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ANALYTICAL METHOD TO PROFILE THE HOB MILL GENERATING AN ORDINATE WHIRL OF SURFACES WITH NON-INVOLUTE PROFILE

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Abstract: *The ordinate whirls of surfaces, cylindrical or helical, are generated by enwrapping with worm type tools. A particular aspect of this problem is to profile the tool primary peripheral surface. The classical solution is offered by Olivier second theorem, referring to a couple of reciprocal enwrapped surfaces, which depend on two independent parameters. We should notice that analytically solving such a problem could involve serious difficulties in calculus and, further, in results numerical analysis A newly developed analytical method to profile the primary peripheral surface of the hob mill used for generating the slitting saw cutter teeth with non-involute profile, used in the iron & steel industry, is presented in this paper. The method was conceived grounded on the helical motion de-composing principle and it works in two steps. The profile of the rack-gear conjugated to the generated profile is firstly determined, according to Intermediary surface method, and then, on this base, the hob mill primary peripheral surface is found. A specific profiling algorithm is suggested, followed by a numerical application, in the case of a slitting saw cutter with triangular tooth profile and having the diameter of 1600 millimeters.*

Key words: *Hob mill profiling, analytical method, helical motion de-composition, Disc-tool, Triangular tooth.*

1. INTRODUCTION

The ordinate whirls of surfaces, cylindrical or helical, are generated by enwrapping with worm type tools. It is the case of involute gears with straight or curved teeth, of helical compressors rotors, flute-shafts, slitting saw cutters with straight teeth, etc.

A particular aspect of this problem is to profile the primary peripheral surface of the hob mill, meaning a cylindrical helical surface with constant pitch, reciprocal enwrapped to whirl surfaces.

The enounced problem consists in finding a helical surface with axis disjoint from the one of the axoid associated to the whirl of surfaces to be generated, a second order contact problem – punctiform contact between enveloped surfaces [1].

The classical solution is offered by Olivier second theorem, referring to a couple of reciprocal enwrapped surfaces, which depend on two independent parameters. We should notice that analytically solving such a problem could involve serious difficulties in calculus and, further, in results numerical analysis, for finding the generator hob mill profile.

Hereby, the second Olivier theorem [1] is referring, in principle, to a surface

$$\Sigma: F(X,Y,Z)=0, \tag{1}$$

which generates a surfaces family

$$(\Sigma)_{\alpha,\beta}: F(X,Y,Z,\alpha,\beta)=0, \tag{2}$$

with α and β independent parameters.

The envelop of this surfaces family results, on the base of the above-mentioned theorem, by associating to family equations the conditions:

$$F_{\alpha}'=0;\ F_{\beta}'=0. \tag{3}$$

The envelop is obtained by eliminating the two parameters from equations (2) and (3), as a surface with punctiform contact to Σ, be it $S(X,Y,Z)=0$.

The analytical solution of the presented problem is hard to find, because of the generating process specific kinematics, which often leads to transcendent equations imposing, eventually, a numerical approach.

Problem solving can be substantially improved if using the Intermediary surface (here the rack-gear associated to profiles ordinate whirl) method [1–3].

The problematic of helical surfaces generation can also use the helical motion de-composition principle [5] and, from here, a specific manner to profile the hob mill can be developed. The approached problematic continues to be studied in the dedicated literature [6–8].

In this paper, we further present, in the section 2, a specific algorithm, laying on the Intermediary surface method and using also the Plain trajectories complemen-

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tary theorem [2, 4], as well as an application of it in profiling the hob mill for generating the teeth of the slitting saw cutter used in iron industry (regarded as a disc-tool), in section 3. In section 4, a numerical simulation is exposed, while the last section is dedicated to conclusions.

2. ANALYTICAL METHOD TO PROFILE THE HOB MILL

2.1. The conjugated rack-gear

The hob mill profiling, in analytical form, has been done based on Gohman theorem, using the Intermediary surface method, in the following steps.

