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APPLICATIONS OF THE MONTE CARLO METHOD IN THE MANUFACTURING PROCESSES

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Abstract: *The accelerated developement of computers and software (particularly free and open-source software) made more accessible, for a large category of researchers and engineers, the use of computer aided simulation techniques. In general terms, the Monte Carlo Method (MCM) or Monte Carlo Simulation (MCS) can be used to describe any technique that approximates solutions to quantitative problems through statistical sampling The Monte Carlo simulation has important applications for discret-event systems (usual in manufacturing), stress/strength stochastic modeling (in design), reliability, maintainability and availability evaluation. Simulation in system reliability analysis is based on the MCS method that generates random failure times from each component's failure distribution. As practical exemplification the paper presents a few Monte Carlo simulation procedures and a manufacturing case study.*

Key words: *simulation, Monte Carlo method, discret-event system, manufacturing*.

1. INTRODUCTION AND HISTORY

The name Monte Carlo was applied to a class of mathematical methods first used by scientists for the development of nuclear weapons in Los Alamos in the 1940s [29]. The Manhattan Project for the atomic bomb used extensively code words: Monte Carlo was used for the solving technique using random numbers (RN). On the Manhattan Project, with complicated partial differential equations, impossible to be solved by hand, were rearranged for RN, then RN tables assisted the problems solving. The earliest computer ENIAC was used with a crude RN generator [2].

While there is no essential link to computers, the effectiveness of numerical or simulated gambling as a serious scientific pursuit is enormously enhanced by the digital computers. Carrying out games of chances or random sampling will produce anything worthwhile.

The Monte Carlo Method (MCM) or Monte Carlo Simulation (MCS) describe any technique that approximates solutions to quantitative problems through statistical sampling [10]. MCS gives a method for propagating (translating) uncertainties in model inputs into uncertainties in model outputs (results MCS relies on the process of explicitly representing uncertainties by specifying inputs as probability distributions). If the inputs of a system are uncertain, the prediction of future performance is necessarily uncertain.

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The result of any analysis based on inputs represented by probability distributions is itself a probability distribution. The probability distribution of performance propagate the input uncertainties into uncertainties in the results. MCS is the most common technique for propagating the uncertainty of a system to the predicted performance [10]. For each realization, all the uncertain parameters are sampled. The system is then simulated in time to compute the system performance. The results of the independent system realizations are assembled into probability distributions of outcomes [10].

The first step in studying a system is to build a model to obtain predictions on the system`s behavior [14].

In Bayesian statistics, it must to integrate over the posterior distribution. Markov Chain Monte Carlo technique is a Monte Carlo integration method which draws samples from the target posterior distribution [8].

2. STRESS/STRENGTH STOCHASTIC MODELING

Stress/Strength interference theory is a technique to quantify the probability that the strength of an item is less than the stress to which it is subjected. If the distribution of the strength is quantified, and the distribution of the stress it is under can be quantified too, the area of intersection of the two stresses represents the probability that the strength is less than the stress [17]. The goal of any design for robustness is to minimize the variance of both distributions, and maximize the separation of the means; the probability of distribution intersection, or failure, is minimized (Fig. 1) [17].

This stress can be modeled using closed-form equations, as a function of dimension, force, deflections, etc. For complex structures, finite element models and analy-

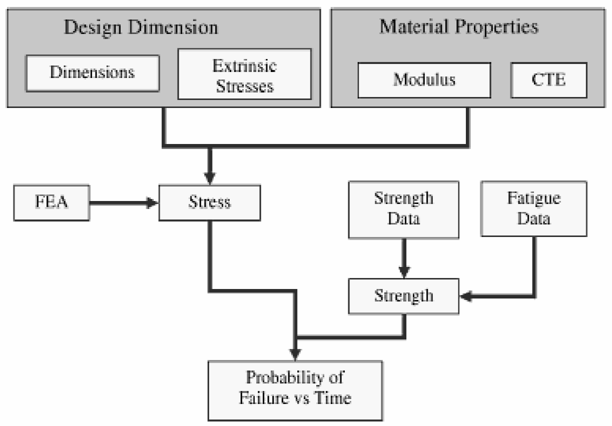

Fig. 1. Stress Strength Methodology [17].

