PROFILING OF THE PLANING TOOL FOR CYCLOIDAL WORMS MACHINING – CAD METHOD VERSUS ANALITYCAL METHOD

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Abstract: *Cycloidal worms are helical-cylindrical surfaces, having a constant pitch. They are used in helical pumps construction, usually as driving screws. This paper introduces a new method for profiling a helical-cylindrical surface, reciprocal enwrapped to the cycloydal worm flanks, developed under CATIA graphical environment. New method quality is assessed by comparing the results of its application to the ones obtained in same purpose by using a numerical method, grounded on the fundamental theorems of the enwrapped surfaces. The comparison is illustrated through numerical examples.*

Key words: *planning tool profiling, CAD method, analytical method, cycloidal worm machining, helical cylindrical surface.*

1. INTRODUCTION

Cycloidal worms are machine parts that represent the working elements of the cycloidal pumps. The number of threads for this type of worms is usually two; however, there are structures when pump driving (cycloidal) worm has more threads.

The problem of profiling the tools used for generating a cycloidal worm draws on surfaces generation by enwrapping problematic, depending on the tool type: disc tool, worm tool, planing tool [1−3, 9]. There are known and frequently applied in this purpose fundamental analytical methods [1] as well as complementary analytical methods [3, 4].

The development of graphical designing environments of CAD type enabled to approach the profiling of tools used to generate the above-mentioned kind of surfaces by specific means [8, 10, 11]. By following this direction, the evolution of CATIA environment opened new perspectives for designing tools generating by enwrapping [5−7].

This paper presents a comparative analysis between the results found when profiling a cylindrical tool (planning tool), used for machining a cylindrical cycloidal worm from a helical pump, by successively applying an analytical method and a CAD method.

2. CYCLOIDAL WORM GEOMETRY

2.1. Reference systems

Fig. 1 illustrates the modality of generating the helical flank of the cycloidal worm.

According to notations from the picture, the following reference systems are considered:
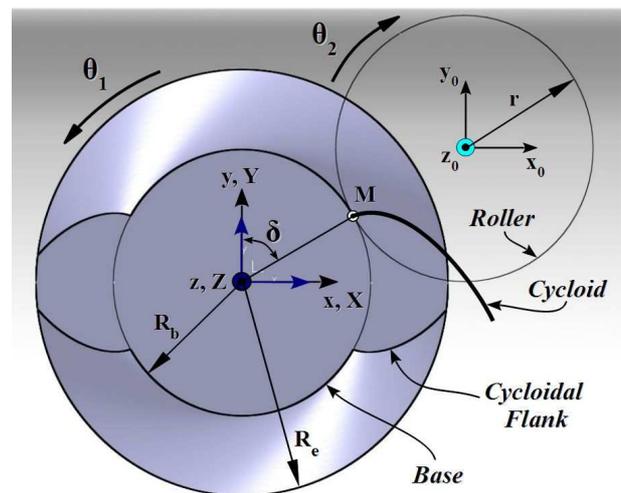
Fig. 1. The cycloidal flank.

- xyz – global reference system, attached to the base (circle of R_b radius).
- $x_0y_0z_0$ – global reference system, with its origin in the roller center.
- XYZ – relative reference system, attached to the cycloidal flank, and initially overlaid to xyz system.
- $X_0Y_0Z_0$ – relative reference system, attached to the generator point from the roller (circle of r radius).

2.2. Equations of the cycloidal worm flank

If θ_1 and θ_2 are the angular parameters characterizing the rotation motions around base center (θ_1), and roller center (θ_2), then due to the rolling without slipping taking place between the two circles the following condition results:

$$R_b \cdot \theta_1 = r \cdot \theta_2 . \qquad (1)$$

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By keeping in view Fig. 1, the relative position between the two fix reference systems is given through

$$x_0 = \alpha(x - a),\qquad(2)$$

where $\alpha = I_3$, because they have parallel and identically oriented axis. The matrix defining the position of $x_0y_0z_0$ system origin relative to xyz system is:

$$a = \left\| \begin{matrix} (R_b + r)\sin\delta \\ (R_b + r)\cos\delta \\ 0 \end{matrix} \right\|;\qquad(3)$$

δ meaning an angular parameter, imposed by constructive reasons.