- The surface to generate being known like:

$$\Sigma(u,v)\left|\begin{aligned}&X = X(u,v);\\&Y = Y(u,v);\\&Z = Z(u,v),\end{aligned}\right. \tag{4}$$

with u, v – independent parameters, in the rolling motion of the centrodes associated to the surfaces whirl Σ and to the future rack-tool (the intermediary surface), the surfaces family $(\Sigma(u,v))_\varphi$ is determined, Fig. 1, φ meaning the rolling motion parameter.

- The enveloping condition in its specific form:

$$\vec{N}_\Sigma \cdot \vec{R}_\varphi = 0\,, \tag{5}$$

is then associated to the family $(\Sigma(u,v))_\varphi$, where $\vec{N}_\Sigma$ is the normal to $\Sigma(u,v)$ surface, and $\vec{R}_\varphi$ – the speed vector in the relative motion between the intermediary surface I (the rack-gear) and $\Sigma(u,v)$ surface. Relation (5) means, in principle a dependence having the general form:

$$Q(u,v,\varphi) = 0\,. \tag{6}$$

- The equations ensemble:

$$(\Sigma(u,v))_\varphi\left|\begin{aligned}&X(u,v,\varphi) = 0;\\&Y(u,v,\varphi) = 0;\\&Z(u,v,\varphi) = 0,\end{aligned}\right. \tag{7}$$

representing, in the reference system of the whirl of surfaces to generate XYZ, attached to its centrode, the enveloping condition (5), enables the elimination of one among the parameters. Therefore, the $(\Sigma(u,v))_\varphi$ family envelope, representing the intermediary surface, will result as:

$$I(u,\varphi)\left|\begin{aligned}&X_1 = X_1(u,\varphi);\\&Y_1 = Y_1(u,\varphi);\\&Z_1 = Z_1(u,\varphi).\end{aligned}\right. \tag{8}$$

The system $X_1Y_1Z_1$ is associated to the reference rack-gear centrode (Fig. 1). The rolling motion between the two centrodes C_1 – circle of R_r radius and C_2 – straight-line tangent in P pole to C_1 circle, lead to the condition:

$$R_r \cdot \varphi = \lambda\,, \tag{9}$$

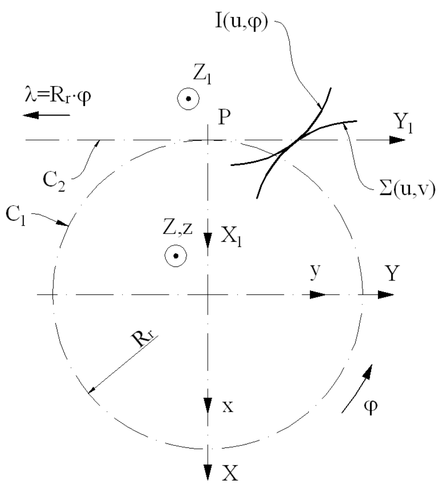

Fig. 1. The centrodes of $\Sigma(u,v)$ and $I(u,\varphi)$ surfaces.

where λ is the translation parameter of C_2 centrode, associated to generator rack-gear.

The contact between $\Sigma(u,v)$ and $I(u,\varphi)$ surfaces is linear, being represented through $C_{\Sigma\text{-}I}$ characteristic curve, which in the XYZ system, has equations like:

$$C_{\Sigma-I}\,\big|\,\Sigma(u,v,\varphi) = 0 \;\big|\; \varphi = ct,\ u = ct\,. \tag{10}$$

2.2. The primary peripheral surface of the hob mill

Once known the equations of the intermediary surface I (8), the next task is to find the primary peripheral surface of the hob mill, solidary with $X_2Y_2Z_2$ reference system, see Fig. 2.