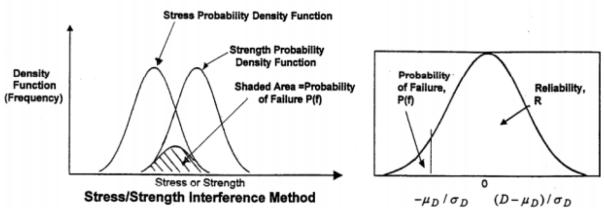

Fig. 2. Stress/strength probability density functions [18].

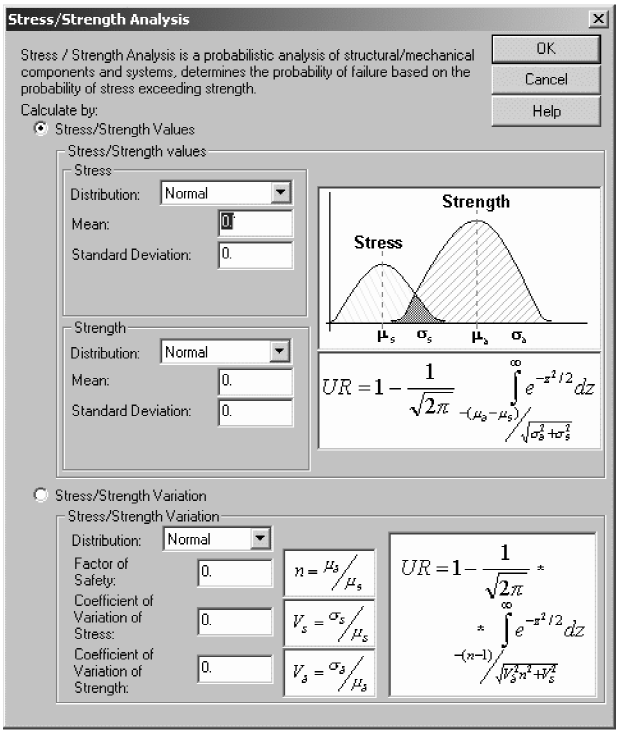

Fig. 3. Computer aided stress/strength evaluation [19].

sis (FEA) simulate stresses. The inherent strength distribution and properties are important as a function of time (the fatigue properties of the material pertain to the strength degradation over time [17]). MCS is the most common approach for fatigue reliability analysis.

Part strength must excced part stress to operate properly. Since strength and stress are both random, there is always a slight chance that stress will exceed the strength of the part, (the intersection -shaded area- of the graphs in the Fig. 2).The expected probability of success (reliability R) , is [16]:

$$R = P[\text{Stress} \le \text{Strength}] = \int_0^\infty f_{Stress}(x) \cdot R_{Strength}(x)dx. \tag{1}$$

The above calculation assumes both stress and strength are in the positive domain. For general cases, the expected reliability uses the following equation:

$$\text{R} = \text{P}[\text{X}_1 \le \text{X}_2] = \frac{1}{\text{F}_1(U) - \text{F}_2(U)} \int_L^U \text{f}_1(x) \cdot \text{R}_2(x)dx, \tag{2}$$

where: $L \le X_1 \le U$; X_1:Stress; X_2:Strength. When $U = \infty$, $L = 0$, the above two equations are the same. The stress and strength distributions can be estimated. Variations in the model parameters and probability values are associated with the calculated probability [16].

If the distributions for stress and strength are known, the reliability (1) is the probability that strength is larger than stress. Since both strength and stress are random variables (RV), the reliability is also a RV. Since stress is a RV, for any stress value x_i, there is a reliability value of $R(x_i)$ calculated from the strength distribution. From these $R(x_i)s$, it can get the mean and variance of the reliability and the two-sided confidence intervals [16].

The software Weibull++ [20] generates RNs based on the distribution of the diameter d and the load F in the part dimensioning. These RN are used to calculate random stress values. A distribution can be fitted to obtain the failure governing stress distribution [20].

The steps to estimate the reliability are:

1. Identify failure modes or failure mechanisms (mechanical static/dynamic, chemical, electrical, physical, structural, or thermal) of the component.
2. Identify appropriate failure model mean stress/strength characteristics.
3. Identify design parameters for stress and strength
4. Collect the appropriate data to calculate the statistics for stress and strength
5. Calculate the reliability and safety margin [15].

3. MONTE CARLO SIMULATION IN RELIABILITY, MAINTAINABILITY AND AVAILABILITY

In reliability analysis the MCS is often employed when the analytical solution is not attainable and the failure domain cannot be approximated by an analytical form. Sampling techniques (variance reduction), improve the computational efficiency by reducing the statistical error inherent in MCS [5].

