Proceedings in Manufacturing Systems, Volume 7, Issue 1, 2012 ISSN 2067-9238

GEOMETRIC AND GENERATION PARTICULARITIES OF THE TOOTH WITH HYPOCYCLOIDAL CURVED FLANKS

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Abstract: *The hypocycloidal flanks of the cylindrical gear teeth have the flank lines defined by lengthened or shortened hypocycloid and the involute profile. The curved and bulged flanks have some advantages, among them being the increased strength in bending and controlled positioning of the contact imprint. The tooth flank generation has on the basis the application of the technological process of milling using a cutting tool with multi-cutters by rolling with mobile line and continuous division. The paper deals with basic aspects regarding kinematics, construction, and running of the multiple cutters milling head for processing teeth with involute profile and hypocycloid at flanks in cylindrical gears. The two curves that define the flank, the involute and hypocycloid, are kinematically and simultaneously generated by rolling. The gears processed by means of the milling head had errors corresponding to the precision steps 8 and 9. The calculation relations and numerical data in the paper contain the basic parameters that define the construction and the adjustment of the tool, generation motions and geometry of the processed teeth.*

Key words: *multiple cutter milling head, cylindrical gear, curved flanks, closed hypocycloid, involute profile, adjust parameters, tooth processing machine, motion parameters, main motion, real cutting speed.*

1. INTRODUCTION

The diversity of gearing types, forms and sizes, precision, productivity, volume and efficiency of production, have determined the existence and utilization of a great number of machine types, processes, and tools for tooth generation.

The increasing exigency regarding the kinematic accuracy, smooth running and load capacity of gearings [6] imposed by their designation leads to the fact that the flank generation to be one of the most complex surface generation modes. Regarding the curved tooth and rack processing as concerns spur gears the technical literature reveals information that proves interest in this field.

The curved teeth in cylindrical gears represent preoccupations of many researchers presented in papers, doctoral thesis and/or patents, among them being [5]. Cylindrical gearing with curved and bulged teeth have been developed and spread from the necessity of increasing the bending resistance and the load capacity of the tooth (25%) in regard with spur gears.

The processing is achieved by milling with monoblock or applied cutters milling heads. The flank lines are circle, helix, cycloid or hypocycloid arcs disposed symmetrical or asymmetrical on the gear width [6].

The cutting process is defined by the cutting tool, generation motion, and method of generation of the involute profile, mode of profile repeating. The main motion is continuous rotation on the trajectory that defines the flank line. The line flank is generated kinematically as trajectory of a point in the mean plane of the generating rack.

Many theoretical and experimental researches supply data of the functioning behaviour of these teeth. It has been proved in many of them that the execution precision and transmissions stiffness are not enough if it is not applied to the flanks some modifications for diminishing the flank and other elements (shafts, bearings, housings) elastic deformation effects.

The hypocycloidal flanks of the gears are defined [3] by two cycloid curves: involute – regarding the profile, and a segment of the closed hypocycloid loop, lengthened or shortened, as flank line. The two curves are generated kinematically through simple motions, strictly correlated by the generation kinematics.

A cutting tool of multiple cutters milling head type subject of a Romanian patent [5] is used. The application of the generation process of the flanks requires a certain kinematic structure of the gearing machine. The machine adjustment is achieved on the basis of some adjusting functions that contain the technological (cutting speed, feed speed), geometric (number of teeth, modulus), and constructive (radius, number of the cutter groups) parametres [7].

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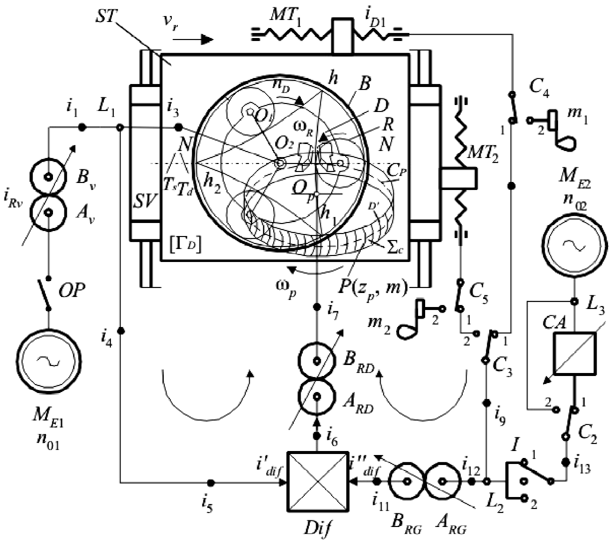

Fig. 1. Generation kinematics of hypocycloidal flank.

2. KINEMATIKS OF HYPOCYCLOIDAL FLANK DIRECTRICE

The flank line D (closed hypocycloid hh_1 – Fig. 1) is generated kinematically in the plane [Γ_D] as trajectory of a point belonging to the rolling curve R, which rolls inside the fixed base B [2]. The flank lines D' are generated by transposing by rolling the plane curves D on the revolution surface Σ_c of the workpiece-gear P. The number of circles R corresponds to the ratio $i_H = R_B / R_R$ (R_B is the base radius and R_R – rolling curve radius).

These are positioned equidistantly inside the base B and revolute simultaneously with the angular speed ω_R. To each rolling curve two cutters are attached by means of a port-tool support. The cutters have principal cutting edges (T_s and T_d) and secondary ones.