In the relative motion of the intermediary surface $I(u,\varphi)$ relative to $X_2Y_2Z_2$ system of the future primary peripheral surface, a family of surfaces is generated as:

$$(I(u,\varphi))_{\varphi_2}\left|\begin{aligned}&X_2 = X_2(u,\varphi,\varphi_2);\\&Y_2 = Y_2(u,\varphi,\varphi_2);\\&Z_2 = Z_2(u,\varphi,\varphi_2).\end{aligned}\right. \tag{11}$$

There is, obviously, the dependence:

$$\lambda = p\cdot\varphi_2\cdot\cos\omega\,, \tag{12}$$

where φ_2 is the rotation motion parameter, around Y_2 axis, while p – the helical parameter of the surface representing the hob mill primary peripheral surface.

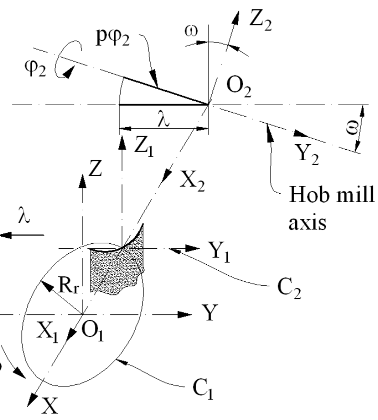

Fig. 2. The reference system of the hob mill, $X_2Y_2Z_2$.

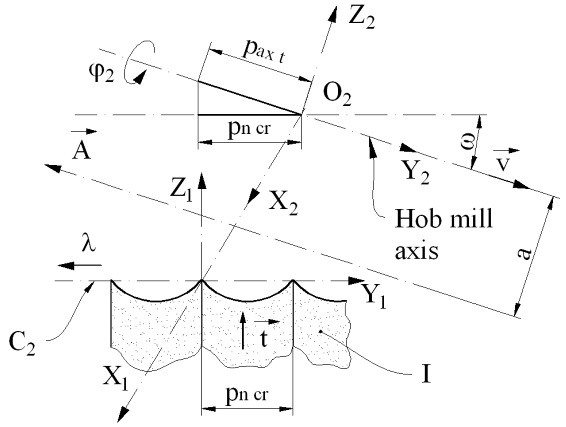

Fig. 3. The rack-gear normal pitch and the hob mill axial step.

The p parameter value can be determined by imposing the identity condition between the normal pitches of the intermediary surface (cylindrical surface) and the helical surface (hob mill surface), Fig. 3.

The normal pitch of the reference rack-gear, $p_{n\,cr}$, is identical to the circular pitch of the cylindrical surfaces ordinate whirl that we intend to generate, see also Fig. 2, and it can be calculated as:

$$p_{n\,cr} = \frac{2\pi \cdot R_r}{z}. \tag{13}$$

In relation (13), z means the number of Σ surfaces from the whirl, equal to the number of "teeth". Hence the hob mill axial step $p_{ax\,t}$ is:

$$p_{ax\,t} = \frac{p_{n\,cr}}{\cos\omega} = \frac{2\pi \cdot R_r}{z} \cdot \frac{1}{\cos\omega}. \tag{14}$$

The inclination angle of the hob mill axis – ω, measured relative to the frontal plane of both the generated surfaces Σ whirl and rack-gear *I*, can be calculated from relation:

$$\sin\omega = \frac{R_r}{R_{rs}} \cdot \frac{1}{z}, \tag{15}$$

where R_{rs} is the radius of the hob mill rolling cylinder (tangent to C_2 centroide) and R_r – the rolling radius of the cylinder solidary to the surfaces whirl (see Fig. 4).

The primary peripheral surface of the future hob mill results as envelop of the surfaces family generated by *I* – rack-gear flank, during its relative motion *p* helical parameter, around Y_2 ($\vec{v}$) axis,

$$p = \frac{p_{ax\,t}}{2\pi} = \frac{R_r}{z} \cdot \frac{1}{\cos\omega}. \tag{16}$$

The helical motion $\left(\vec{v}, p\right)$ can be de-composed in two elementary motions:

- Translation along *I* surface generatrices, having $\vec{t}$ as versor – $T\left(\vec{t}\right)$, and
- Rotation around an axis parallel to $\vec{v}$ and placed at the distance a from it – $\left(\vec{A}, a\right)$,

$$\left(\vec{v}, p\right) = T\left(\vec{t}\right) + \left(\vec{A}, a\right). \tag{17}$$

The distance a has the expression:

$$a = p \cdot \tan\theta, \tag{18}$$

where θ is the angle between $\vec{v}$ and $\vec{t}$ axis, $\theta = \frac{\pi}{2} - \omega$.

