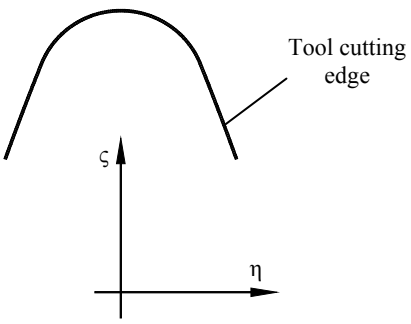

Fig. 12. Gear–type tool profile – analytical method.

with the specific enveloping condition,

$$\varphi = -\theta + \arcsin\left(\frac{R_0}{R_{rp}} \cdot \sin\theta\right). \tag{19}$$

In the case of the rectilinear segment *BC*, we have

$$\Sigma_{BC} \left| \begin{array}{l} Y = -r\cos\varepsilon - u\sin\varepsilon; \\ Z = R_0 + r\sin\varepsilon - u\cos\varepsilon, \end{array} \right. \tag{20}$$

with the conditions $u_{min} = 0$ and u_{max} resulting from the relation $Y^2 + Z^2 = R_i^2$. The profile family is:

$$(\Sigma_{BC})_{\varphi_1} \left| \begin{array}{l} \eta = (-r\cos\varepsilon - u\sin\varepsilon)\cos[(1-i)\varphi_1] - \\ -(R_0 + r\sin\varepsilon - u\cos\varepsilon)\sin[(1-i)\varphi_1] - A_{12}\sin(i\varphi_1); \\ \zeta = (-r\cos\varepsilon - u\sin\varepsilon)\sin[(1-i)\varphi_1] + \\ +(R_0 + r\sin\varepsilon - u\cos\varepsilon)\cos[(1-i)\varphi_1] - A_{12}\cos(i\varphi_1); \end{array} \right. \tag{21}$$

with the specific enveloping condition

$$\varphi_1 = \varepsilon - \arccos\left(\frac{R_0\cos\varepsilon - u}{R_{rp}}\right). \tag{22}$$

In Table 7 the co–ordinates of points from tool cutting edge and in Fig. 12 – its graphical representation are presented.

5. CONCLUSIONS

The present comparative study between the results of profiling the gear–type tool by using two methods – the kinematical method (based onto CATIA designing environment) and the analytical method – in the cases of two simple profiles (rectilinear and arc of circle), proves the new CAD method capacity of rigorously describing the gear–type tool profile. The differences between the points' co–ordinates, in the considered examples, are smaller than $1 \cdot 10^{-3}$ mm.

Same time, by using CATIA designing environment facilities, interference trajectories between the worked profiles and the generator tool can be easily drawn. The gearing line shape can be also found, by only changing the element referred to whom the contact point between the two enwrapped profiles is defined.

The problematic of the suggested validation might be extended for the other known enwrapping generation methods by rolling: rack–tool generation or worm–tool generation.

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