The assembly formed by the simple planetary mechanism (the base and the i_H rolling curves) and the corresponding cutter couples attached to each rolling curve constitutes a cutting tool of multi-cutter (i_H =3, 4, 5 or 6) milling head (*CF*) type [5]. The cutting tool can be adapted on the tooth milling machine FD 320A-Cugir [7]. The gearing formed from the base B and the i_H rolling curves R and the revolution couples created in a driving disk D_a with the revolution axis in O_2 and in which the rolling curves axes are materialized corresponding to certain specific accuracy conditions: rigidity, running without noise and vibrations, smooth motion, backlash free and modularization.

The involute profile of the flanks is also generated kinematically [1] as envelope of the successive positions of the two cutting edges T_d and T_s [3], which materialize in the median plane of the plane wheel P and in parallel planes to this one the generating rack profile with the reference line *NN* tangent to the rolling circle C_P of the gear P. The reproduction of the profile is obtained by continuous division [1], in which case the revolution motions of the tool with the speeds ω_R and ω_B and of workpiece ω_P are continuous. Between the three motions and their angular speeds some relations has to be established, which are imposed by the generation kinematics, one of them being of form: $\omega_R = i_H \cdot \omega_D$.

The rolling speed v_r between the workpiece-gear (surface Σ_c) and the plane [Γ_D] expresses the generating speed of the involute profile. The speed v_r is from the technological point of view the speed of the tangential feed motion of the saddle *ST* that supports the tool.

The generation kinematics of the flanks imposes the existence of the three circular motions and linear continuous motion (v_r) supplied by four generating kinematic chains represented in Fig. 1, namely: main, tangential feed, rolling for generation of the curve D, rolling for the involute profile. The trajectories of the generating motions are supplied by simple kinematic couples: shaft-bearing and saddle-guide respectively existent in the kinematic structure of the teeth processing machine.

The main kinematic chain adjusted through the change gears A_v / B_v ensures the motion of each group of cutters on hypocycloidal trajectories. Thus, the cutting edges T_d and T_s effectuate the main motion with the cutting speed tangent to the shortened hypocycloids, respectively elongated, which are flank directrices.

The feed motion with the speed v_r is obtained at the end of the tangential feed kinematic chain consisting of: $M_{E\,2}$ – *CA* – C_2 (pos. 1) – L_2 – C_3 (1) – MT_1 and tangential saddle *ST*.

The rolling kinematic chain for generating the directrix hh_1 in plane [Γ_D] is formed by: *CF* (*Da*) – L_1- *Dif*. - A_{RD}/B_{RD} and workpiece P (joint O_P, n_{Pd}). Hence, a second condition of correlation of the revolution motions is fulfilled, having the form:

$$\varepsilon_{Pd} = \varphi_D \cdot \frac{i_H}{z_p} \text{ [degrees]}, \tag{1}$$

where ε_{Pd} represents the revolution angle of the workpiece for continuous division and φ_D – revolution angle of the driving disk of the milling head *CF*.

The rolling kinematic chain for generation of the involute profile is formed by the generating rack (line *NN*) attached to the plane [Γ_D], and consists of: L_2 – A_{RG} / B_{RG} – *Dif* – A_{RD} / B_{RD} and P (n_{Pr}). The *Botez mechanims* is formed by workpiece-gear and generating rack defined in the mean plane. On the basis of the closing condition of this kinematic chain the following relation is established:

$$\varepsilon_{\mathrm{Pr}} = \frac{\varphi_D \cdot s_T \cdot i_H}{2\pi \cdot z_P \cdot R_r} \text{ [rad]}, \tag{2}$$

where s_T represents the tangential feed of the saddle *ST*, in mm/(rev. P); R_r – rolling radius of the workpiece, in mm; z_P – tooth number of the workpiece. From relations (1) and (2) the revolution angle ε_P of the workpiece results, in radians, corresponding to a certain angle φ_D, in radians, expressed by the relation:

$$\varepsilon_P = \varepsilon_{Pd} + \varepsilon_{\mathrm{Pr}} = \varphi_D \frac{i_H}{z_P}\left(1 + \frac{s_T}{2\pi \cdot R_r}\right) \text{ [rad]}, \tag{3}$$

on which basis the workpiece speed n_P is determined.

The generation motion parameters are considered constant in the cutting process. The rolling motion between workpiece and *CF* is continuous and motion along the flank profile is discontinuous.

3. DEFINING THE REFERENCE PROFILE OF THE COUNTERPART RACK

The reference profile of the counterpart rack is represented in Fig. 2 in the Cartesian co-ordinate system $S_c(x_c,y_c,z_c)$. This profile has two cutting edges relative to the axis $O_c y_c$ of the reference space width. The form of profile is given very generally [8]; it includes a profile correction on reference root (that generates a correspondent tip profile correction at generated wheel) and a protuberance portion at the reference tip. The addendum of tooth of the generating rack tool h_{aP0} is used for the purpose of having the tip radius of the generating rack tool ρ_{aP0} conform to the functional and technological data. The same notations of the particular points *A*, *B*, *C'*, *C*, *D*, *E*, *F*, *G*, *H* are mentioned on the both right and left profiles, but the position of this profile will be considered by means of a specific parameter (factor) k_c. This factor has a value of +1 for the right reference profile and -1 for the left reference profile. The co-ordinates of these points are given by the following expressions (4), (5) and (6):