MCS is usually coupled with genetic algorithm to optimize maintenance policy. A simulation tool optimized block replacement as a maintenance policy and the spare provisioning policy; now three decision variables are optimized simultaneously: preventive maintenance time plan, labor workforce size, and inventory level [6 and 7].

The main features of the MCS-based maintenance model are: ranking of repairs, preventive maintenance policy, objective function (the total maintenance cost plus economic loss, to be minimized). The MCS procedure is:

- Failure times of equipment are sampled using the "current" reliability function of equipment. Due to imperfect maintenance assumption, the reliability function changes with time.
- At failure times of equipment, the type of failure modes that caused equipment failure is sampled in accordance with the probability of occurrence.
- The cost of corrective maintenance, the repair time, and the economic losses are determined corresponding to the type of identified failure modes.
- Preventive maintenance requests for equipment are in accordance with the predetermined preventive maintenance schedule (PM policy) [6, 7].

In the availability study, based on the availability of related subsystems, are presented two methods of assessing the availability of a turbogenerator group [1]. The starting element is the block diagram for parametric reliability analysis of the group and a simulation program, based on the operating values, appreciate the number of simulation for the accuracy of results [1].

Many complex systems cannot be broken down into groups of series and parallel components [23]. If the system can be broken down into series/parallel configurations, it is a relatively simple analytical formula for the system's reliability, for example for the exponential distribution; in all other cases (even for normal distribution) the solutions are far more complicated. The reliability values for the components are determined with standard or accelerated life data analysis techniques and the simulation determine the reliability of the entire system [23].

Simulation in system reliability analysis is based on the MCS method that generates random failure times from each component's failure distribution. The overall system reliability is obtained by simulating system operation and calculating the reliability values for a series of time values. As drawbacks: the results depend on the number of simulations, and most of the reliability optimization and allocation techniques cannot be applied.

The system simulation is different from the analytical methodology. While one can perform a MCS based on the results of the analytical system reliability solution, the methodology described below uses MCS of the *individual components* to estimate the overall system reliability [23].

BlockSim simulation software [Reliasoft 20], has as inputs the end time for the estimated reliability and the number of increments. The *Use Seed* allows to choose the seed value for the generation of RNs (Fig.4) [23].

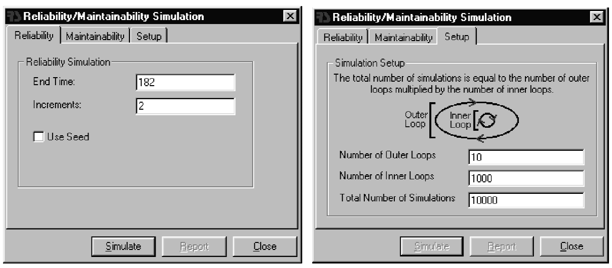

Fig. 4. Reliability simulation inputs and setup [23].

The next inputs are the number of inner loops (IL) and of outer loops (OL). The product of the two values gives the total number of simulations. The number of IL indicates the number of simulation points for each component. The number of OL indicates the number of repetitions of the ILs [23].

The simulation procedure has the following steps[23]:

1 Fix the number of points to generate (IL).
2 For each run, generate a RN between 0 and 1.
3. Obtain a failure time for each component based on this RN.
4. Keep the smallest time-to-failure with the corresponding component (i.e., time-to-failure with a value less than the desired mission time).
5. Check which components or combination of components cause system failure.
6. The unreliability of the system is the number of times the system was found to have failed divided by the total number of runs. The reliability of the system is 100% minus the unreliability.
7. Return to Step 2 and repeat the procedure for the desired number of cycles (OL).
8. The reliability of the system is the summation of the reliabilities of the OL divided by the number of OL (the average reliability). The process is repeated, and results averaged to system reliability each time [23].

The system reliability-cost simulation uses the probability density function (PDF) of the time between failures (TBF) for each component [22]. The user supplies a MTBF and a coefficient of variation (COV) for each component, and the model calculates the component PDFs/CDFs of the TBFs. A Monte Carlo simulation estimates the system failure rate, λ (t), and R(t). The number of failures for each component and the cost to replace failed components gives the cost for a reliability level. The Reliability-Cost Tradeoff is based on a user-supplied relationship between reliability and cost for each component, to determine the reliability of each component to maximize the system reliability and minimize the acquisition and repair cost. A heuristic algorithm has been developed to calculate the Pareto front between reliability and associated cost [22]. Each point on the Pareto front is mapped to a target component-level reliability and cost resulting the given system level reliability and cost [22].

