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EXPERIMENTAL INVESTIGATION OF TANGENTIAL CONTACT STIFFNESS AND EQUIVALENT DAMPING

Iuliana PISCAN[1,*], Thierry JANSSENS[2], Farid AL-BENDER[3], Cristina PUPĂZĂ[4]

[1)] PhD. ing., Department of Machines and Production Systems, POLITEHNICA University of Bucharest , Bucharest, Romania
[2)] Dr. ir., post-doctoral researcher, Department of Mechanical Engineering, KU Leuven, 3001 Heverlee, Belgium
[3)] Professor Dr. ir., Department of Mechanical Engineering, KU Leuven, 3001 Heverlee, Belgium
4)Associate Professor, Dr.-Ing., Department of Machines and Production Systems, POLITEHNICA University of Bucharest, Romania

Abstract: *In this paper, the phenomena of hysteretic behaviour of frictional contacts observed during reciprocating experiments are discussed. The hysteretic behaviour is determined theoretically based on the expression of hysteresis virgin curve. On the basis of the experimental analysis, an analytical method for determining the energy dissipation, equivalent damping and tangential contact stiffness is proposed. For all material contacts the same tendencies of the energy dissipation and equivalent damping can be observed which are increasing with load and displacement and, also for the contact stiffness which is increasing with increasing load and weakening with the displacement.*

Key words: *contact stiffness, damping, hysteresis, friction, energy dissipation.*

1. INTRODUCTION

The study of frictional contact has been the subject of scientific research since Coulomb's hypothesis [1]. Frictional contact appears in various mechanical systems commonly found in machine tools applications, including gears, slides, bearings, bolted joints and others.
The most important characteristic of friction which occurs during the process of sliding is the hysteretic effect. The hysteretic effect can be perceived as sticking and sliding phases being caused by the behaviour of the friction force which acts as a nonlinear spring before sliding. This phenomenon which appears at the microscopic level is called pre-sliding displacement with non-local memory and results from the tangential contact stiffness between contacting bodies [2, 3, 4].

The hysteresis characteristic can be perceived in many mechanical systems from various applications such as civil, mechanical, and electrical. The hysteresis models can be divided into rate-independent and rate-dependent models, which in the mechanical engineering context are referred to as static and dynamic hysteresis [5]. This paper will focus on rate-independent hysteresis from mechanical engineering perspective. In studying the hysteretic behaviour, several physical or mathematical models are available in the literature. The Masing's rules [6] and their generalizations such as from Fan [7], distributed-element models (DEMs) [8], Pisarenko's method [9], Bouc–Wen model [10], Maxwell Slip model [2, 11], are examples used in the modelling of hysteresis in mechanical structures.

Hysteresis phenomena influence the dynamic behaviour of machine tools mechanical structures with moving parts, which is not comprehensively examined in the literature yet. However, in other fields of engineering, where the hysteresis phenomena occur, further research was conducted [4].

In this paper, we propose a simple approach to model the hysteretic effects of a frictional contact. An analytical description of hysteresis loop has been described on the basis of experimental studies. Experimental studies for flat on flat frictional contacts were concentrated on the pre-sliding zone.

2. ANALYTICAL DETERMINATION OF THE HYSTERESIS

This section presents an analytical computation of hysteresis loops behaviour for two bodies moving relative to each other (see Fig.1).

In Fig. 2 a typical measured hysteresis loop is shown where $x = -A_m, \ldots, A_m$ is the input (displacement amplitude) and h_m is the output, the break-away force or the static friction force limit. The expression of the virgin curve is given by the exponential equation [12].

$$f(x) = h_0(1 - e^{-a_c(x-x_0)}) . \tag{1}$$

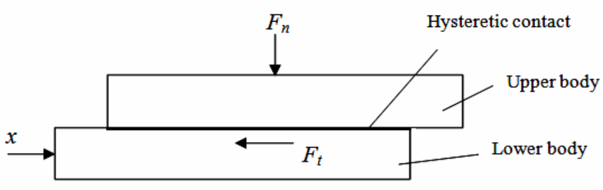

Fig. 1. Frictional contact of flat on flat connection.