$$x_{cA} = k_c \frac{p}{2} = k_c \frac{\pi\, m}{2};$$
$$x_{cB} = k_c \left[\frac{\pi}{4} - \frac{pr^*_{P0}}{\cos\alpha_{P0}} + h^*_{aP0} + \rho^*_{aP0} \tan\left(\frac{\pi}{4} - \frac{\alpha_{P0}}{2} \right) \right] m;$$
$$x_{cC'} = x_{cB} - k_c\, \rho^*_{aP0}\, m \cos\alpha_{P0};$$
$$x_{cC} = x_{cB} - k_c\, \rho^*_{aP0}\, m \cos\alpha_{prP0};$$
$$x_{cD} = \frac{-k_c (y_{cE} - y_{cC}) + x_{cC} \cot\alpha_{prP0} - x_{cE} \cot\alpha_{P0}}{\cot\alpha_{prP0} - \cot\alpha_{P0}}; \quad (4)$$
$$x_{cE} = k_c \frac{\pi\, m}{4};$$
$$x_{cF} = x_{cE} - k_c \left(h^*_{fP0} - h^*_{FfP0} \right) m \tan g\alpha_{P0};$$
$$x_{cG} = x_{cF} - k_c\, h^*_{FfP0} \tan\alpha_{FfP0}\, m;$$
$$x_{cH} = 0;$$

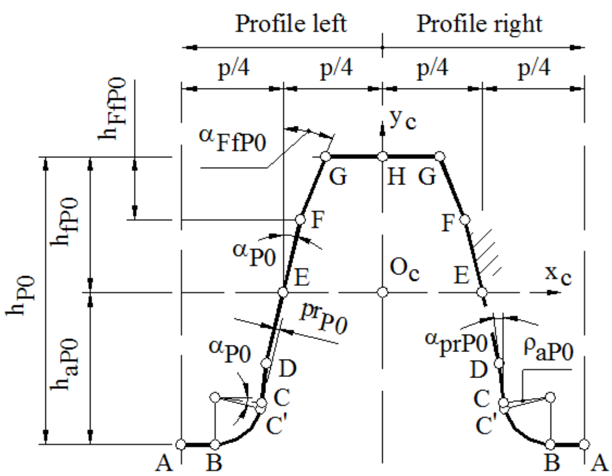

Fig. 2. Geometrical elements of the counterpart rack profiles in the plane $x_c O_c y_c$ of the co-ordinate system $S_c(x_c,y_c,z_c)$.

$$y_{cA} = -h^*_{aP0}\, m;$$
$$x_{cB} = y_{cA};$$
$$y_{cC'} = y_{cB} + \rho^*_{aP0}\, m\, (1 - \sin\alpha_{P0});$$
$$y_{cC} = y_{cB} + \rho^*_{aP0}\, m\, (1 - \sin\alpha_{Pr\,P0});$$
$$y_{cD} = \frac{k_c (x_{cC} - x_{cE}) + y_{cC} \tan\alpha_{prP0} - y_{cE} \tan\alpha_{P0}}{\tan\alpha_{prP0} - \tan\alpha_{P0}}; \quad (5)$$
$$y_{cE} = 0;$$
$$y_{cF} = h^*_{FfP0}\, m;$$
$$y_{cG} = h^*_{fP0}\, m;$$
$$y_{cH} = y_{cG};$$

$$z_c = 0 \quad (6)$$

The independent parameter *u* defines each portion of the profile delimited by the particular points. It is in the range $u \in [0, u_{max}]$, where the maximum value is given by the expression:

$$\begin{aligned} u_{\max} &= k_c (x_{cA} - x_{cB})\, k_1 + \\ &+ \left(\frac{\pi}{2} - \alpha_{prP0} \right) \rho_{aP0}\, k_2 + \\ &+ \sqrt{(x_{cD} - x_{cC})^2 + (y_{cD} - y_{cC})^2}\; k_3 + \\ &+ \sqrt{(x_{cF} - x_{cD})^2 + (y_{cF} - y_{cD})^2}\; k_4 + \\ &+ \sqrt{(x_{cG} - x_{cF})^2 + (y_{cG} - y_{cF})^2}\; k_5 + \\ &+ k_c (x_{cG} - x_{cH})\, k_6, \end{aligned} \quad (7)$$

The values of the selective parameters k_I are given in Table 1.

Using these notations, the co-ordinates of the reference profiles of the counterpart rack are:

$$\begin{aligned} x_c = x_c(u^*) &= k_c[(x_{cA} - u^* \cdot m)\, k_1 + \\ &+ \left(x_{cB} - \rho^*_{aP0} \cdot m \cdot \sin\frac{u^*}{\rho^*_{aP0}} \right) k_2 + \\ &+ \left(x_{cC} - u^* \cdot m \cdot \sin\alpha_{prP0} \right) k_3 + \\ &+ \left(x_{cD} - u^* \cdot m \cdot \sin\alpha_{P0} \right) k_4 + \\ &+ \left(x_{cF} + u^* \cdot m \cdot \sin\alpha_{FfP0} \right) k_5 + \\ &+ \left(x_{cG} + u^* \cdot m \right) k_6]; \end{aligned} \quad (8)$$