The *I* surface characteristic, in its composed motion (17), does not depend on the motion component in the course of which the surface *I* is self-generating, so it does not depend on the translation motion along $\vec{t}$ generatrix. Therefore, only the rotation motion around $\vec{A}$ axis determines this characteristic.

As it is known [5], the characteristic on *I* surface results by projecting $\vec{A}$ axis on this surface. In this way, the characteristic curve results as:

$$C_{I-S}\begin{vmatrix} X_1 = X_1(u,\varphi); \\ Y_1 = Y_1(u,\varphi); \\ Z_1 = Z_1(u,\varphi), \end{vmatrix} \tag{19}$$

to whom we associate the analytical condition of $\vec{A}$ axis projection onto *I* surface, see Fig. 4,

$$\left(\overrightarrow{N_I}, \vec{r_1}, \vec{A}\right) = 0, \tag{20}$$

equivalent to an analytical form:

$$K = K(u,\varphi) = 0. \tag{21}$$

The vectors from (20) mixed product are defined as:

- $\overrightarrow{N_I}$ – the normal to *I* surface, from (8), in $X_1Y_1Z_1$ reference system

$$\overrightarrow{N_I} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dot{X}_{1u} & \dot{Y}_{1u} & \dot{Z}_{1u} \\ \dot{X}_{1\varphi} & \dot{Y}_{1\varphi} & \dot{Z}_{1\varphi} \end{vmatrix} = N_{X_1} \cdot \vec{i} + N_{Y_1} \cdot \vec{j} + N_{Z_1} \cdot \vec{k}. \tag{22}$$

- $\vec{A}$ – the hob mill axis versor,

$$\vec{A} = \cos\omega \cdot \vec{j} - \sin\omega \cdot \vec{k}. \tag{23}$$

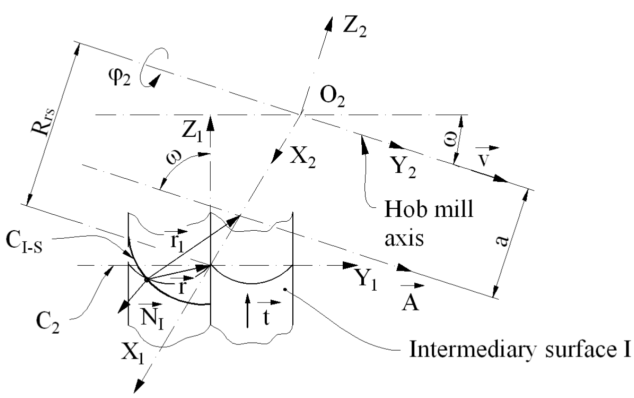

Fig. 4. The $\vec{A}$ axis projection onto *I* surface.

- $\vec{r}_1$ – the position vector of I surface current point, respect an arbitrary point from $\vec{A}$ axis,

$$\vec{r}_1 = -(R_{rs} - a)\vec{i} + \vec{r}, \qquad (24)$$

with $\vec{r}$ – the position vector of I surface (8) current point, respect the origin of $X_1Y_1Z_1$ reference system.

Condition (21), in association with equations (19), determines the characteristic curve in the helical motion of I surface relative to $\vec{v}$, having in principle the form:

$$C_{I-S}\begin{vmatrix} X_1 = X_1(\varphi); \\ Y_1 = Y_1(\varphi); \\ Z_1 = Z_1(\varphi). \end{vmatrix} \qquad (25)$$

The curve (25) must be further transposed in $X_2Y_2Z_2$ system, by applying the transformation:

$$X_2 = \alpha(X_1 - b), \qquad (26)$$

with the following notations:

$$\alpha = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{pmatrix} \text{ and } b = \begin{pmatrix} -R_{rs} \\ 0 \\ 0 \end{pmatrix}. \qquad (27)$$