Kinematics of cycloidal profile generation imposes to correlate two rotation motions:

$$x = \omega_3{}^T(\theta_1)\cdot X\qquad(4)$$

and

$$x_0 = \omega_3{}^T(-\theta_2)\cdot X_0 .\qquad(5)$$

Relation (4) regards base motion, referred to xyz system, while relation (5) – roller motion, referred to $x_0y_0z_0$ system.

If now in relation (2) we introduce relations (3), (4), and (5), then the equation of relative motion between roller and base results as

$$x = \omega_3(\theta_1)[\omega_3(-\theta_2)\cdot X + a].\qquad(6)$$

A generic point from the roller, M (see also Fig. 1), having the co-ordinates:

$$M \left| \begin{matrix} X_0 = -r\cdot\sin\delta; \\ Y_0 = -r\cdot\cos\delta; \\ Z_0 = 0, \end{matrix} \right.\qquad(7)$$

is generating, during (6) motion, the cycloid:

$$C \left| \begin{matrix} X^C = -r\cdot\sin(\theta_1 + \theta_2 + \delta) + (R_b + r)\sin(\theta_1 + \delta); \\ Y^C = -r\cdot\cos(\theta_1 + \theta_2 + \delta) + (R_b + r)\cos(\theta_1 + \delta); \\ Z^C = 0. \end{matrix} \right.\qquad(8)$$

The surface of worm helical flank results by giving to the cycloid the helical motion of Z-axis and p helical parameter:

$$\left\| \begin{matrix} X \\ Y \\ Z \end{matrix} \right\| = \omega_3{}^T(\varphi)\cdot \left\| \begin{matrix} X^C \\ Y^C \\ Z^C \end{matrix} \right\| + p\cdot\varphi\cdot\vec{k} .\qquad(9)$$

Here φ means a variable angular parameter, while the vector including X^C, Y^C and Z^C – the parametric equations of the helical flank.

If we define the transmission ratio as

$$i = \frac{\theta_2}{\theta_1},\qquad(10)$$

then, after developing, the helical flank Σ equations become:

$$\Sigma \left| \begin{matrix} X = -r\cdot\sin[(1+i)\theta_1 + \delta - \varphi] + (R_b + r)\sin(\theta_1 + \delta - \varphi); \\ Y^C = -r\cdot\cos[(1+i)\theta_1 + \delta - \varphi] + (R_b + r)\cos(\theta_1 + \delta - \varphi); \\ Z^C = p\cdot\varphi. \end{matrix} \right.\qquad(11)$$

The rest helical flanks with cycloidal generatrix result similarly, obviously after considering the generatrix appropriate position.

3. ANALYTICAL METHOD FOR CYLINDRICAL TOOL PROFILING

The condition to be applied for finding the characteristic curve (the curve of tangency between the cylindrical surface, generated by the cutting edge of the planing tool, and the helical surface, according to Gohman theorem [1] is

$$\overrightarrow{N_\Sigma}\cdot\dot{t} = 0 .\qquad(12)$$

In relation (12) $\overrightarrow{N_\Sigma}$ means the normal in the current point of Σ surface, while $\dot{t}$ is the versor of cylindrical surface generatrix, defined as versor of the tangent to the helical line from Σ surface (11), corresponding to the cylinder having R_e radius (see also Fig. 2).

Normal $\overrightarrow{N_\Sigma}$ equation issues from (11), as

$$\overrightarrow{N_\Sigma} = p\cdot\dot{Y}_{\theta1}\cdot\vec{i} - p\cdot\dot{X}_{\theta1}\cdot\vec{j} + (\dot{X}_{\theta1}\dot{Y}_\varphi - \dot{X}_\varphi\dot{Y}_{\theta1})\cdot\vec{k} ,\quad(13)$$

where:

$$\begin{aligned} \dot{X}_{\theta1} &= -r(1+i)\cos[(1+i)\theta_1 + \delta - \varphi] + \\ &\quad + (R_b + r)\cos(\theta_1 + \delta - \varphi); \\ \dot{Y}_{\theta1} &= r(1+i)\sin[(1+i)\theta_1 + \delta - \varphi] - \\ &\quad - (R_b + r)\sin(\theta_1 + \delta - \varphi); \\ \dot{X}_\varphi &= r\cdot\cos[(1+i)\theta_1 + \delta - \varphi] - (R_b + r)\cos(\theta_1 + \delta - \varphi); \\ \dot{Y}_\varphi &= -r\sin[(1+i)\theta_1 + \delta - \varphi] + (R_b + r)\sin(\theta_1 + \delta - \varphi). \end{aligned}\qquad(14)$$
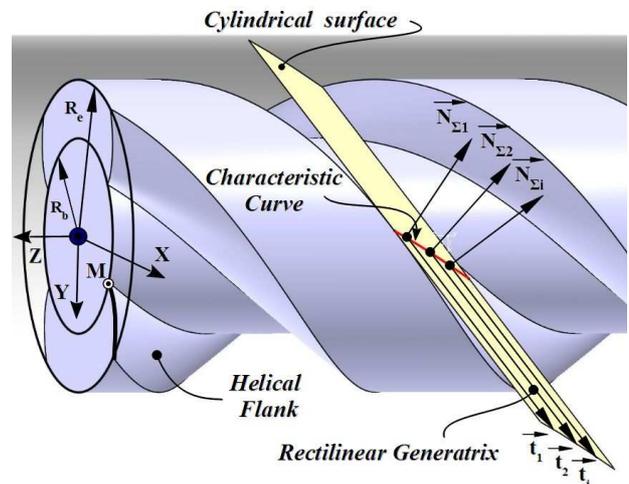
Fig. 2. Cylindrical surface generatrix & helical flank normals.

The helix corresponding to the cylinder having R_e radius can be found by using the last two equations from (14), when θ_1 becomes θ_{1max}, calculated as

$$\theta_{1max} = \frac{r}{R_b} \cdot arccos\left[\frac{(R_b + r)^2 + r^2 - R_e^2}{2(R_b + r)r}\right]. \quad (15)$$

Thus, $\dot{t}$ versor equation is

$$\dot{t} = \dot{X}_{\varphi|\theta 1max} \cdot \vec{i} + \dot{Y}_{\varphi|\theta 1max} \cdot \vec{j} + p \cdot \vec{k}. \quad (16)$$

It follows that the condition for finding the characteristic curve on Σ surface (11) results, in principle, by having in view the relations (12), (13), (14), (15) and (16) as

$$q(\theta_1, \varphi) = 0, \text{ or } \varphi = \varphi(\theta_1). \quad (17)$$

The ensemble formed by joining relations (11) and (17) gives the characteristic curve shape, referred to *XYZ* system, having, in principle, the form:

$$C_\Sigma \left| \begin{array}{l} X_C = X(\theta_1); \\ Y_C = Y(\theta_1); \\ Z_C = Z(\theta_1). \end{array} \right. \quad (18)$$

4. 3-D CAD METHOD FOR CYLINDRICAL TOOL PROFILING

The 3-D method to profile the disc-tool uses the facilities of *Generative Shape Design*, *Assembly*, and *Kinematics* modules from CATIA environment.

We started from the simplifying hypothesis that the cylindrical tool could be considered as a revolution surface of very large diameter. The 3-D CAD method was developed in the case of a cycloidal worm like the one depicted in Fig. 1. The worm has two starts, and its generatrix, in the frontal plane, are the cycloids generated by a point from the roller of radius r, rolling onto the base circle of radius R_b.

By starting from here, the two elements defining the cycloid – Base and Roller – were modeled in CATIA designing environment, with the help of specific commands from the *Generative Shape Design*. Model parameters are (as denoted in Fig. 1):

- Base circle radius, R_b.
- Roller radius, r.
- Worm exterior radius, R_e.
- Worm helix pitch, p.
- Angular parameter, δ, defining the position of cycloid starting point onto the base circle.