Table 1

Selective parameters values

Part of the profile (Fig. 2)	k_1	k_2	k_3	k_4	k_5	k_6
AB	1	0	0	0	0	0
BC	0	1	0	0	0	0
CD	0	0	1	0	0	0
DF	0	0	*0*	1	0	0
FG	0	0	0	0	1	0
GH	0	0	0	0	0	1

$$\begin{aligned} y_c = y_c(u^*) = y_{cA}\, k_1 &+ \\ +\left[y_{cB} + \rho^*_{aP0}\, m \cdot \left(1-\cos\frac{u^*}{\rho^*_{aP0}}\right)\right] k_2 &+ \\ +\left(y_{cC} + u^*\cdot m\cdot \cos\alpha_{prP0}\right) k_3 &+ \\ +\left(x_{cD} + p^*\cdot m\cdot \cos\alpha_{P0}\right) k_4 &+ \\ +\left(x_{cF} + u^*\cdot m\cdot \cos\alpha_{FfP0}\right) k_5 &+ \\ + y_{cG}\, k_6&; \end{aligned} \qquad (9)$$

$$z_c = z_c(u^*) = 0. \qquad (10)$$

4. MULTICUTTER MILLING HEAD ADAPTED ON THE GEARING MACHINE FD

This has on the basis the generation kinematics of the hypocycloid curve generated in the plane Γ_D.

The tangential speed vector to the hypocycloidal trajectory v_H is given by relation

$$|v_H| = \sqrt{v_{Hx}^2 + v_{Hy}^2}\,, \qquad (11)$$

where $v_{Hx} = \frac{\mathrm{d}X_D}{\mathrm{d}\varphi_D}\cdot\frac{\mathrm{d}\varphi_D}{\mathrm{d}t}$ and $v_{Hy} = \frac{\mathrm{d}Y_D}{\mathrm{d}\varphi_D}\cdot\frac{\mathrm{d}\varphi_D}{\mathrm{d}t}$.

The final form of speed v_H is

$$|v_H| = 2\pi\cdot n_D\cdot(R_B - R_R)\cdot\sqrt{1+\left(\frac{R_M}{R_R}\right)^2 + 2\frac{R_M}{R_R}\cdot\cos\frac{R_B}{R_R}\varphi_D}\,. \qquad (12)$$

The instantaneous direction of the vector $\bar{v}_H$ depends on angle φ_D and is expressed by its director cosines. The vector $\bar{v}_H$ is tangent to the hypocycloid loop generated in point M_{ds} and having the direction of the rotation motion n_D.

5. TOOL CUTTING EDGE GEOMETRY

The linear cutting edges of the tool wedges are conjugated curves to generated curve (involute). The involute profile is generated as envelope of the successive positions of the cutting edge (method of generation through tangents).

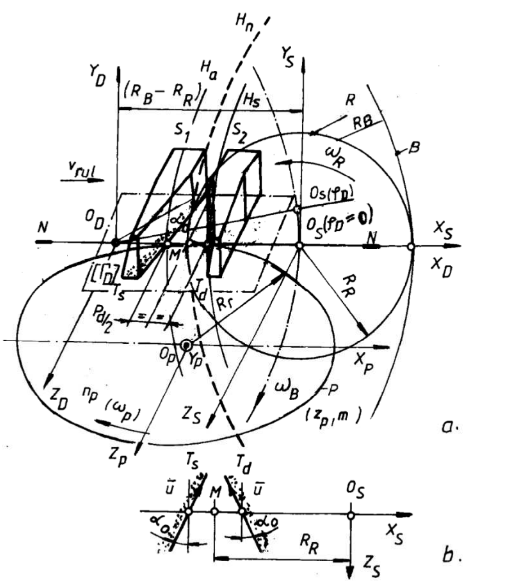

Fig. 3. Cutters adapted on the milling head rolling curve.

The cutting tool of milling head type with multiple cutters *CF* consists of a group of cutter couples (S_1 and S_2) that equals to $i_H = R_B/R_R = 3$, 4, 5 or 6, this aspect representing an innovation in this field. Each cutter group is attached to a rolling curve *R* (Fig. 3) that rolls inside the basis *B* adapted through a device to the saddle S_T of the hobbing machine, which has a tangential feed motion. During a rotation of O_S in regard with O_{CF}, each rolling curve describes i_H full rotations, the groups of cutters getting in contact with a consecutive corresponding number of workpiece teeth.

5.1. Elements regarding construction and grinding of milling head cutters

Each cutter has three wedges. One of them corresponds to the cutting edge $T'_{s,d}$ for roughing and one to the cutting edge $T_{s,d}$ for finishing. The cutting edge on the point is for roughing and generates the root surface of the tooth. The cutting edges T_s and T_d of the cutters generates the flanks of a clearance of the generating rack. The edge T_d generates the concave flank and T_s generates the convex one. Therefore, their active angles are different.

Each cutter group processes in two closed clearances that delimitate the tooth flanks.

The cutting edges belong to the rack angle plane A_γ.

