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FORWARD AND INVERSE KINEMATICS STUDY OF INDUSTRIAL ROBOTS TAKING INTO ACCOUNT CONSTRUCTIVE AND FUNCTIONAL PARAMETER'S MODELING

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Abstract: *Forward and inverse kinematic studies of industrial robots (IR) have been developed and presented in a large number of papers. However, even general mathematic formalization is usually almost correct, (basically following up general Hartenberg - Denavit (H-D) conventions and associated homogenous transformation matrix), only few papers presents kinematic models ready to be directly implemented on a real scale industrial robot or as well able to evaluate kinematics behavior of a real scale IR specific model. That is usually due to some inconsistencies in modeling, the most frequently of these referring on: the incomplete formalization of the full set of constructive and functional parameters (that mandatory need to be considered in case of a specific real IR's model), avoidance of considering IR's specific design features, (as joint dimensions and links dimensions are) leading to wrongly locating the reference frames used for expressing homogenous coordinate transformations, as well as missing of the validation procedures able to check the correctitude of the mathematical models, previously to its implementing in a real scale IR's controller. That is why present paper shows first a completely new approach for IR's forward an inverse kinematics, in terms of IR's analytical modeling by taking into account the full set of IR's constructive and functional parameters of two different IR's models. Then, for both direct and inverse mathematical models complete symbolic formalization and full set of solutions for forward and inverse kinematics are presented for both IR types. In order to study mathematical models applicability on the real scale IR, two specific IR models were studied: an ABB serial-link open chain kinematics IR and a Fanuc serial-link closed chain kinematics IR. Numerical results were verified by cross validation using both analytically calculations results and by mean of a constrained 3D CAD model used to geometrically verify the results. The parametric form of the model elaborated in PTC Mathcad 14 allows a quick reconfiguration for other robot's models having similar configurations. Results can be also used for solving dynamics, path planning and control problems in case of real scale IR.*

Key words: *industrial robot, extended parametric modeling, homogenous transformation matrix, forward kinematics, inverse kinematics.*

1. INTRODUCTION

Industrial robots have long been used to replace the necessity of the human operator in repetitive tasks or dangerous environments. From the early design stages, with respect to the specificity of the application each specific IR model may be efficiently integrated into a flexible manufacturing cells by primary taking into account its maximum reach ability and payload. Both of these major functional features are basically depending by the length of each IR's link, the available limits for each IR's joint orientation as well as some specific constructive features of each IR's subassemblies [1]. From this point of view the IR's work space can be defined as a relationship between the length of the arms, the number of degrees of freedom and the type of the joint (rotational, translational or combinations between the two) [2, 9]. The kinematics of the robot is ussually represented using a symbolic structure describing each joint and link and the relationship between the two in the path of the motion. Modeling the chains of bodies connected by joints is done using the Denavit-Hartenberg (D-H) conventions [2, 3, 7, 8, and 9]. The representation of D-H convention results in a 4×4 matrix called the homogenous transformation matrix. For an n degrees of freedom robot, the resulting number of homogenous transformation matrix is $n + 1$. The products of the coordinates frames transformations matrices for each link are used to determine the forward kinematics. By this mean the position and orientation of the tool attached to the robot can be computed using a given set of joint angles. Also, the form and limit of the robot's working envelope can be analyzed using the position vector from the transformation matrix describing the forward kinematics [3]. The inverse kinematics problem is solved to achieve a desired position and orientation of the tool relative to the workstation. The inverse kinematics problem r equires solving at first the inverse kinematics

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equations for the position fallowed by the equations for the orientation of the tool relatively to the robot's base frame. The number of solutions for the joint angles is affected by the number of IR specific configurations and number of degrees of freedom. While the problem of inverse kinematics may have 8 solutions for the most 6 DOF robots, the problem for 7 DOF robots is more complex because an infinity of solutions may be identified. Therefore, sometimes one DOF in suppressed when solving the kinematics equations. To analytically validate the inverse kinematics results, the value of joint angles determined for a specified pose of the IR's end effector should correspond with the value of joint angles from the inverse kinematics [15]. In order for this purpose a geometric validation may be used too by mean of constrained 3D models of the studied real scale IR achieved by taking into account their complete set of constructive parameters and design features [4, 5, and 6]. After defining each value corresponding to each joint angle, the final tool frame should correspond with the desired pose frame. Many kinematic studies for industrial robots behavior evaluation were recently published. Several analytical and numerical approaches are available for solving forward and inverse kinematics problem for industrial robots. Forward and inverse kinematics was analytically computed in [7] for a serial-link opened kinematics KUKA KR60 6 DOF IR, the accuracy of the results being verified using a simulation program which uses the Unified System for Automation and Robot Simulation (USARSim). The forward kinematics, inverse kinematics, workspace and joint accelerations and velocities were determined and results were also verified using Robo-analyzer software [8]. Similar work was done for a closed kinematics 4 DOF palletizing robot type including specific of its specific design features and constraints regarding mechanical behavior when solving the forward kinematics problem, the validation of the results being made in MATLAB [9]. Also, complete direct kinematic models were presented for the real case of Kawasaki FS10E industrial robot, by using an extended set of constructive and functional parameter's modeling [4] and the conventional D-H formalization algorithm [10 and 12], as well as the quaternion mathematical formalization algorithm [11]. For both previously approaches, the validation of analytic calculus results was performed using Kawasaki PC Roset Offline programming software and Dynalog experimental measuring system [10], and respectively Microsoft Robotics Developer Studio software [11]. A wide range of numerical approaches have been elaborated, with focus on fuzzy logic inverse kinematics mapping model for redundant manipulators [13], adaptive neuro-fuzzy interference system for inverse kinematics modeling [14] and approximation method for inverse kinematics modeling using MLP training [15]. Also, combined analytical and numeric approaches are available for modeling the kinematics of redundant manipulators having more than 6 DOF [16].