In principle, it will result in the form:

$$C_{I-S}\begin{vmatrix} X_2 = X_2(\varphi); \\ Y_2 = Y_2(\varphi); \\ Z_2 = Z_2(\varphi). \end{vmatrix} \qquad (28)$$

During C_{I-S} characteristic (28) helical motion of $\vec{v}$ axis and p parameter, the shape of the hob mill helical surface can be now determined:

$$\begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix} = \omega_2^T(\psi)\cdot\begin{pmatrix} X_2(\varphi) \\ Y_2(\varphi) \\ Z_2(\varphi) \end{pmatrix} + \begin{pmatrix} 0 \\ p\cdot\psi \\ 0 \end{pmatrix}, \qquad (29)$$

representing a right-worm of p helical parameter. In relation (29), ω_2 means the transformation matrix corresponding to rotation around Y_2 axis, while ψ – the helical motion angular parameter. After calculus in (29) result:

$$S\begin{vmatrix} X_2 = X_2(\varphi)\cdot\cos\psi + Z_2(\varphi)\cdot\sin\psi; \\ Y_2 = Y_2(\varphi) + p\cdot\psi; \\ Z_2 = -X_2(\varphi)\cdot\sin\psi + Z_2(\varphi)\cdot\cos\psi. \end{vmatrix} \qquad (30)$$

The equations (30) represent, on principle, the hob mill primary peripheral surface.

3. PROFILING OF THE HOB MILL FOR THE SLITTER SAW CUTTER TEETH

At the iron mills, the profiles are often cut by using high diameter disc-tools ("slitter saw cutters"), working at very high cutting speeds (100–120 m/s). Their teeth profiles are diverse. In the approached application, we considered a triangular tooth profile (see Fig. 5).

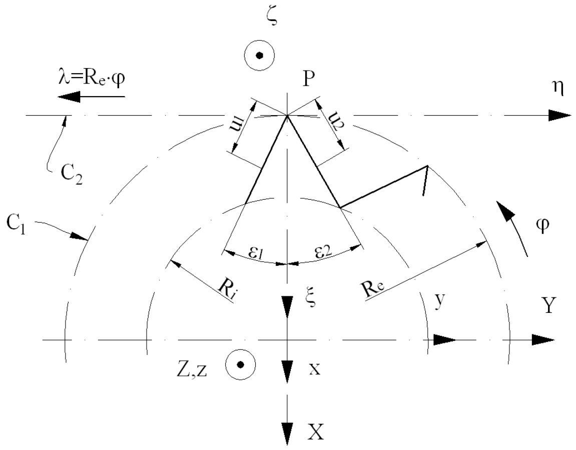

Fig. 5. The slitting saw cutter tooth profile.

3.1. The generator rack-gear

In Fig. 5, the reference systems required for solving the approached problem are represented:

- xyz is the global system, having its z axis in common with the centrode associated to the tool;
- XYZ – relative system, attached to C_1 centrode and to the profile to generate;
- $\xi\eta\zeta$ – relative system, attached to the profile generator rack-gear.

Regarding the two centrodes motions, the rotation of worked piece centrode, C_1, is expressed through φ parameter, while C_2 centrode translation – through λ. Between the two parameters there is the dependence:

$$\lambda = R_e \cdot \varphi. \qquad (31)$$

The equations of the two segments composing the cutter tooth profile are:

$$\Sigma_{\overline{PA}}\begin{vmatrix} X = -R_e + u_1\cdot\cos\varepsilon_1; \\ Y = -u_1\cdot\sin\varepsilon_1 \end{vmatrix} \qquad (32)$$

and

$$\Sigma_{\overline{PB}}\begin{vmatrix} X = -R_e + u_2\cdot\cos\varepsilon_2; \\ Y = u_2\cdot\sin\varepsilon_2, \end{vmatrix} \qquad (33)$$

with u_1 and u_2 variable parameters. The Σ profiles families have, in the reference system attached to the generator rack-gear, the following equations:

$$(\Sigma_{\overline{PA}})_\varphi\begin{vmatrix} \xi = (-R_e + u_1\cos\varepsilon_1)\cos\varphi + u_1\sin\varepsilon_1\sin\varphi + R_e; \\ \eta = (-R_e + u_1\cos\varepsilon_1)\sin\varphi - u_1\sin\varepsilon_1\cos\varphi + R_e\cdot\varphi; \end{vmatrix} \qquad (34)$$

$$(\Sigma_{\overline{PB}})_\varphi\begin{vmatrix} \xi = (-R_e + u_2\cos\varepsilon_2)\cos\varphi - u_2\sin\varepsilon_2\sin\varphi + R_e; \\ \eta = (-R_e + u_2\cos\varepsilon_2)\sin\varphi + u_2\sin\varepsilon_2\cos\varphi + R_e\cdot\varphi. \end{vmatrix} \qquad (35)$$

The enveloping condition is attached to the two profiles families, according to the Plane trajectories method [2]:

$$\frac{\xi'_{u_1}}{\eta'_{u_1}}=\frac{\xi'_{\varphi}}{\eta'_{\varphi}}\text{, and }\frac{\xi'_{u_2}}{\eta'_{u_2}}=\frac{\xi'_{\varphi}}{\eta'_{\varphi}}\text{ respectively.}\tag{36}$$

From relations (36), after calculus, the following dependences resulted:

$$u_1=R_e\left[\cos\varepsilon_1-\cos(\varphi-\varepsilon_1)\right]\tag{37}$$

and

$$u_2=R_e\left[\cos\varepsilon_1-\cos(\varphi+\varepsilon_2)\right].\tag{38}$$

The ensembles formed by the pair of equations (34) and (37), and (35) and (38) respectively give the profiles composing the generator rack-gear flank, I_{PA} and I_{PB}:

$$I_{PA}\left|\begin{array}{l}\xi=-R_p\cos\varphi+u_1\cos(\varphi-\varepsilon_1)+R_e\,;\\ \eta=-R_p\sin\varphi+u_1\sin(\varphi-\varepsilon_1)-R_e\cdot\varphi;\\ \zeta=t\,;\\ \qquad u_1=R_e\left[\cos\varepsilon_1-\cos(\varphi-\varepsilon_1)\right],\end{array}\right.\tag{39}$$

$$I_{PB}\left|\begin{array}{l}\xi=-R_p\cos\varphi+u_2\cos(\varphi+\varepsilon_2)+R_e\,;\\ \eta=-R_p\sin\varphi+u_2\sin(\varphi+\varepsilon_2)+R_e\cdot\varphi;\\ \zeta=t\,;\\ \qquad u_2=R_e\left[\cos\varepsilon_2-\cos(\varphi+\varepsilon_2)\right].\end{array}\right.\tag{40}$$

3.2. Hob mill profiling

Figure 6 shows the following, on principle:

- the rack-gear flank surfaces, into $\xi\eta\zeta$ system;
- hob mill axis, $\vec{v}$;
- axis $\vec{A}$ of the rotation motion, component of the couple of motions equivalent to the helical one, $(\vec{v},p)$.

The following reference systems are necessary:

- $\xi\eta\zeta$, representing a mobile system, attached to the generator rack-gear, and
- $X_2Y_2Z_2$ – relative system, attached to the hob mill surface and having X_2 axis overlaid to ξ axis.

In the previous section, the relations for calculating the values of angle ω, helical parameter p and distance a were mentioned – the relations (15), (16) and (18).

In the mentioned conditions, the characteristic on the surface I – the rack-gear flank (here the couple of surfaces (39) and (40)) is defined as the projection of $\vec{A}$ axis onto it, in the form (20), see also Figs. 4 and 6.