The hypocycloid $h'Mh'_1$ (Fig. 4) generated in the plane S_D results out-of-phase with the angle φ_0 in regard with the hypocycloid hMh_1 (the geometric one) [1]:

$$\tan\varphi_0 = \frac{(R_B-R_R)\cdot\sin\varphi_D - \left[-\left(R_R + k_{II}\cdot\frac{m\pi}{4}\right)\cdot\sin\frac{R_B-R_R}{R_R}\varphi_D\right]}{(R_B-R_R)\cdot\cos\varphi_D + \left[-\left(R_R + k_{II}\cdot\frac{m\pi}{4}\right)\cdot\cos\frac{R_B-R_R}{R_R}\varphi_D\right]}. \qquad (13)$$

The angle φ_0 is the parameter of out-of-phase of the active loops of the hypocycloid and is used for determining the correction angle (adjusting angle) of the milling head.

For determining the angle φ_0 for hypocycloids generated by points belonging to the cutting edges, a calculation program was achieved and run. The followings were considered: $i_H = 3$, 4, 5, and 6, $m = 1.5$; 2.0; 2.5; 3.0, and 4.0 mm.

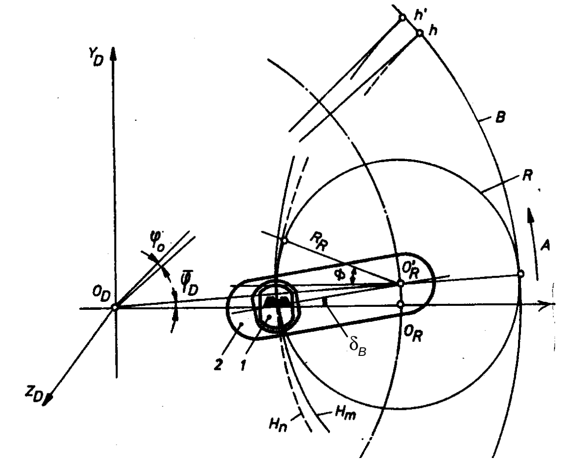

Fig. 4. Kinematics of the two cutters in tooth generation.

For the milling head adjustment, the rotation center O_R of the rolling curve R was positioned (Fig. 4) in regard with the axis $O_D X_D$ (direction of the tangential feed) with distance $O_R D'_R = 2(R_P - R_R)\sin\frac{\overline{\varphi}_0}{2}$, in mm.

At the same time, the basis B rotation in direction of arrow A with the angle δ_B must be achieved considering the relation

$$\delta_B = \frac{1}{i_H}\left(\frac{R_B - R_R}{R_R}\zeta + \arctan\frac{O_R D'_R}{R_R}\right), \text{ in degrees.} \quad (14)$$

For the mentioned data the angle δ_B varies between 5' and 42'.

Further, for bringing the four cutting edges of the cutters in the plane $X_D O_D Z_D$, the port-cutters 1 is rotated in regard with the plate 2 of the element attached to the rolling curve R. This last adjustment is achieved for each group i_H of cutters.

The length of the tangential saddle stroke is given by relation:

$$L_{tg} = m\left(\pi + h_{gf}\cdot\cot\alpha_0\right) + \frac{1}{2}\sqrt{\left(z_p + 2\right)^2 - \left(z_p\cdot\cos\alpha_0 - 2c_0^*\right)^2}, \quad (15)$$

where h_{gf} = 1.25 and $c_0^* = 0.25$.

For rapid establishing of the stroke length, the diagram shown in Fig. 5 was achieved on the basis of relation (15).

Figure 5 shows the milling head having four groups of cutters adapted on the machine FD 320A. The machine enables the generating motion executed by the tool and workpiece-gear and the interconnection between these motions. The machine kinematics supplies two rolling kinematic chains for generating on kinematic way the flank line and profile, and also their adjustment with change wheels. The i_H groups of cutters of the milling head that follow hypocycloidal trajectories achieve the main motion. This motion is obtained as output of the main kinematic chain driven by the electric motor ME_1.

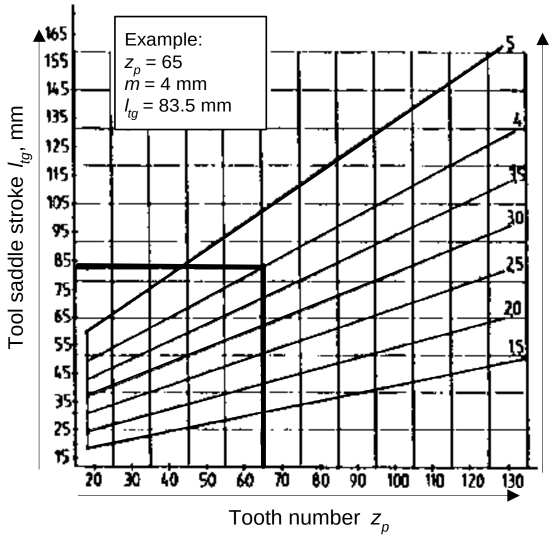

Fig. 5. Diagram for rapid establishing of the stroke length (l_{tg}).