The mechanism that generates the cycloidal curve was then created in *Assembly* & *Kinematics* environment. This is possible with the help of *Trace* command, by tracking the trajectory of cycloid start point M, relative to the reference system of *Base* element. The entire surface of cycloidal worm is then modeled by using this curve.

The *X*-axis of the worm reference system (considered at an infinite distance along *Y*-axis) is projected onto the helical flank surface, by using *Projection* command, in order to obtain the characteristic curve (see Fig. 3).
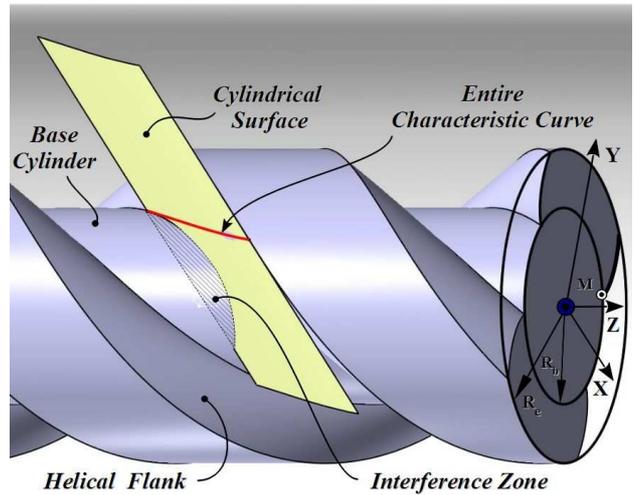
Fig. 3. The characteristic curve and the peripheral surface of the cylindrical tool – 3-D CAD method.

The primary peripheral surface of the cylindrical tool is modeled through *Extrude* command, in the direction of the planing tool system *X*-axis, Fig. 3.

We should notice that the peripheral surface of the cylindrical tool frequently intersects the surface of the cycloidal worm base (meaning surfaces interference). Therefore it is necessary to eliminate from the characteristic curve the section among the tangency point with the worm base cylinder and the intersection point between the tangent to this cylinder and the cycloidal flank. The procedure for characteristic curve trimming is illustrated in Fig. 4.
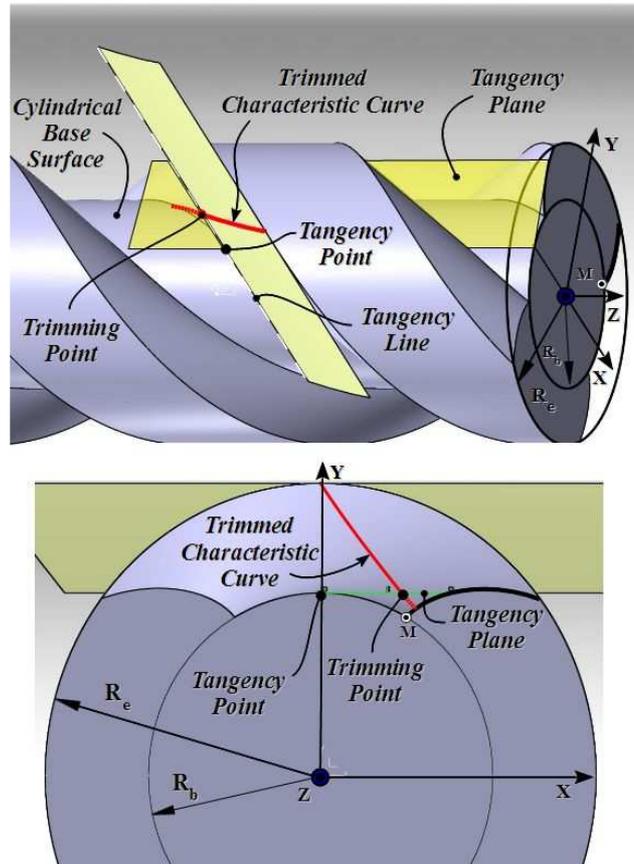
Fig. 4. The characteristic curve trimming.