* Corresponding author: 313 Splaiul Independentei, Bucharest, Romania
Tel.: + 4 021-402 91 00;
Fax: + 4021-318 10 01;
E-mail addresses: *iuliana.piscan@yahoo.com* (I. Piscan), *Thierry.Janssens@mech.kuleuven.be* (T.Janssens), *Farid.Al-Bender@mech.kuleuven.be* (F. Al-Bender), *cristinapupaza@yahoo.co.uk* (C. Pupăză),

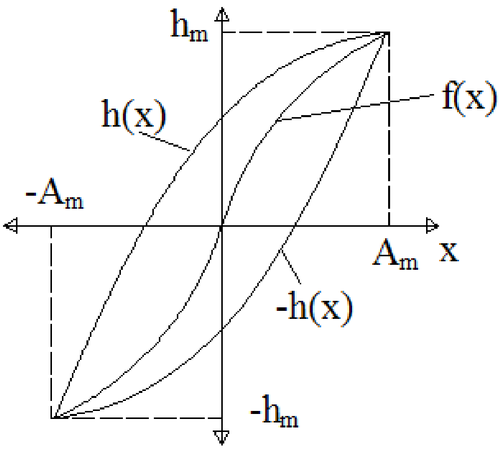

Fig. 2. Typical measured hysteresis loop.

The model parameters, h_0 and a_c, are used to describe and to evaluate the hysteresis behaviour. The first parameter h_0, representing the saturation value of the hysteresis loop can be seen as the static friction force limit, or the break-away force. The parameter a_c represents a measure of the curvature of the hysteresis. The parameter x_0, represents a variable which allows to shift the starting point of the hysteresis half towards the origin of the hysteresis. Other model equations are also possible [4].

Based on the Masing's rules the hysteresis curves can be expressed by using the equations presented in this section. If no relative displacement between the two contacting bodies has occurred, such that there is no history of motion, and then the friction force follows the virgin curve, $f(x)$

$$h(x) = f(x) \text{ with } f(x) = \begin{cases} h_{virg}(x), x \geq 0 \\ -h_{virg}(-x), x \leq 0 \end{cases}. \qquad (2)$$

where

$$h_{virg} = h_0[1 - \exp(-a_c x)]. \qquad (3)$$

As can be seen in Fig. 2, if the direction of the motion changes at $x = A_m$, the friction force becomes:

$$h(x) = -h_m + 2f\left(\frac{x + A_m}{2}\right). \qquad (4)$$

$$h_m = f(A_m), \qquad (5)$$

calculated with the formula of $h(x)$ before reversal point where displacement amplitude range = $-A_m$ to A_m

Substituting the hysteresis curve equation into Eq. (4) one obtains Eq. (6)

$$h(x) = 2 \cdot h_0\left[1 - \exp\left(-a_c \frac{(x + A_m)}{2}\right)\right] - h_m. \qquad (6)$$

The amount of energy dissipation in one cycle of a hysteresis loop can be calculated based on the area it encloses. The area of the hysteresis loop is calculated by using two methods: in the first method the expression of the virgin curve is integrated and in the second method the expression of the outer hysteresis loop is integrated.

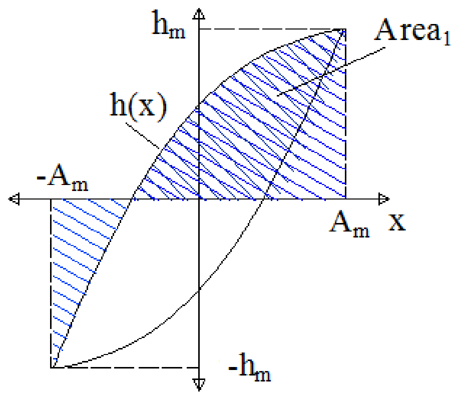

Fig. 3. Schematic representation of hysteresis area.