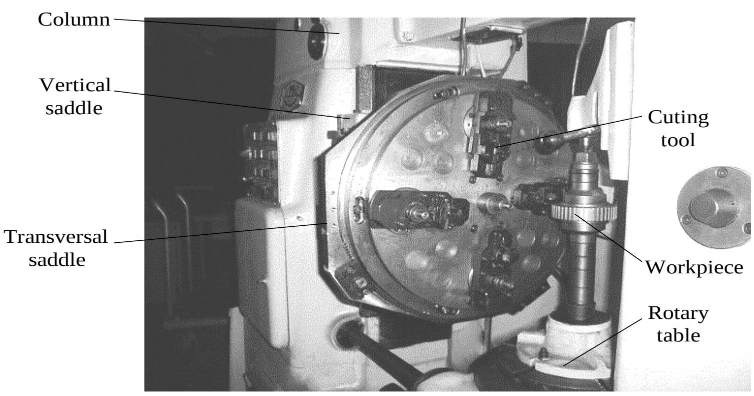

Fig. 6. Milling head with 4 groups of cutters.

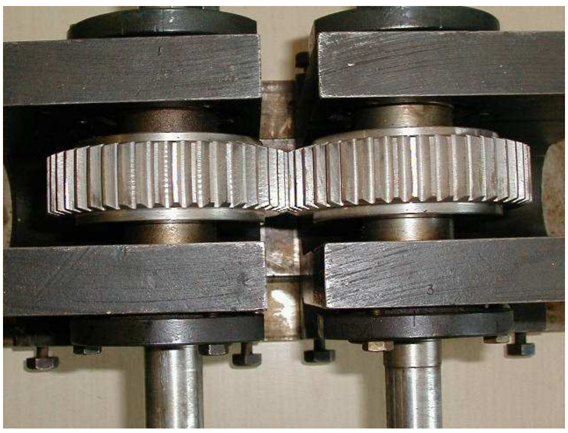

Fig. 7. Gearing with polyhypocycloidal teeth processed using the milling head.

In Fig. 7 a cylindrical gear having curved gearing teeth is presented. The contact spot is formed in the center of the adjoined flanks.

5.2. Machine kinematic chain adjustments

The adjusting function of the main kinematic chain (Fig. 8) has the form:

$$\frac{A_V}{B_V} = C_V \frac{n_R}{i_H}, \qquad (16)$$

where C_V = 1/54.282 is the kinematic chain constant.

For the constant C_V calculation the transmission ratii i_1, …, i_9 on the diagram (Fig. 8) are presented in Table 2. The speed of the rolling curves is adjusted in 32 steps in the field 72, …, 720 rot/min (FD 320A machine) corresponding to the four values of the ratio $i_H = i_{10}$ = 3, 4, 5 or 6.

The rolling kinematic chain for kinematic generation of the hypocycloidal directrice of the flanks, considered between milling head and workpiece, has the adjustment function

$$\frac{A_{RD}}{B_{RD}} = C_{RD} \frac{i_H}{z_p} \frac{f}{e}, \qquad (17)$$

where $C_{RD} = 9.6$ and e / f = 36/6 or 24/48.

Table 2

Transmission ratii

Nr. crt.	1	2	3	4	5	6	7	8	9
Tooth number	125/ 168	24/ 45	24/ 41	40/ 40	44/ 35	24/ 24	24/ 24	28/ 88	12/ 30
Nr. crt.	10	11	12	13	14	15	16	17	18
Tooth number	3;4; 5;6	i_{dif}	1/60	28/ 59	1/24	28/ 28	15/ 15	21/ 20	21/ 21
Nr. crt.	19	20	21	22	23	24	25		
Tooth number	1/24	67/ 55	18/ 36	27/ 27	35/ 48	29/ 29	1/40		

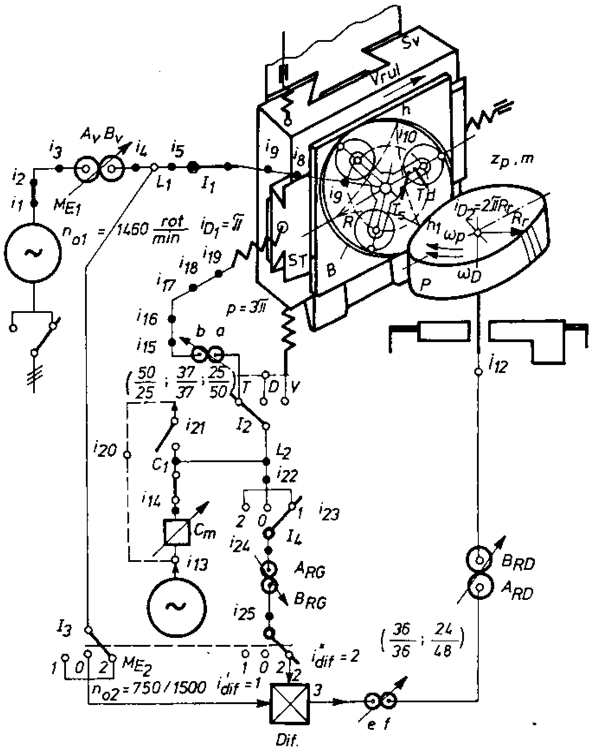

Fig. 8. Structural diagram of the machine and milling head.

The rolling kinematic chain for kinematic generation of the involute generatrice of the flanks, constituted also between the tool and workpiece, has the adjustment function

$$\frac{A_{RG}}{B_{RG}} = C_{RG} \frac{1}{i_H \cdot m} \frac{A_{st}}{B_{st}}, \qquad (18)$$

where $C_{RG} = 7.5$ and A_{st}/B_{st} = 2; 1 and 1/2.