However the presented approach use PTC Mathcad 14 for parametric modeling of forward and inverse kinematics for two serial-link industrial robots, the first one having an open-chain kinematics (ABB IRB 6620) [5] and the second one a closed-chain kinematics (Fanuc m2000iA 900L) [6]. For both IR models the full set of constructive and functional parameters have been take account into modeling [4]. In both models the IR's work space limits are determined and results for extremity points are validated using by mean of a constrained 3D CAD geometric model. Thus, the presented parametric model can be further used for dynamic analysis, path planning and control of presented IR's real scale models, as well as for other IR type's (with similar configuration) modeling due to facilities offered for their quick reconfiguration.

2. TECHNICAL SPECIFICATIONS AND FUNCTIONAL PARAMETERS OF THE STUDIED ROBOTS

The studied robots are ABB IRB6620 (Fig. 1) and Fanuc m2000iA (Fig. 2).

- ABB IRB6620 is a medium payload serial link robot with open-chain kinematics and 6 DOF, [5];
- Fanuc m2000iA 900L is a heavy payload serial link robot with closed-chain kinematics and 6 DOF [6].

The technical specifications and functional parameters of the ABB IRB6620 and Fanuc m2000iA robots are detailed in Table 1.

Table 1

Technical specifications of the ABB IRB6620 Industrial robot [5, 6]

Manufacturer and model		**ABB IRB6620**	**Fanuc m2000iA 900L**
Maximum payload (kg)		150	900
Number of NC axis		6	6
Repeatability (mm)		0.03	0.5
Reach (mm)		2200	4638
Maximum speed	1	100 °/s	45 °/s
	2	90 °/s	30 °/s
	3	90 °/s	30 °/s
	4	150 °/s	50 °/s
	5	120 °/s	50 °/s
	6	90 °/s	70 °/s
Working range	1	+170° / -170°	+165° / -165°
	2	+140° / -65°	+100° / -60°
	3	+70° / -180°	+35° / -130°
	4	+300° / -300°	+360° / -360°
	5	+130° / -130°	+120° / -120°
	6	+300° / -300°	+360° / -360°
Weight (kg)		900	9600

3. IR'S CONSTRUCTIVE PARAMETERS AND D-H MODIFIED CONVENTION

The D-H convention is used for the modeling of chains of bodies connected by joints. Originally the D-H convention only was applied to single-loop chains, but now is almost universally applicable to most serial chains structures.

For representing the frames of the studied industrial robots, the modified convention of the D-H parameters will be used herein [17]. To attach the link frames the fallowing procedure has been fallowed (Figs. 3 and 4) [4]:

1) Identification of the joint axes and drawing of infinite lines along them. For steps 2 to 5 below, two of

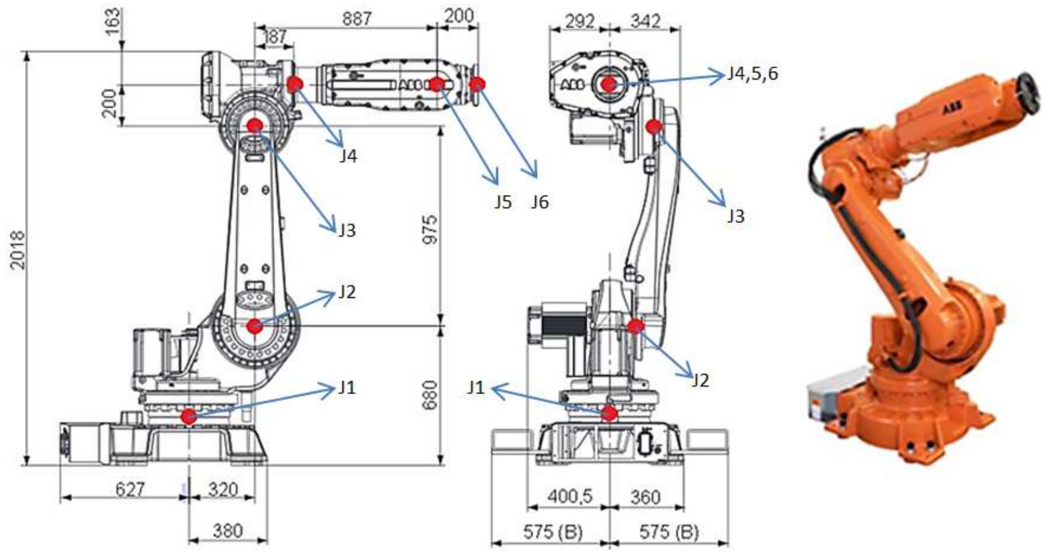


Fig. 1. Dimensional specifications and joint numbering of the ABB IRB6620 Industrial robot [5]
$J1$ to $J3$ - Corresponding joints for position and $J4$ to $J6$ corresponding joints for orientation.