2.1. The determination of the area by integrating the outer hysteresis loop expression

The area enclosed in the loop (see Fig. 3) gives the energy dissipation during micro-slip per cycle which can be obtained from integration. This integration of the hysteresis curve expression gives the area of the upper part as:

$$Area_1 = \int_{-A_m}^{A_m} h(x)dx. \qquad (7)$$

$$Area_1 = 2h_0\left(2A_m - \frac{1}{a_c} + \frac{1}{a_c}\exp(-a_c A_m)\right) - h_m A_m. \qquad (8)$$

The area of the upper part of the hysteresis loop, above the x-axis, is the same as the area of the lower part thus resulting in a total area of the hysteresis loop, which is the energy dissipation, as:

$$Area_t = 2 \cdot Area_1 \quad . \qquad (9)$$

By replacing Eq. (8) in Eq. (9) gives:

$$Area_t = 8\left[h_0\left(A_m - \frac{1}{a_c} + \frac{1}{a_c}\exp(-acA_m)\right) - \frac{h_m A_m}{2}\right]. \qquad (10)$$

2.2. The determination of the area by integrating the virgin curve expression

The hysteresis curve (Fig.2) can be converted to the virgin curve, based on the Masing's rules, by dividing one half of the curve by 2 in the two dimensions which results in the virgin curve (Fig.4).

The area within the virgin curve loop (Fig. 4) is the difference of the integral of the virgin curve expression and its linear representation which is the global contact stiffness expression (see equations below).

$$Area = \int_0^{A_m} f(x)dx - \int_0^{A_m} y(x)dx. \qquad (11)$$

where $f(x)$ is the virgin curve expression and $y(x)$ is the function described by the following expression

$$y(x) = \frac{h_m}{A_m}x. \qquad (12)$$

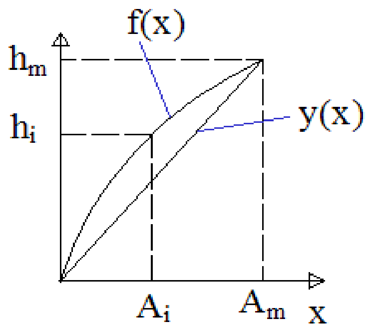

Fig. 4. Schematic representation of the virgin curve area.

By rearranging and integrating Eq. (11) one can get to Eq. (13). According to the Masing's rules, the virgin curve is extended over both dimensions, which means that the area is multiplied by the scaling factor to the power two.

$$Area = 8\cdot\left[h_0\left(A_m - \frac{1}{a_c} + \frac{1}{a_c}\exp(-a_c A_m)\right) - \frac{A_m}{2}\cdot\frac{h_m}{A_m}\right]. \quad (13)$$

Based on the Masing's rules, the resulting friction force can be calculated for any input motion trajectory of the body. This kind of hysteresis is called "hysteresis with non-local memory", since every velocity reversal has to be remembered until an internal loop is closed [4].

Therefore, the area of the virgin curve in function of the amplitude for any distribution of points in the *x* and *y* direction can be determined based on the upper equations

$$Area = \int_0^{A_i} f(x)dx - \int_0^{A_i} y(x)dx . \quad (14)$$

$$y(x) = \frac{h_i}{A_i}x . \quad (15)$$

$$Area = 8\cdot\left[h_0 A_i - \frac{1}{a_c} + \frac{1}{a_c}\exp(-a_c x) - \frac{A_i^2}{2}\cdot\frac{h_i}{A_i}\right]. \ 16)$$

where $A_i = 0, ..., A_m$.

The equivalent damping is given by:

$$c_e\cdot\omega = \frac{Area}{\pi\cdot A_i^2} . \quad (17)$$

in which ω is the frequency, c_e is equivalent damping and A_i is the displacement amplitude. As can be noticed from the equivalent damping equation the amount of energy dissipation per cycle is not dependent on the velocity, as in viscous damping, but is dependent on the amplitude of the motion. In the literature, this type of energy dissipation is called hysteretic damping, solid damping, or structural damping. Another alternative approach is describing functions for determining stiffness and damping given in [4].

The tangential contact stiffness can have two expressions: the first one is the global contact stiffness (19) is the ratio between force and displacement at each instance and the second one, the local contact stiffness, is the derivation of the virgin curve (18) (see related equations below):

$$k_l = h_0\cdot a_c\cdot\exp(-a_c\cdot x) . \quad (18)$$

$$k_g = \frac{h_{virg}}{x} . \quad (19)$$

3. EXPERIMENTAL DETERMINATION OF HYSTERESIS LOOPS

Experimental investigation is performed in order to determine the characteristics of the frictional contact. The measurements are performed on a previously developed tribometer [12] with some minor adjustments. Tribometers can be used for investigating rolling or sliding friction in dry or lubricated contacts. In this paper, the used tribometer is for sliding friction in dry conditions.