The tangential feed kinematic chain driven by the electric motor ME_2 is adjusted through the feed gearbox C_a. The mechanism C_a enables 12 steps of feed speeds in the field w_T = 0.25, …, 22.4 mm/min. Corresponding to

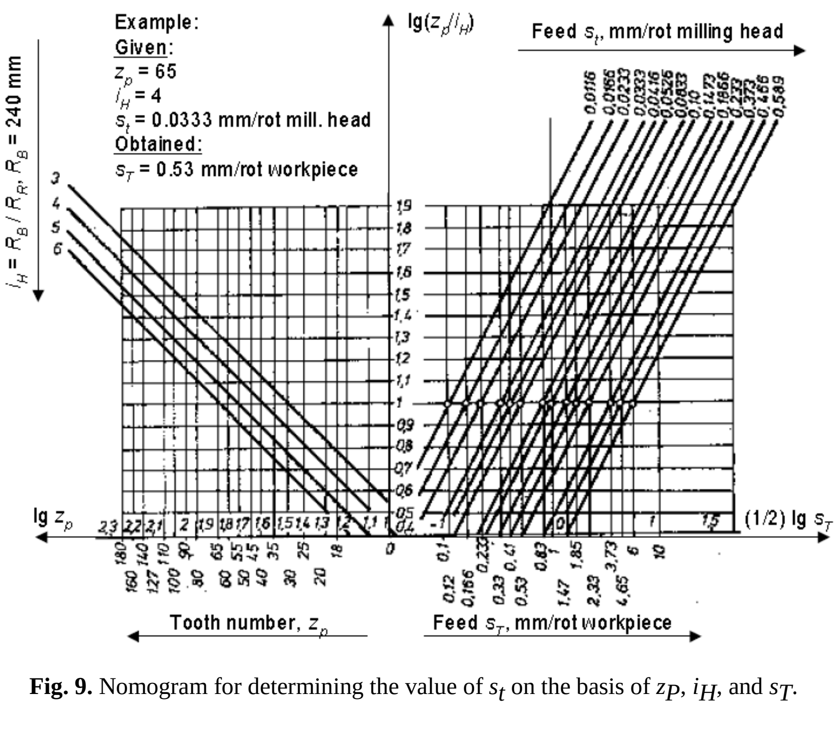

Fig. 9. Nomogram for determining the value of s_t on the basis of z_P, i_H, and s_T.

the eight speed steps of the milling head, values in the field 0.0104, …, 0.1866 mm/rot of milling head results for the feed s_T in 96 steps.

Knowing the parameters z_P, i_H, and s_T the feed gearbox and change wheels are adjusted for obtaining the proper value of the feed speed w_T.

The radial speed kinematic chain is driven manually or hydro-mechanically, the motion being transmitted to the saddle S_P by means of the screw-nut mechanism S_r.

Using this milling head as experimental model adapted on the machine FD 320A, one can process gears with external diameter between 50 mm and 125 mm having modulus between 1.5 mm and 4 mm.

The method and milling head were tested in the Machine Tools Laboratory of the Machine and Cutting Tools Department, University "Politehnica" of Bucharest. The milling head could be adapted on other tooth processing machines of the FD family [4].

The displacement of the tangential saddle of the machine on which the milling head is adapted, is done with a fedd continuous lienar motion.

The feed is considered $s_{\varphi D} = \frac{\varphi_D}{2\pi} \cdot s_{(\varphi_D = 2\pi)}$ cores-sponding to the disk rotation with a certain angle φ_D, in rad. For a complete rotation of the milling head (φ_D = = 2(rad) i_H workpiece teeth are processed, while the saddle moves with the feed $s_t = \frac{s}{\varphi_D} = 2\pi$ mm/rot. of milling head. If the milling head achieves z_p / i_H revolutions, the cutter groups take contact with the z_P teeth of the workpiece.

Therefore, the saddle feed is expressed by relation

$$s_T = s_t \frac{z_P}{i_H}, \text{ mm/rot of workpiece,} \tag{19}$$

and is the repeated position value along the reference lines of the generating rack and corresponding to a workpiece revolution.

On the basis of relation (19), the nomogram shown in Fig. 9 was realised. Knowing the parameters z_P, i_H, and s_T, the value of feed s_t can be easily determined. This is necessary in some technological calculations and for establishing the value of the profile error (roughness).

The adaptation of the process and milling head on the tooth processing machine of FD–Cugir and Phauter family is done without significant kinematic or constructive modifications [3]. The milling head is considered as a machine device.

6. MODELING AND SIMULATION OF THE DEVICE ADAPTED ON MACHINE FD 320 A

The milling head was designed and executed as an experimental model in four variants having i_H = 3, 4, 5, and 6 [4]. This enables processing gears (precision class 8) with diameters between 50 and 125 mm and modulus m = 1.5, …, 3.5 mm.

For gearing running in gear testing of the processed cylindrical gearing with polyhypocycloidal teeth, a stand with open mechanical energetic flow was designed and used. The researches emphasized the formation and position of the contact spot and the level of noise. The milling head adapted on the tooth processing machine was modelled as a multi-body system in order to emphasize the kinematic and dynamic behaviour.