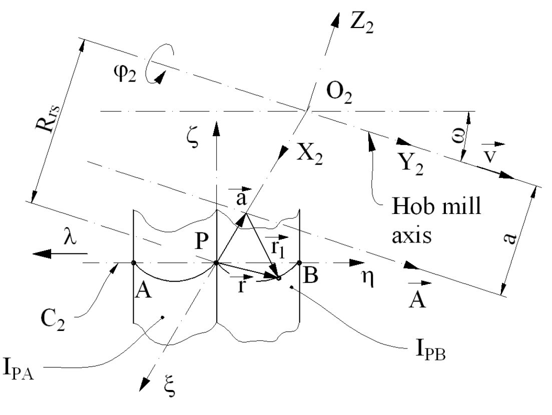

Fig. 6. The hob mill profiling.

Hence, relation (22) takes the specific form:

$$\overrightarrow{N_I}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \dot{\xi}_\varphi & \dot{\eta}_\varphi & \dot{\zeta}_\varphi\\ \dot{\xi}_t & \dot{\eta}_t & \dot{\zeta}_t\end{vmatrix}=N_{I\xi}\cdot\vec{i}+N_{I\eta}\cdot\vec{j}+N_{I\zeta}\cdot\vec{k}\,.\tag{41}$$

The expressions of the derivatives from this determinant result, after calculus, as it follows:

$$\begin{aligned}&\dot{\xi}_\varphi=R_e\sin\varphi+\frac{du_1}{d\varphi}\cdot\cos(\varphi-\varepsilon_1)-u_1\sin(\varphi-\varepsilon_1);\\ &\dot{\eta}_\varphi=-R_e\cos\varphi+\frac{du_1}{d\varphi}\cdot\sin(\varphi-\varepsilon_1)+u_1\cos(\varphi-\varepsilon_1)+R_e;\\ &\dot{\zeta}_\varphi=0,\end{aligned}\tag{42}$$

in the case of I_{PA} surface, and

$$\begin{aligned}&\dot{\xi}_\varphi=R_e\sin\varphi+\frac{du_2}{d\varphi}\cdot\cos(\varphi+\varepsilon_2)-u_2\sin(\varphi+\varepsilon_2);\\ &\dot{\eta}_\varphi=-R_e\cos\varphi+\frac{du_2}{d\varphi}\cdot\sin(\varphi+\varepsilon_2)+u_2\cos(\varphi+\varepsilon_2)+R_e;\\ &\dot{\zeta}_\varphi=0,\end{aligned}\tag{43}$$

for I_{PB} surface. For both surfaces we have

$$\begin{aligned}&\dot{\xi}_t=0;\\ &\dot{\eta}_t=0;\\ &\dot{\zeta}_t=1,\end{aligned}\tag{44}$$

The position vector $\vec{r}$, necessary to express $\vec{r_1}$ (23), Figure 6 has, according to (40) and (41), the expression:

$$\begin{aligned}\vec{r}=&\left[-R_e\cos\varphi+u_1\cos(\varphi-\varepsilon_1)+R_e\right]\cdot\vec{i}+\\ &\left[-R_e\sin\varphi+u_1\sin(\varphi-\varepsilon_1)-R_e\cdot\varphi\right]\cdot\vec{j}+t\cdot\vec{k},\end{aligned}\tag{45}$$

in the case of I_{PA} surface, for I_{PB} surface respectively:

$$\begin{aligned}\vec{r}=&\left[-R_e\cos\varphi+u_2\cos(\varphi+\varepsilon_2)+R_e\right]\cdot\vec{i}+\\ &\left[-R_e\sin\varphi+u_2\sin(\varphi+\varepsilon_2)+R_e\cdot\varphi\right]\cdot\vec{j}+t\cdot\vec{k}.\end{aligned}\tag{46}$$

The $\vec{A}$ axis versor can be expressed as:

$$\vec{A}=\cos\omega\cdot\vec{j}-\sin\omega\cdot\vec{k}\,.\tag{47}$$

The locus of points satisfying the condition (20) represents, in principle, the characteristic on the rack-gear flanks surfaces – $C_{I\text{-}S}$ – see (25), in $X_2Y_2Z_2$ system.

In the motion given by relation (29), the characteristic $C_{I\text{-}S}$ generates the helical surface of $\vec{v}$ axis and p parameter, see equations (30).