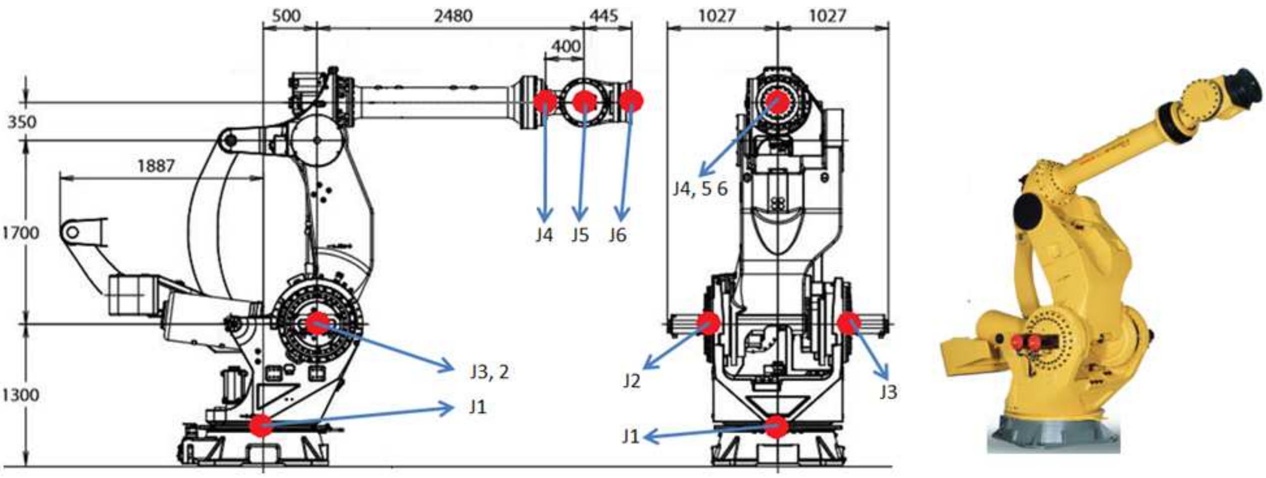


Fig. 2. Dimensional specifications and joint numbering of the Fanuc m2000iA Industrial robot [6]
$J1$ to $J3$ = Corresponding joints for position and $J4$ to $J6$ corresponding joints for orientation.

these neighboring lines are considered (at axes i and i + 1);

2) Identification of the common perpendicular between them, or point of intersection. The link frame origin is assigned at the point of intersection, or at the point where the common perpendicular meets the i^{th} axis;

3) Assignment of the Z_i axis pointing along the i^{th} joint axis;

4) Assignment of the X_i axis pointing along the common perpendicular, or if the axes intersect, the X_i is assigned to be normal to the plane containing the two axes;

5) Assignment of the Y_i axis to complete a right hand coordinate system;

6) Assignment of {0} to match {1} when the first joint variable is zero.

For {N} an origin location is chosen. The X_n direction is determined freely, but generally so as to cause as many linkage parameters as possible to become zero;

4. LINK PARAMETERS

The link parameters describe the dimensions of the robot arm, the joint offsets (resulting from D-H convention) and the position and orientation of each joint referencing the previous joint. The link parameters for ABB IRB6620 and Fanuc m2000iA 900L robot are detailed in Tables 2 and 3. The sets of parameters will define the Homogenous transformation matrix.