On the main spindle of machine (Fig. 10) a shaft it was mounted supporting a bevel gear in contact with another one on a perpendicular shaft (driving spindle II, ratio 1:5). This is the shaft that moves the driving disk D carrying the satellite axes (III, IV, and V) on which the satellites rotates – spur gears in contact with the fixed crown. Together with the satellite axes there are the port-tools devices, on which the couples of cutters are mounted and adjusted in position to reduce the error factor. The bevel gear meshing was modelled as a joint of rolling without sliding type (cone on cone for the bevel gearing or cylinder in cylinder for the cylindrical gearing). The model tree is an open chain.

As input at the main spindle an imposed motion was supplied, which corresponds in rad/s to the chosen cutting speed vc = 58.3 m/min ∈ (56.95 m/min, 72.1 m/min). This goes to a torque at the main spindle of 70 Nm. The cutting force was considered 100 N acting about full contact position between tool and workpiece in the range (−3°, +3°).

During simulation, the program provides information concerning positions, velocities, accelerations, point trajectories, the forces and moments applied to the articulations, the energies, as well as other data concerning the system, pre-defined by the software or defined by the user [9]. In Fig. 11 the torque variation on the body 1 (disc D) at the cutting impact is shown. The maximum torque is 126.8 Nm, with an increase of 77 Nm. The variation of cutting speed at the cutting contact (*vctot*) is shown in Fig. 12.

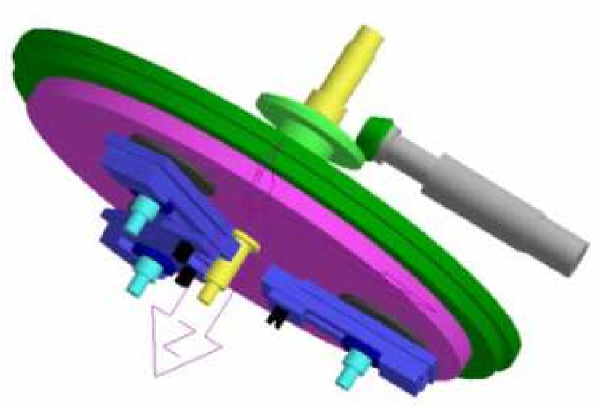

Fig. 10. Milling head with three groups of cutters.

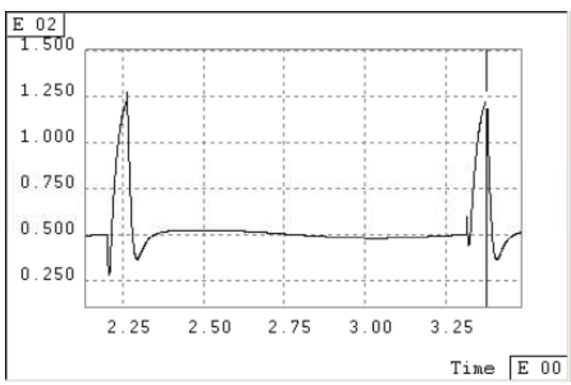

Fig. 11. Variation of moment on the disc D axis (body 1) at the cutting impact.

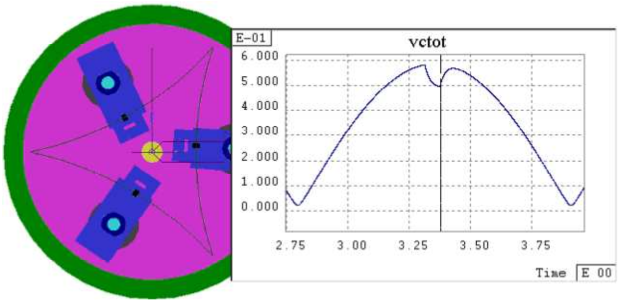

Fig. 12. Image with hypocycloid and variation of cutting speed at the contact tool-workpiece.

7. CONCLUSIONS

The tooth generated with the described milling head is involute with curved hypocycloidal flanks in cylindrical gear. The flank line and profile of the hypocycloidal curved teeth are generated kinematically by rolling with mobile line and continuous division. For processing a milling head *CF* is used, which has more couples of cutters. This tool is adapted on the tooth processing machine of FD Cugir type.

The paper presents the necessary motion for flank generation, parameters that define the motions, milling head kinematics, the kinematic chains of the tooth processing machine on which the milling head was adapted, adjusting possibilities of the machine and milling head.

The parameters that define the generated flank form are grouped as follows: technological (feed), constructive and of motion.

The calculation relations and numerical data in the paper contain the basic parameters that define the construction and the adjustment of the tool, generation motions and geometry of the processed teeth.

The gears processed by means of the milling head had errors corresponding to the precision steps 8 and 9.

The kinematic structure of the machine for polyhypocycloidal teeth processing is based on the kinematic process generation of the tooth flanks. A milling head with cutters arranged in 3, 4, 5 or 6 groups of two was used. There are four generating kinematic chains in the machine structure. The kinematic accuracy of the gearing of the milling head, disposing of the i_H groups of cutters, precision of adjusting, and rigidity of the kinematic structure of the machine determine the processed gear accuracy.

The design of the multi-cutter milling head has on its basis a simple planetary mechanism of high accuracy. For diminishing the errors some kinematical, constructive and adjusting solutions were designed. The 3D model of the milling head was achieved in SolidDynamics program for obtaining the kinematic and dynamic behaviour of the device under variable loads given by the cutting force.

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