If the 3-D CAD method is applied for cylindrical tool profiling, the trimmed characteristic curve is found directly by using CATIA environment facilities. If the analytical method is used, the trimming procedure can be applied as it follows:

- The full-length characteristic curve is found as a set of *n* points (X_k, Y_k, Z_k), $k = 1, 2 \ldots n.$, by giving values between 0 and θ_{1max} to θ_1 angular parameter from equations (18).
- Both characteristic curve projection and trimming plane trace are represented into *XY* plane (see Fig. 4), by using a graphical environment.
- The trimming point co-ordinates are determined with the help of graphical environment facilities.
- The distance between the trimming point and *XY* system origin is calculated and denoted by R_x.
- The value θ_{1x} of θ_1 angular parameter, which corresponds to the trimming point, is calculated with a relation similar to (15):

$$\theta_{1x} = \frac{r}{R_b} \cdot arccos\left[\frac{(R_b + r)^2 + r^2 - R_x^2}{2(R_b + r)r}\right]. \quad (19)$$

- The trimmed characteristic curve is found then, also by using (18), but the values of θ_1 angular parameter are now considered only between θ_{1x} of θ_{1max}.

5. COMPARISON BETWEEN THE RESULTS OF CAD METHOD VERSUS THE RESULTS OF ANALYTICAL METHOD APPLICATION

We further present a comparison between the results obtained when profiling the planing tool for cycloidal worms machining, by using two different methods (analytical, respectively CAD), in order to assess the 3-D CAD method performance.

Both methods were applied in the case of cycloidal worm, characterized by the input values of its geometrical parameters specified in Table 1.

The analytical method was implemented by realizing a dedicated MatLab application. Its algorithm grounds on the following main steps:

- For each value of θ_1 from above, the interval $[\varphi_{min}, \varphi_{max}]$, meaning the domain of possible values for φ angular parameter, is also divided in a (very high) number of points, n_φ.
- For each couple of values (θ_1, φ), the condition

$$|q(\theta_1, \varphi)| \le \varepsilon, \quad (20)$$

which replaces (17) condition, is tested.

- If condition (20) is satisfied, the couple (θ_1, φ) is then used for finding, with relations (11), the corresponding point from the planing tool surface.

We must notice that there is a direct connection between the values of ε and n_φ: closer to zero it is ε – higher should be n_φ in order to enable finding the solutions of condition (20).

The input set of values for the calculus parameters, used by the MatLab application in the approached case, is presented in Tab. 2.

Table 1

Input values of worm geometrical parameters

Crt. No.	Parameter	Value
1	R_b	10 [mm]
2	r	10 [mm]
3	R_e	16 [mm]
4	p	80 [mm]
5	δ	30 [°]

Table 2

Input values of calculus parameters

Crt. No.	Parameter	Value
1	R_x	11.047683 [mm]
2	φ_{min}	−10 [°]
3	φ_{max}	72 [°]
4	ε	0.01
5	n_θ	1000 points
6	n_φ	500000 points

Table 3

Characteristic curve points co-ordinates [mm]