For the redundancy allocation are considered the stochastic reliabilities of the components and to arrive at the optimal solution is employed Monte Carlo simulation technique [21].

4. MANUFACTURING MODELING AND SIMULATION

The manufacturing industry operates in an unpredictable environment characterized by increasing global competition and price erosion: innovative technologies such as manufacturing simulation assist manufacturers make economical use of resources and materials, for process analysis assistance and improve quality and prepare for fluctuations in demand.

Table 1 provides illustrations of systems and areas and the types of design, planning, and operational issues, using modeling and simulation [3].

Table 1

Modeling and Simulation of Manufacturing Areas [3]

Type of System	**Design, Planning, and Operational Issues**
Manufacturing Systems	Plant design and layout
	Continous improvement
	Capacity management
	Agile manufacturing evaluation
	Scheduling and control
	Materials handling
Transportation systems	Railroad system performance
	Truck scheduling and routing
	Air traffic control Terminal and depot operations
Computer and communication systems	Performance evaluation
	Work-flow generation and analysis
	Reliability assessment
Project planning and control	Product planning
	Marketing analysis
	Research and development performance
	Construction activity planning
	Scheduling project activities
Financial planning	Capital investment decision making
	Cash flow analysis
	Risk assessment
	Balance sheet projections
Environmental and ecological studies	Flood control
	Pollution control
	Energy flows and utilisation
	Farm management
	Pest control
	Reactor maintainability
Health care systems	Supply management
	Operating room scheduling
	Manpower planning
	Organ transplantation policy evaluation

Table 2

Manufacturing components [4]

Product	**Resources**	**Demand**	**Control**
Parts/pieces	Equipment layout	Customers orders	Warehouse management
Routings	Number of machines	Start date	Inventory control
Process times	Downtime	Due date	Shop floor control
Setup times	Preventive maintenance	WIP inventory	WIP tracking
Bill of materials	Storage areas		PLCs
Yeld	Tools/fixtures		Station rules
Rework	Labor-classification Shift schedules		

Process improvement starts with measurement, data collected, and simulation (a decision-making tool for system understanding) [4]. The "as built" models provide manufacturers an evaluation of the capacity of the system for new orders, equipment downtime, etc. and schedule to run the facilities. Table 2 shows how details are added as the model validation process proceeds. During model validation, details are added as the model approaches an acceptable level of accuracy [4].

4.1. Manufacturing Modeling Features

Manufacturing simulation models are developed with general-purpose and manufacturing-focused tools, with ease of use and flexibility. Manufacturing-oriented packages are Arena, AutoMod, AutoMod, HyperMesh, ProcessModel, ProModel, Witness, etc. [9].

Most manufacturing processes have random or unpredictable variables in their environment or components; then, stochastic modeling simulates the system. Output data of the simulation are random too and estimates of the true characteristics of the model: multiple runs are necessary, and the results across replications provide an estimate of the system expected performance.

In a continuous simulation, state variables change continuously; in a discrete one, variables change only at a finite number of points in time. In discrete-event systems the state variables change instantaneously through jumps at discrete points in time, as: traffic systems, flexible manufacturing systems, inventory systems, production lines, etc. [11]. Most manufacturing systems are modeled as dynamic and discrete event simulations (DES) in industries including automotive, military, etc. DES is used to analyze overall manufacturing environments (adding new equipment, upgrading existing machines, changing factory layout), specific issues, and individual measures of performance. The simulation models were developed with general-purpose programming languages (FORTRAN, C, BASIC, PASCAL, etc.) and special-purpose simulation languages such as SLAM II (with manufacturing modules for conveyors and automated guided vehicles), SIMAN, and GPSS [12]. They offer RN generation, use different probability distributions, modeling elements, etc. Other class of software, simulators, with a graphical interface, model system randomness (interarrival times, processing times, downtimes, etc.) with theoretical probability distributions. The software contains standard distributions and a multiple-stream pseudorandom number generator and it make independent replications of the model. At present [13] it develops the Open Source software (Facsimile -open-source discrete-event simulation/emulation library, SimPy, Tortuga -discrete-event simulation in Java, etc.), and many commercial software: AutoCAST (for casting technologies), GoldSim (system dynamics of discrete event simulation, embedded in a Monte Carlo framework), MATLAB, NEi Nastran (simulation of stress, dynamics, and heat transfer), SIMUL8, Simulink, Vensim (system dynamics, discrete elements; optimization and Monte Carlo) [13].