Three material pairs are used for the experiments:

- Aluminium on aluminium (Aluminium Alloy 6082);
- Plastic on plastic (TECAVINYL PVC, grey);
- Steel on steel (St 1.1730).

Many experiments were performed for different normal load cases and for different amplitudes, in order to determine the evolution of the frictional force in function of the normal load, the contact stiffness and equivalent damping. The experiments were restricted to the pre-sliding regime.

3.1. Experimental results for aluminium on aluminium

The experimentally determined hysteresis loop and its fit for one material contact pair (Aluminum on Aluminum) and for one normal load, *W* = 109 N, at the frequency, *f* = 1 Hz is shown in Fig. 5. The hysteresis curves are fitted using Eq. (1).

The friction force in function of the relative displacements, or the hysteresis loops, determined experimentally were averaged over the periods in order to eliminate noise and random behaviour effects.

The area of the hysteresis loop is calculated by using two methods: in the first method, the expression of the virgin curve is integrated and in the second method the expression of the outer hysteresis loop is integrated (see related equations below). In Fig. 6 the area as function of normal load and displacement is presented. The equivalent damping $c_e\cdot\omega$ plotted in function of the normal load and displacement is showed in Fig. 7.

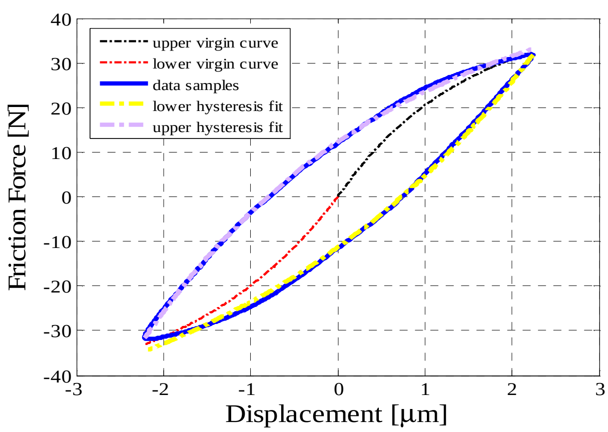

Fig. 5. Hysteresis in pre-sliding:Aluminum on aluminum, *W* = 109 N, *f* = 1 Hz.

In Figs. 8 and Fig. 9 the global contact stiffness and, respectively the local contact stiffness are presented, which were determined based on the Eq. 18 and, respectively Eq. 19 .

One can notice an increase of contact stiffness with the increasing load and a decrease with the increasing displacement amplitude.

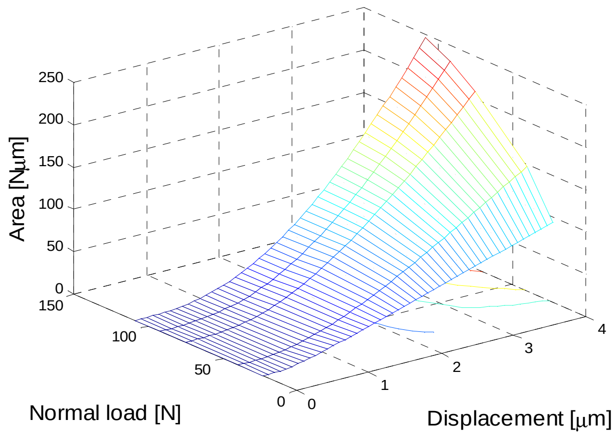

Fig. 6. The area of the hysteresis for all loads.