Point crt. no.	Analytical method			3-D method		
	X	Y	Z	X	Y	Z
1	4.6977	9.9991	1.4905	4.6958	10.0000	1.4930
2	4.6927	10.0050	1.5006	4.6908	10.0059	1.5030
3	4.6877	10.0110	1.5107	4.6858	10.0119	1.5131
4	4.6826	10.0170	1.5208	4.6808	10.0178	1.5231
…	…	…	…	…	…	…
501	2.2435	12.9759	6.5447	2.2427	12.9765	6.5464
502	2.2387	12.9819	6.5550	2.2379	12.9826	6.5568
503	2.2340	12.9879	6.5652	2.2330	12.9887	6.5672
504	2.2292	12.9939	6.5754	2.2282	12.9947	6.5776
505	2.2244	12.9999	6.5856	2.2235	13.0007	6.5876
506	2.2197	13.0059	6.59590	2.2187	13.0066	6.5978
507	2.21497	13.0119	6.6061	2.2140	13.0126	6.6080
508	2.2102	13.0179	6.6163	2.2092	13.0186	6.6183
509	2.2054	13.0238	6.6265	2.2044	13.0246	6.6285
510	2.2007	13.0298	6.6367	2.1997	13.0306	6.6387
…	…	…	…	…	…	…
1000	0.0011	15.9999	11.7156	−0.0008	16.0000	11.7170
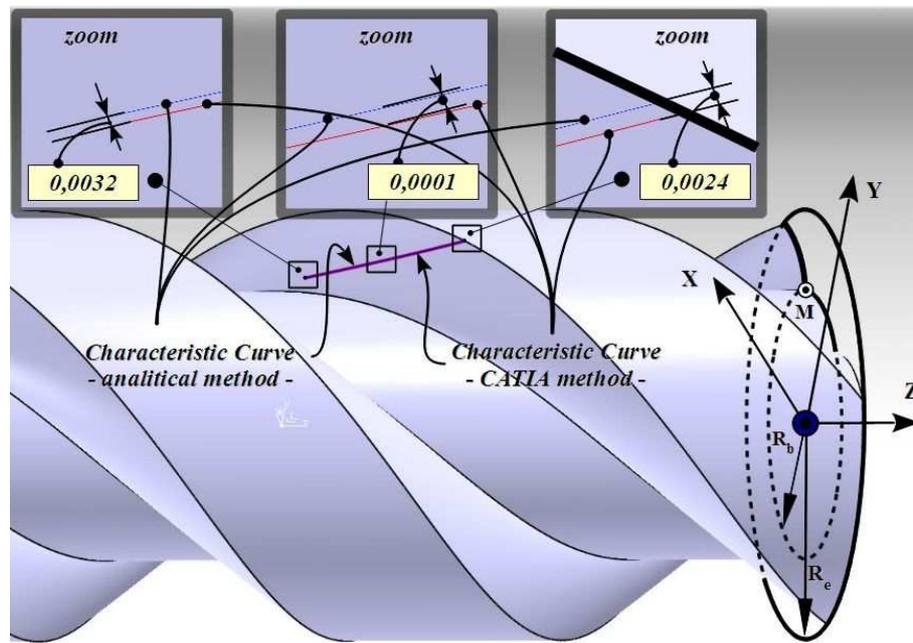
Fig. 5. Evaluation of the distance between the characteristic curve obtained by analytical method and the characteristic curve obtained by CATIA method (graphical approach).

The co-ordinates of the points from the characteristic curve, calculated by using the two different methods are presented (partially, for exemplification) in Table 3.

We must notice that, despite their identical current number, the two points positioned on the same line of the table are not corresponding to the same generic point from the characteristic curve. Because of the differences between the two methods approaches, in the analytical method case the points from the characteristic curve does not result equidistant (because of the harmonic functions presence in (18)), while the 3-D CAD method always finds equidistant points. For this reason, a direct comparison between the co-ordinates of points exemplified Table 3 is not relevant.

We further present two approaches, which can be used to find the difference between the characteristic curve obtained by analytical method and the characteristic curve obtained by CATIA method: a graphical approach and an analytical approach.

The graphical approach uses the CATIA environment facilities dedicated to calculate the distance between two points, applied after representing both characteristic curves in the same picture (referred to the same co-ordinates system), and after zooming-in them several times for a better resolution. The results obtained in this case are illustrated in Fig. 5.

The analytical approach is based on finding, for each among the points from one characteristic curve (having no matter which one is considered as reference) the closest point from the other characteristic curve. This search can be made by calculating the Euclidian distance between two points,

$$d = \sqrt{\left(X_i - X_j\right)^2 + \left(Y_i - Y_j\right)^2 + \left(Z_i - Z_j\right)^2} . \quad (21)$$

In our case, (X_i, Y_i, Z_i), $i = 1, 2, \dots n_\theta$, are the co-ordinates of the generic point from the characteristic curve analytically found, while (X_j, Y_j, Z_j), $j = 1, 2, \dots n$,

are the co-ordinates of the points from the characteristic curve found by using the 3-D CAD method.

For each point from the first curve, d_{min} parameter is initially set at a very high value (e.g. 1000). Afterwards, the distances from this point to all the points from the other curve are successively calculated with (21), and every time a value smaller than the current d_{min} is found, d_{min} takes the respective value. Thus, after running over all the points from the second curve, d_{min} means the minimum distance between the current point from the first curve and the second curve. Obviously, this approach is appropriate only when at least the second curve is given through a high enough number of points, relative to the followed calculus precision.