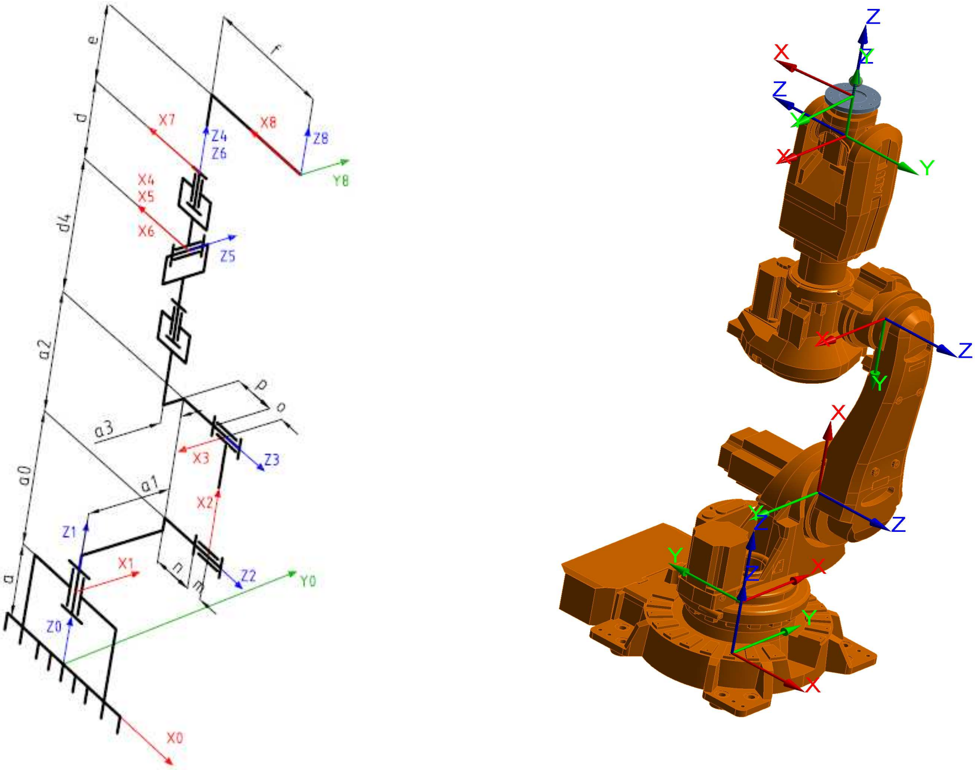


Fig. 3. Assignment of link frames for ABB IRB6620 Robot [4]:
a_i = the distance from Z_i to Z_{i-1} measured along X_i; α_i = the angle between Z_i and Z_{i+1} measured along X_i; d_i = distance from X_{i-1} to X_i measured along Z_{i-1} and θ_i = angle between X_{i-1} and X_i measured about Z_i; *n*, *m*, *p* and *o* represent robot arm joint offset parameters.

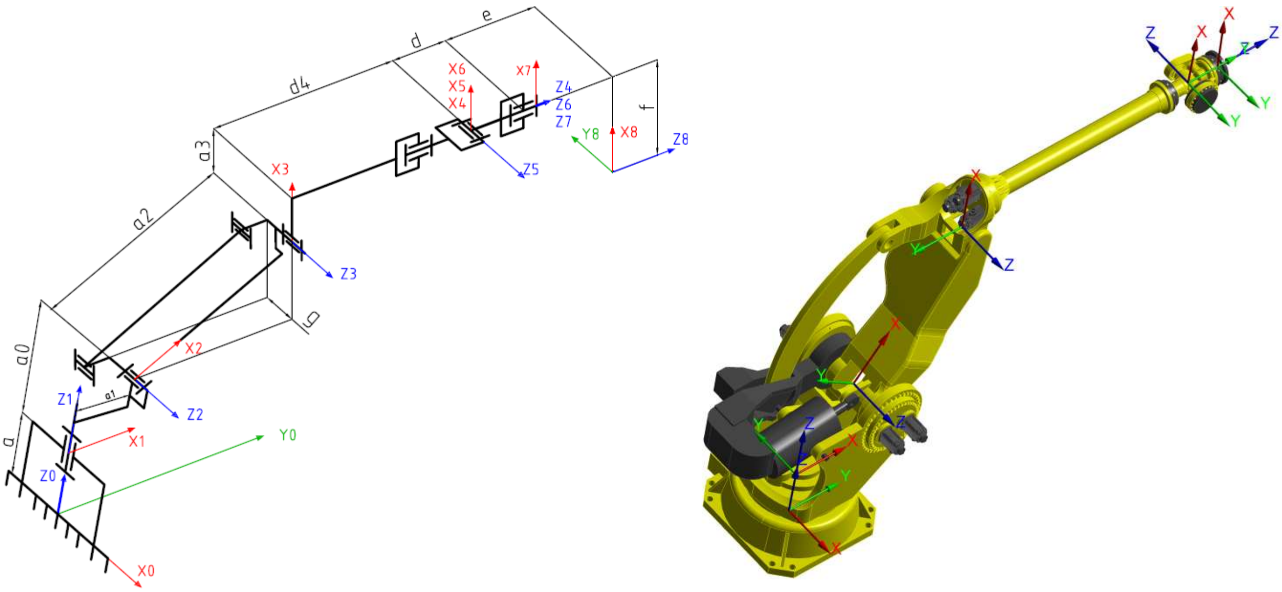


Fig. 4. Assignment of link frames for Fanuc m2000iA 900L Robot [4];
all the notations are the same as in Fig. 2, the only difference being the *g* robot arm joint offset parameter.