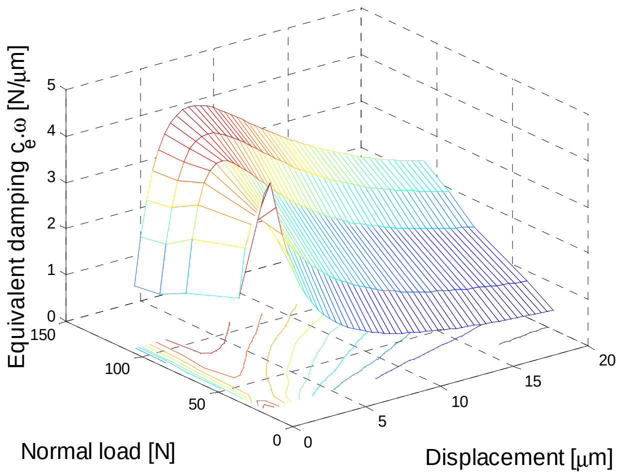

Fig. 7. Equivalent damping in function of normal load and displacement.

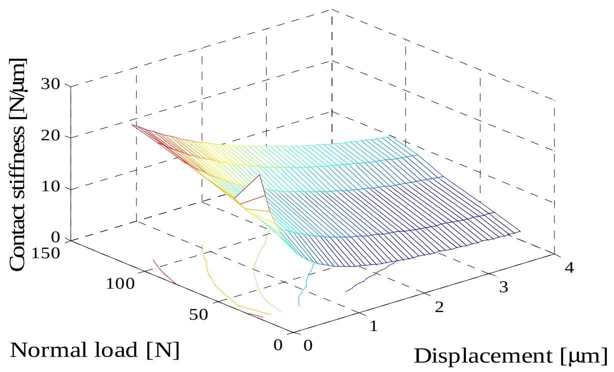

Fig. 8. Global stiffness as function of normal load and displacement.

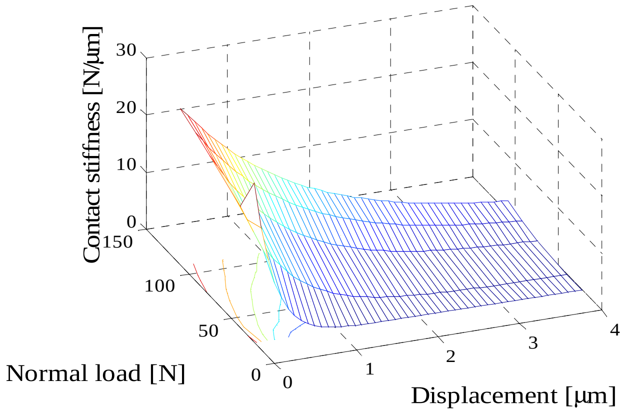

Fig. 9. Local stiffness as function of normal load and displacement.

3.2. Experimental results for PVC on PVC

The results of the experiments for PVC are presented in this section. In Fig. 11 the hysteresis loops area for PVC contacting materials in function of normal load and displacement is presented which has the same increasing trend as in the previous material. Fig. 12 presents the equivalent damping in function of the normal load and displacement. The equivalent damping, defined by Eq. (17), is increasing with the load and decreasing with the displacement amplitude.

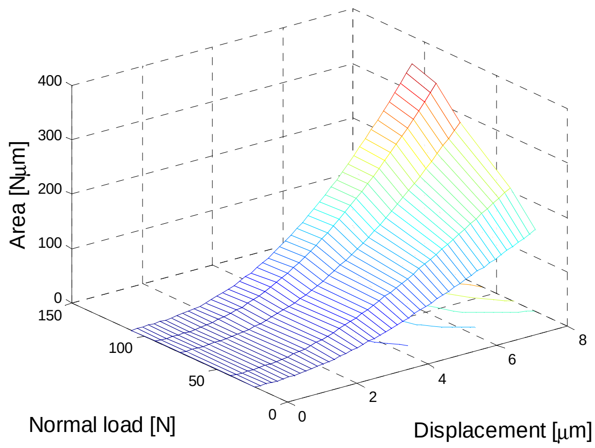

Fig. 11. Area in function of the normal load and displacement.