In manufacturing is important the time between two consecutive breakdowns: the system reliability requires evaluation of individual machine reliabilities and the line reliability. Techniques as reliability network reduction, minimum cut set, etc. are not enough. MCS estimate the probability of non-interrupted operation of a manufacturing system (with m machines arranged in series) for a specified period of time; i-machine index, n-initial number of iterations, T – lifetime of a manufacturing system, p – system probability to fails before t, DLT – desired lifetime of the system, RE – relative simulation error, DRE – desired relative error, t_i – machine i lifetime, $f(x_i)$ – PDF of machine i lifetime, X_i – RV having a PDF $f(x_i)$. The Monte Carlo algorithm is [24]:

Step 0. Initialize: number of failures $f = 0$, iteration number $k = 0$, total number of iterations = n.
Step 1. Set $k = k + 1$. Generate random varieties X_i from a given $f(x_i)$ for all i; set $t_1 = X_1\sim f(x_1)$, $t_2 = X_2\sim f(x_2)$,..., $t_m = X_m\sim f(x_m)$.
Step 2. Calculate the system lifetime T=min $\{t_1, t_2,..., t_m\}$.
Step 3. If $T < DLT$, set $f = f + 1$, else $f = f$; if $k \le n$, go to Step 1, otherwisw go to Step 4.
Step 4. Calculate a relative error of the simulation run P = Prob(System fails before DLT) = f / n

$$\mathrm{RE} = \mathrm{C:V.}(\hat{\mathrm{p}}) = \frac{\sqrt{Var(\hat{\mathrm{p}})}}{|\hat{\mathrm{p}}|} = \frac{\sqrt{\frac{\mathrm{p}(1-\mathrm{p})}{\mathrm{n}}}}{\mathrm{p}} . \tag{3}$$

If $RE < DRE$, then $p = f / n$, Stop. Otherwise go to Step 5.
Step 5. Calculate the number of additional iterations required to achieve DRE.

Set n = additonal number of iterations = $\dfrac{p(1-p)}{DRE^2 p^2}$

$f = 0$, $k = 0$, go to Step 1 [24].

5. MONTE CARLO SIMULATION CASE STUDY

Let us consider two work places in a machine tool repair shop, where a main shaft is disassembled. The bearings are dismantled in the first work place, A, and the gears are dismantled in the work place B. The work place A dismantle operation is the first one in the work flux and then follows the transfer of the shaft to the B work place for the next disassembly work. The transport takes 15 minutes, and from now on the gears are dismantled from the shaft I. The necessary mean time for the bearings dismantle is 55.5 minutes, respectively 44.5 minutes for the gears.

The shafts succeed one after other during the work and made a row, but during the evolution of the work flow it is possible to interpose waiting times.

The goal of the Monte Carlo simulation is to find the mean length of the waiting queue for shafts and the average yield of the dismantling line.

The field data for the dismantling times of the two work places are RV: the histograms of these times are plotted in Fig. 5 (work place A-200 values) and in Fig. 7 (work place B-180 values). Supplementary the frequencies of work place A are detailed in Table 3.

Table 3

The field data frequencies for the dismantling times of the work place A

t_i	22.5 - 27.5	27.5 - 32.5	32.5 - 37.5	37.5 - 42.5	42.5 - 47.5	47.5 - 52.5	52.5 - 57.5	57.5 - 62.5
f_i	1	2	4	4	18	43	47	39

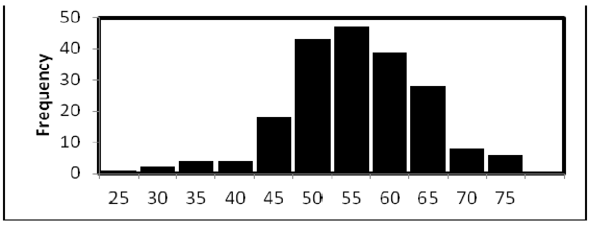

Fig. 5. Histogram of the field data for the work place A.