Table 2

Link parameters derived for ABB IRB6620 Robot

i	a_{i-1}	α_{i-1} (mm)	d_i	θ_i
1	0°	0	0	θ1
2	-90°	320	0	θ2
3	0°	975	0	θ3
4	-90°	280	887	θ4
5	90°	0	0	θ5
6	-90	0	0	θ6

Table 3

Link parameters derived for Fanuc m2000iA Robot

i	a_{i-1}	α_{i-1} (mm)	d_i	θ_i
1	0°	0	0	θ1
2	-90°	500	0	θ2
3	0°	1700	0	θ3
4	-90°	180	2850	θ4
5	90°	0	0	θ5
6	-90	0	0	θ6

5. HOMOGENOUS TRANSFORMATION MATRIX

The homogenous transformation matrix represents each coordinate frame of the link with respect to the coordinate system of the previous link. The resulting matrix is a 4×4 matrix:

$$ {}^{i-1}A_i = \begin{bmatrix} \cos\theta i & -\sin\theta i\cos\alpha i & \sin\theta i\sin\alpha i & ai\cos\theta i \\ \sin\theta i & \cos\theta i\cos\alpha i & -\cos\theta i\sin\alpha i & ai\sin\theta i \\ 0 & \sin\alpha i & \cos\alpha i & di \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (1) $$

$$ {}^{0}T_{i-1}{}^{i-1}Ai \,, \quad (2) $$

where ${}^{0}T_i$ is the homogenous transformation describing the pose of coordinate frame i with respect to the world coordinate system 0.

The resulting homogenous transformation matrix are detailed in Eq. (3):

$$ T01 = \begin{pmatrix} \cos(\theta1) & -\sin(\theta1) & 0 & 0 \\ \sin(\theta1) & \cos(\theta1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \quad (3) $$

$$ T12 = \begin{pmatrix} \cos(\theta2) & -\sin(\theta2) & 0 & a1 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta2) & -\cos(\theta2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; $$

$$ T23 = \begin{pmatrix} \cos(\theta3) & -\sin(\theta3) & 0 & a2 \\ \sin(\theta3) & \cos(\theta3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; $$

$$ T34 = \begin{pmatrix} \cos(\theta4) & -\sin(\theta4) & 0 & a3 \\ 0 & 0 & 1 & d4 \\ -\sin(\theta4) & -\cos(\theta4) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; $$

$$ T45 = \begin{pmatrix} \cos(\theta5) & -\sin(\theta5) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin(\theta5) & \cos(\theta5) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; $$

$$ T56 = \begin{pmatrix} \cos(\theta6) & -\sin(\theta6) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -\sin(\theta6) & -\cos(\theta6) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; $$

$$ TCPF = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{pmatrix}; \; TB = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a+a0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; $$

$$ TEF = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & f \\ 0 & 0 & 1 & e \\ 0 & 0 & 0 & 1 \end{pmatrix}. $$

6. FORWARD KINEMATICS

For an n-axis rigid-link manipulator, solving the forward kinematics problem gives the position and orientation of the robot's end effector relative to its base. The solution is obtained by repeated application of equation (4):

$$ {}^{0}T_n = {}^{0}A_1 {}^{0}A_2 \ldots {}^{n-1}A_n = K(q) \,. \quad (4) $$

which represent the multiplication of the frame transformation matrices for each link.

The matrix of transformation between the final frame and the base frame can be defined by equation (5):

$$ T = T01 \cdot T12 \cdot \ldots \cdot T56 \cdot TCPF \cdot TB \cdot TEF = \begin{pmatrix} R11 & R12 & R13 & Px \\ R21 & R22 & R23 & Py \\ R31 & R32 & R33 & Pz \\ 0 & 0 & 0 & 1 \end{pmatrix} \,. \quad (5) $$

Forward kinematics is used to determine the form and limit of the working envelope.

The functional limits for θ1, θ2 and θ3 are defined base on IR's working ranges that may be identified from the technical specifications Table 1.

A constant step value is defined for all joint limits. Using the position vector from the homogenous transformation matrix corresponding to the forward kinematics, a vector describing the position vectors for *Px* and *Pz* can be generated. Each row of the vector represents the coordinates for the given position in the Cartesian space. From the points describing the shape of the working envelope corresponding to the XOZ plane, a 3D model can be obtained revolving the resulting spline around the rotation axis of θ1. For each of the studied robots, the resulting plots are presented in Fig. 5,*a* for ABB IRB6620 and Fig. 5,*b* for Fanuc m2000iA 900L. Comparing the volume of the working envelope for the two robots, the first robot uses all the combinations of joint limits defined because of its serial-link open kinematics. On the other hand, the second robot can only use a limited combination of joint limits. This is due to the existence of a link corresponding to the lever that requires a special geometrical constraint (6):