The equations of the axial section through hob mill peripheral primary surface result by associating to (30) the condition:

$$Z_2=0\,,\tag{48}$$

equivalent to

$$-X_2(\varphi)\cdot\sin\psi + Z_2(\varphi)\cdot\cos\psi = 0. \qquad (49)$$

In another form, the S surface axial section – the hob mill flank is described by the equations:

$$S_A \left| \begin{array}{l} R = \sqrt{X_2^{\,2}(\varphi) + Z_2^{\,2}(\varphi)}; \\ H = Y_2(\varphi) + p\cdot\psi, \end{array} \right. \qquad (50)$$

where, according to (49),

$$\tan\psi = \frac{X_2(\varphi)}{Z_2(\varphi)}. \qquad (51)$$

4. NUMERICAL APPLICATION

Further, we present a numerical application developed in MATLAB on the base of the above-presented profiling algorithm. The values of the input parameters defining the generated surface geometry (see Fig. 5) were: ε_1 = 20º; ε_2 = 40º; R_e = 800 mm; R_i = 788 mm; R_s = 60 mm; z = 200 teeth; p = 4 mm.

The determined coordinates of the points belonging to S surface axial section are sampled in Table 1, while the graphical representation of the hob mill profile in axial section together with the generated tooth profile are shown in Fig. 7.

Table 1

Hob mill axial profile (co-ordinates)

Item no.	Left flank		Right flank	
	R **[mm]**	***H*** **[mm]**	***R*** **[mm]**	***H*** **[mm]**
1	60.000	0.001	60.000	0
2	60.121	–0.041	60.121	0.102
3	60.242	–0.084	60.242	0.204
4	60.364	–0.127	60.363	0.307
5	60.486	–0.170	60.484	0.410
6	60.607	–0.213	60.606	0.512
7	60.729	–0.257	60.727	0.615
8	60.852	–0.300	60.848	0.718
9	60.974	–0.344	60.970	0.821
10	61.096	–0.388	61.091	0.924
. . . .				
91	71.412	–4.339	70.955	9.500
92	71.543	–4.393	71.077	9.609
93	71.675	–4.446	71.199	9.717
94	71.807	–4.500	71.321	9.826
95	71.939	–4.554	71.444	9.935
96	72.071	–4.608	71.566	10.044
97	72.203	–4.662	71.688	10.153
98	72.335	–4.716	71.810	10.262
99	72.467	–4.770	71.932	10.372
100	72.600	–4.825	72.055	10.481

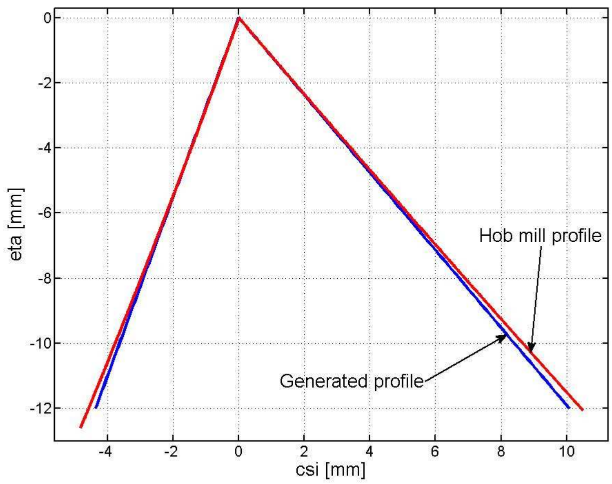

Fig. 7. The hob mill profile in axial section – S_A.

5. CONCLUSION

A new type of algorithm for profiling the hob mill to generate an ordinate whirl of profiles, associated to a rolling centrode is introduced in this paper. This algorithm lays on the principle of de-composing the helical motion, on this way enabling to establish more easily the enveloping condition.

The presented numerical example validates the new method quality. The required algorithm proves to be easy to apply for solving the approached type of problem and opens the gate for a future development of graphical profiling methods.

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