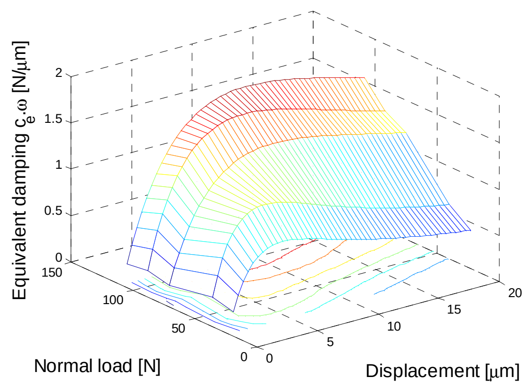

Fig. 12. Equivalent damping in function of normal load and displacement.

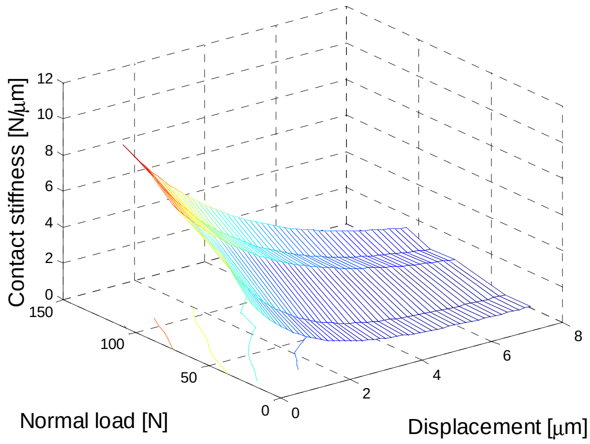

Fig. 13. Local stiffness in function of normal load and displacement.

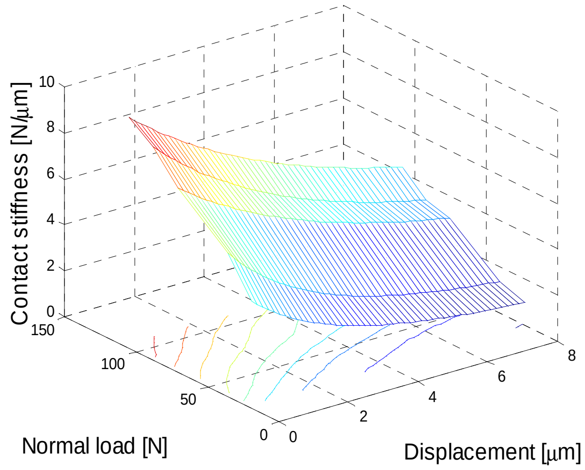

Fig. 14. Global stiffness in function of normal load and displacement.

In Figs. 13 and Fig. 14 the local contact stiffness and global contact stiffness are shown. Also, for this material contact, the tangential contact stiffness is increasing with load and weakening with displacement amplitude.

3.3. Experimental results for steel on steel

The experimental results for steel on steel prove that the system is not powerful enough to perform the friction identification in pre-sliding regime as function of the load conditions. Due to the limitation in the power of the actuator only a small portion of the hysteresis could be identified which is almost behaving as a pure stiffness and hardly any damping is perceived for this amplitude of motion. The highest damping is observed at the lowest load.

In Fig. 15, the fit of hysteresis data of the two bodies and the relative displacement of them are plotted for one normal load W =75 N. It can be seen that for higher loads there is not hysteresis loop due to the stiffness in the contact which is increasing with the normal load.

In Fig. 16 the area enclosed in the hysteresis loop for Steel in function of normal load and displacement is presented. The area is increasing with the normal load and displacement.

Fig. 17 shows the equivalent damping for all loads in function of normal load and displacement. Due to a higher pre-sliding distance, the equivalent damping has almost the same value as for the other two contacting materials.

The local contact stiffness is shown in Fig. 18. The contact stiffness has the same increasing trend with load as for the two contacting materials investigated in this paper.

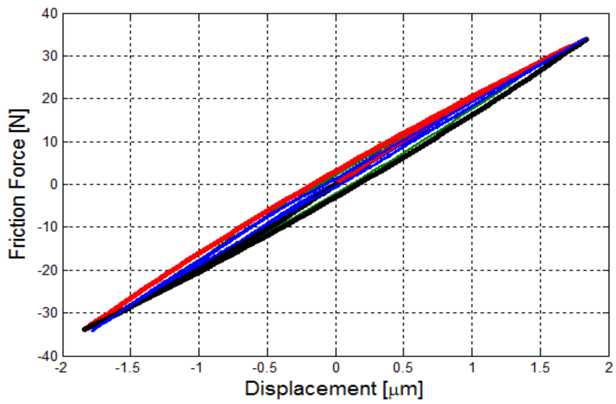

Fig. 15. Fitted hysteresis data: W = 75 N, f = 1 Hz.