$$ \theta2 + \theta3 + 90^\circ \geq -\theta3\max \,. \quad (6) $$

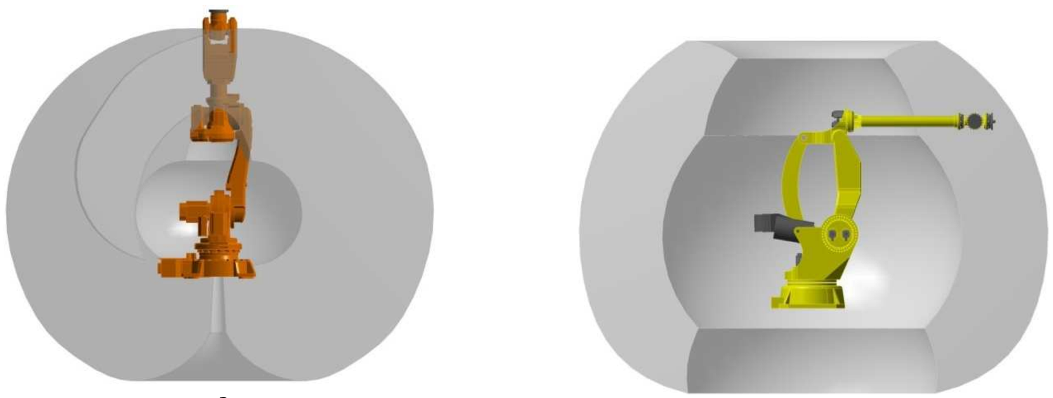
This is why the working limit of the lever's joint should not exceed the inferior limit of θ3 (Fig. 6).

7. INVERSE KINEMATICS

Inverse kinematics is used to compute the joint angles which will achieve a desired position and orientation of the end-effector relative to the base frame.

Both studied robots have two joint offsets (on *X* and *Z* axis).

The problem of inverse kinematics results in sets of joint angles, depending on the configuration of the robot. A total of 8 sets of joint angles can be achieved for a given position for the ABB IRB6620 robot while 6 solutions can be achieved for the Fanuc m2000iA 900L robot.



a b

Fig. 5. Working envelope: *a* – ABB IRB6620; *b* – Fanuc m2000iA.

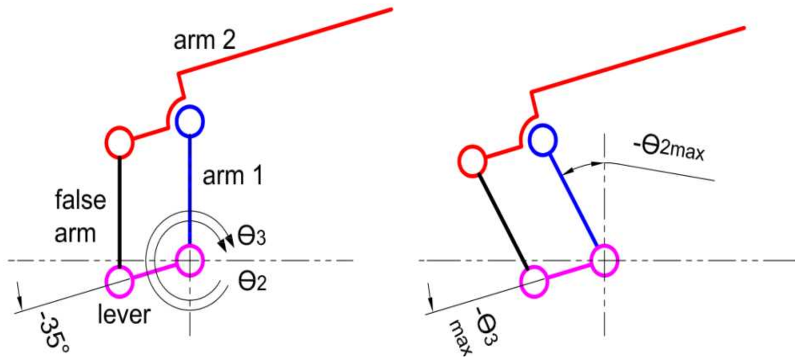


Fig. 6. Geometrical constraint applied to the Fanuc m2000iA Robot.

Analytically solving the inverse kinematics problem can be done by two steps:

1. Computation of θ1, θ2 and θ3 using the elements of the position vector from the T matrix (5).

$$\begin{aligned} Px &= c1(a3\cdot c32 - d4\cdot s23 + a1 + a2\cdot c2);\\ Py &= s1(a3\cdot c32 - d4\cdot s23 + a1 + a2\cdot c2);\\ Pz &= -a3\cdot s23 - d4\cdot c23 - a2\cdot s2. \end{aligned} \quad (7)$$

To choose the optimal equations for each joint, a study of the motion plane is done. To compute the solutions for each joint, the form of the equations can be simplified using the inverse kinematics identities proposed by John Craig. The resulting equations are presented below:

For θ1:

$$\begin{aligned} C1 &= Px\,,\\ S1 &= Py\,,\\ t1 &= \frac{\pi}{2} - a\tan 2(Py, Px)\,. \end{aligned} \quad (8)$$

For θ3:

$$K = \frac{[Px^2 + Py^2 + Pz^2 - 2a1(Px\cdot\cos(t1)]}{2a2} + \frac{[Py\cdot\sin(t1)] + a1^2 - (a3^2 + d4^2 + a2^2)]}{2a2};$$

$$t3 = a\tan 2(d4, a3) - a\tan 2[(\sqrt{a3^2 - d4^2} - \sqrt{K^2}), K] \quad (9)$$

For θ2:

$$\begin{aligned} a &= -d4\cdot\sin(t3) + a2 + a3\cdot\cos(t3);\\ b &= a3\cdot\sin(t3) + d4\cdot\cos(t3);\\ c &= \frac{Px}{\cos(t1)} - a1;\\ d &= -Pz;\\ t2 &= a\tan 2[(a\cdot c + b\cdot d),(a\cdot d - b\cdot c)]. \end{aligned} \quad (10)$$

2. For θ4, θ5 and θ6, 3 equations are used (7):

$$\begin{aligned} T03^{-1}\cdot T06 &= T36;\\ T04^{-1}\cdot T06 &= T46;\\ T05^{-1}\cdot T06 &= T56. \end{aligned} \quad (11)$$