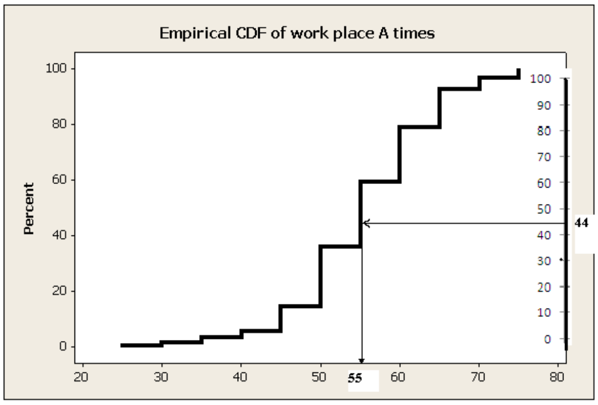

Fig. 6. Empirical cumulative function of the field data for the dismantling times of the work place A.

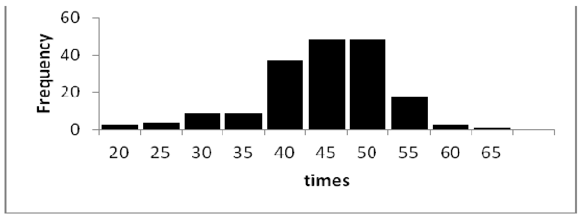

Fig. 7. Histogram of the field data for the work place B.

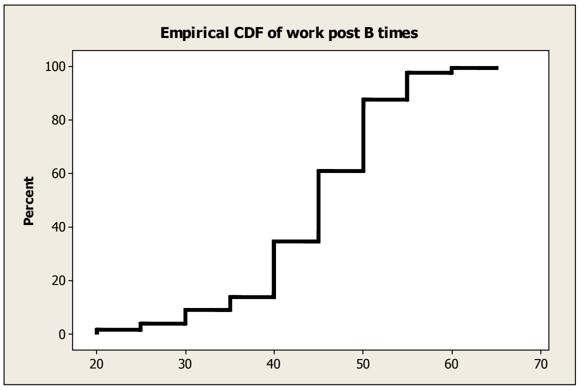

Fig. 8. Empirical cumulative function (ECF) of the field data for the dismantling times of the work place B.

The cumulative distributions were calculated through summation of the relative frequencies with a dismantling length smaller then a fixed value (Figs. 6 and 8).

The number 100 corresponds to the maximal value of the cumulative frequency. Figure 6 is cumulative plotted based on the histogram from Fig. 5. Number 100 corresponds to the maximal value 200; it is easy to read as example that 74% corresponds to the time 60min: in 74% of the work operation the dismantle takes less than 60 min.

The properly simulation starts with generation of a set of RNs (here was used the RND function from MSExcel, and a simple Macro to obtain only values between 0 and

Table 4

The dissasembly times for work places A and B, on the basis of RNs

No. Exp.	Work place A		Work place B	
	RN	Disman-tle time	RN	Disman-tle time
1	44	55	72	50
2	89	65	69	50
3	48	55	81	50
4	35	50	70	50
5	66	60	75	50
6	87	65	10	35
7	30	50	22	40
8	39	55	63	50
9	8	40	11	35
10	12	45	51	45
11	28	40	73	50
12	45	45	18	40
13	67	45	35	45
14	62	45	9	35
15	36	45	65	50
Total		760		675

100). These RN should then be converted in dismantling times using the EDF plot. As illustration (Fig. 6) for the work place A the RN is connected through a horizontal line with the EDF plot: the intersection point has the abscise 55, which give a dismantling time of 55 minutes. Appling the same graphical solution was produced a big number of dismantling times for A and B work places (it can be developed an analytical solution too!). Two set of 15 RN and dismantling times are presented in Table 4.

Comparing the dismantling times, it will be selected the most efficient order of the work places A and B.

6. CONCLUSIONS

Most manufacturing processes have random or unpredictable variables in their environment or components; then, stochastic modeling is used to simulate the system in question. The accelerated developement of computers and software (particularly free and open-source software) made more accessible, for a large category of researchers and engineers, the use of computer aided simulation techniques. In general terms, the Monte Carlo Method or Monte Carlo Simulation can be used to describe any technique that approximates solutions to quantitative problems through statistical sampling The Monte Carlo simulation has important applications for discret-event systems (usual in manufacturing), stress/strength stochastic modeling (in design), reliability, maintainability and availability evaluation. Simulation in system reliability analysis is based on the MCS method that generates random failure times from each component's failure distribution. As practical exemplification the paper presents a few Monte Carlo simulation procedures a manufacturing case study.

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