This algorithm was implemented through a MatLab application. The results of its application for comparing the two characteristic curves considered in our example are presented in Table 4 and Fig. 6.

Table 4

The distance between the two characteristic curves

Crt. No.	d_{min} [mm]
1	0.003218
2	0.003195
3	0.003133
4	0.003030
...	...
501	0.001978
502	0.002106
503	0.002383
504	0.002537
505	0.002374
506	0.002321
507	0.002243
508	0.002324
509	0.002348
510	0.002351
...	...
1000	0.002415
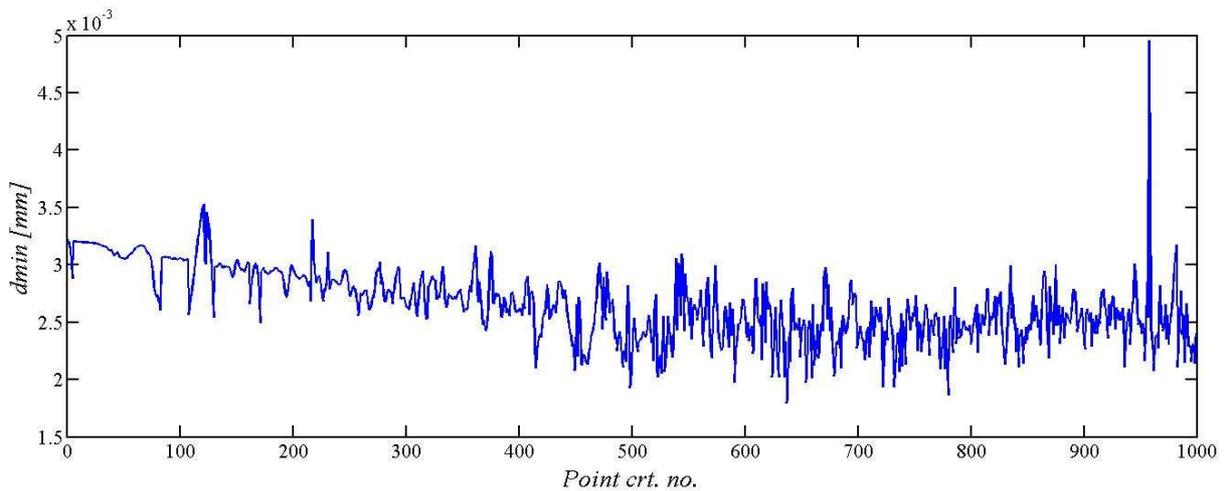
Fig. 6. Evaluation of the distance between the characteristic curve obtained by analytical method and the characteristic curve obtained by CATIA method (analytical approach).

Both approaches for assessing the difference (distance) between the characteristic curves found by using the two mentioned methods lead to similar conclusions: 3-D CAD method gives very precise results, in total concordance with the analytical method. The average distance between the two curves is about 2 ... 3 microns, which is totally acceptable if referred to machining process normal accuracy.

At the same time, we must observe that a higher number of points to define the two curves should reveal a even smaller distance between them, because the precision of distance calculus algorithm depends on the considered number of points.

6. CONCLUSIONS

A new 3-D method for profiling a helical-cylindrical surface, reciprocal enwrapped to the cycloydal worm flanks, was developed under CATIA graphical environment. Because of the enhanced facilities offered by CATIA, the method is easy to apply. The new method accuracy was tested by making a comparison between the results obtained for profiling a cylindrical tool when using the new method and the results offered, in the same case, by the classical analytical method. The difference (distance) between the characteristic curves found by using the two mentioned methods was assessed on two ways, the first based on CATIA specific tools, and the second, grounded on numerical calculus. Both approaches highlighted a remarkable precision for the 3-D method.

The new profiling method also enable to avoid the interference phenomenon appearance, by finding and eliminating from the characteristic curve the section among the tangency point with the worm base cylinder and the intersection point between the tangent to this cylinder and the cycloidal flank. A procedure for characteristic curve trimming was imagined and tested in this purpose.

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