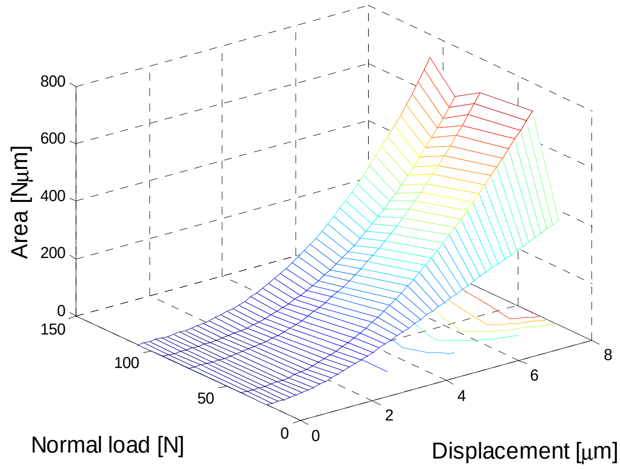

Fig. 16. Area in function of the normal load and displacement.

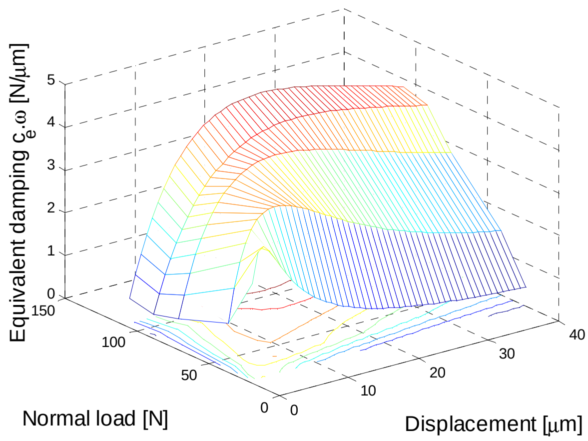

Fig. 17. Equivalent damping in function of normal load and displacement.

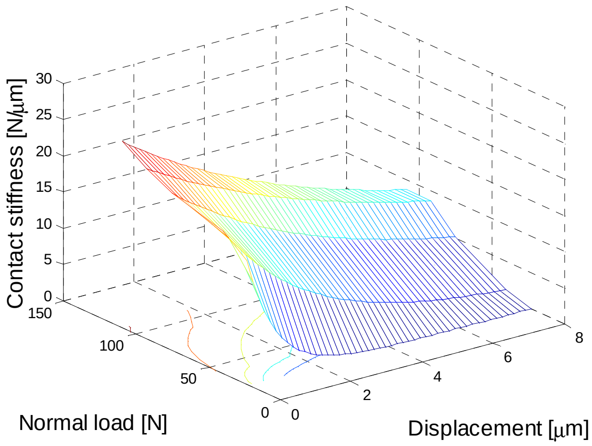

Fig. 18. Local stiffness in function of normal load and displacement.

4. CONCLUSIONS

This paper has considered an existing friction model for the analytical computation of tangential contact stiffness and equivalent damping based on the experimental measurements of three different contact material pairs in a flat on flat frictional dry contact. For this type of configuration, the friction force in function of the displacement and normal load was investigated. A typical hysteresis behaviour was observed which is material, normal load and displacement amplitude dependent.

Two analytical approaches for determining the energy dissipation of hysteresis loops are proposed both of them agree and lead to the same results. The contact stiffness and equivalent contact damping are determined from the virgin curve expression. For all material contacts the contact stiffness is increasing with the load and weakening with the displacement. The equivalent damping shows an increasing trend with the normal load and is also increasing at lower amplitude displacement but decreasing at higher amplitude displacement.

As future work, a theoretical model of the tangential and normal contact stiffness of flat on flat frictional contacts will be developed based on the experimental investigation.

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