For $T0n^{-1}$ the values are known. Each solution uses the $a\tan 2$ geometrical identity that transforms a specific position from the Cartesian space to the joint space using (8):

$$\theta = a\tan 2(x, y)\ [\text{rad}]. \quad (12)$$

To convert from rad to deg, Eq. (13) is used:

$$\theta_{\text{deg}} = \theta\cdot\frac{180}{\pi}\,. \quad (13)$$

For θ4:

$$S4 = [-R13 \cdot \sin(t1) + R23 \cdot \cos(t1)];$$
$$C4 = [-R13 \cdot \cos(t1) \cdot \cos(t2+t3)] - [R23 \cdot \sin(t1) \cdot \cos(t2+t3)] + [R33 \cdot \sin(t2+t3)]; \quad (14)$$
$$t4 = \frac{\pi}{2} - a\tan 2(S4, C4)\ .$$

For θ5:

$$s5 = -\{R13 \cdot [\cos(t1) \cdot \cos(t2+t3) \cdot \cos(t4) \cdot \sin(t1) \cdot \sin(t4)] + R23 \cdot [\sin(t2+t3) \cdot \cos(t4) \cdot \cos(t2+t3) \cdot \cos(t4) - \cos(t1) \cdot \sin(t4)] - R33 \cdot [\sin(t2+t3) \cdot \cos(t4)]\};$$
$$c5 = \{R13 \cdot [\cos(t1) \cdot \cos(t2+t3) \cdot \cos(t4) + \sin(t1) \cdot \sin(t4)] + R23 \cdot [\sin(t2+t3) \cdot \cos(t4) \cdot \cos(t2+t3) \cdot \cos(t4) - \cos(t1) \cdot \sin(t4)] - R33 \cdot [-\cos(t2+t3)];$$
$$t5 = \frac{\pi}{2} - a\tan 2(s5, c5). \quad (15)$$

For θ6:

$$s6 = R11 \cdot [\cos(t4) \cdot \sin(t1) - \cos(t1) \cdot \cos(t2+t3) \cdot \sin(t1) \cdot \sin(t4)] + R31 \cdot \sin(t4) \cdot \sin(t2+t3);$$
$$c6 = R12 \cdot [\cos(t4) \cdot \sin(t1) - \cos(t1) \cdot \cos(t2+t3) \cdot \sin(t4)] - R22 \cdot [\cos(t1) \cdot \cos(t4) + \cos(t2+t3) \cdot \sin(t1) \cdot \sin(t4)] + R31 \cdot \sin(t4) \cdot \sin(t2+t3);$$
$$t6 = \frac{\pi}{2} - a\tan 2(s6, c6). \quad (16)$$

8. RESULTS VALIDATION

To verify the results of the inverse kinematics, an arbitrary point within the working limit of the robot is defined. The inverse kinematics will compute the set of joint angles. An optimal set of joints is defined when the first arm is positioned perpendicular to the ground. Results can be validated analytically and geometrically.

8.1. Analytical validation

For the studied industrial robots the fallowing are defined:

- A position in the Cartesian space of the robot (the point of extremity from the robot's working range)
- The rotation matrix for the positioning of the End-effector given by θ 4, 5 and 6.

The rotation matrix is denoted using the direct kinematics by defining a set of joint angles that will bring the robot in an optimal position (the first arm in vertical position and the second arm parallel to the ground). Values are assigned for θ 4, 5 and 6 to complete the rotation matrix (for example θ4 = 0, θ5 = 0 and θ6 = 0 so that the axis of the robot's output flange to coincide with the motion axis of the second wrist). The components of the resulting matrix are described in (17):

$$R = \begin{pmatrix} R11 & R12 & R13 \\ R21 & R22 & R23 \\ R31 & R32 & R33 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \quad (17)$$

After defining this set of joint angles, the numeric result for the homogenous transformation matrix can be achieved. These results are used as input data for the solutions of the inverse kinematics for the orientation angles.

- The homogenous transformation matrix for the base of the robot, tool center point frame and end-effector.

The homogenous transformation matrixes have the same rotation matrix (the unit matrix). Different values are defined for the components of the position vectors. The parameters for the position vectors for the base, TCPF and end-effector are presented in Table 4.

After defining these parameters the solutions for the inverse kinematics are solved with 8 and 6 solutions being achieved. In this case, only one solution identical to the solutions computed by the direct kinematics is chosen (Table 5 for ABB IR and Table 6 for Fanuc IR).

8.2. Geometrical validation

The geometrical validation requires the 3D CAD model for each robot. The joints are constrained according to the D-H convention. Values corresponding to each joint angle are inserted and the model is updated. The desired position is achieved (Fig. 8) the TCPF coincides with the defined point.

Table 4

Position vector parameters for ABB IRB6620 Robot

	ABB IRB6620	Fanuc m2000iA
a + *a*0	680	1300
D	200	445
E	200	100
F	100	200

Table 5

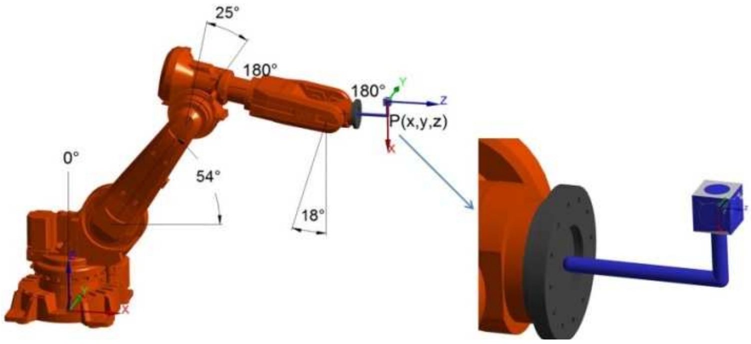
ABB IRB6620 (Position of effector *Px* = 1975, *Py* = 0, *Pz* = 1100)

	Radians	Degrees
θ1	0	0°
θ2	-1.089302	-62.412413°
θ3	0.381833	21.877473°
θ4	3.141592	180°
θ5	0.863328	49.465059°
θ6	3.141592	180°

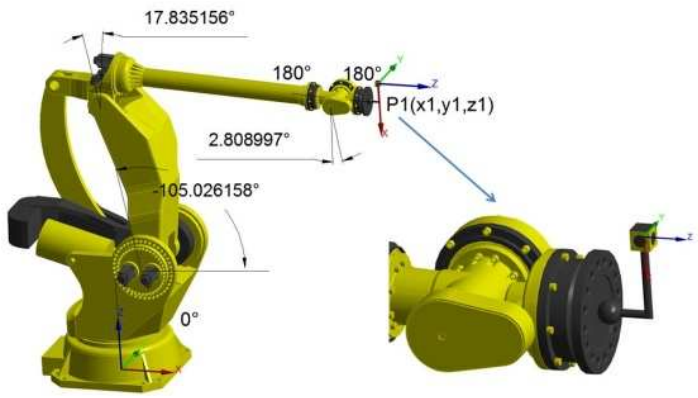
Table 6

Fanuc m2000iA (Position of effector *Px* = 2875, *Py* = 0 and Pz = 4909)

	Radians	Degrees
θ1	0	0
θ2	1.57132945	-90.030548°
θ3	-0.610337	-34.964052°
θ4	0	0°
θ5	0.6107707	-35.9946°
θ6	0	0



a



b

Fig. 8. Geometrical validation for: *a* – ABB IRB6620; *b* – Fanuc m2000iA 900L.

9. CONCLUSIONS

In the present work, a forward an inverse kinematics parametric analytical model was discussed. Symbolic solutions for the inverse kinematics are presented. Two industrial robots were studied, ABB IRB6620 serial-link open chain kinematics and a Fanuc m2000iA 900L.

REFERENCES

[1] A.F. Nicolescu, *Roboți Industriali* (Industrial Robots), EDP Publishing House, Bucharest, 2005.

[2] G. Khushdeep, D. Sethi, *An analytical method to find workspace of a robotic manipulator*, Journal of Mechanical Engineering, Vol. 41, No. 1, 2010, pp. 25–30.

[3] A. Srikanth, M. Sravanth, V. Sreechand, K.K. Kumar, *Kinematic Analysis of 3 dof of serial robot for industrial applications*, International Journal of Engineering Trends and Technology (IJETT), Vol. 4, Iss. 4, 2013, pp. 1000–1004.

[4] A.F. Nicolescu, *Proiectarea și operarea roboților industriali* (Industrial robots design and operation), course notes, University "Politehnica" of Bucharest, available at: `http://imst.curs.pub.ro /2015/login/index.php`

[5] *** ABB Robotics, *Product specification IRB 6620*, Sweden, 2004.

[6] *** Fanuc Robotics, *M-2000iA Series*, available at: `http://www.fanucrobotics.com/cmsmedia/ datasheets/M-2000iA%20Series_23.pdf`, accessed: 2015-09-04.

[7] A. Khatamian, *Solving Kinematics Problems of a 6-DOF Robot Manipulator*, Proceedings of the International Conference on Scientific Computing (CSC), pp. 228−233, Las Vegas – Nevada, 07.2015.

[8] K.K. Kumar, A. Srinath, G. Jugalanvesh, P. Premsai, M. Suresh, *Kinematic Analysis and Simulation of a 6 DOF KukaKR5 Robot for Welding Application*, International Journalof Engineering Research and Applications (IJERA), Vol. 3, Issue 2, 2013, pp. 820−827.

[9] T. Yong, F. Chen, X. Hegen, *Kinematics and Workspace of a 4-DOF Hybrid Palletizing Robot*, Advances in Mechanical Engineering, Vol. 2014, 2014.

[10] A.M. Ivan, *Cercetari privind optimizarea exploatarii robotilor industriali pentru prelucrari prin aschiere* (PhD Thesis), University "Politehnica" of Bucharest, 2011.

[11] S. Popa, A. Dorin, A.F. Nicolescu, A.M. Ivan, *Quaternion-Based Algorithm for Direct Kinematic Model of a Kawasaki FS10E Articulated Arm Robot*, Applied Mechanics and Materials, Vol. 762, 2015, pp. 249−254.

[12] A.F. Nicolescu, A.M. Ivan, *Advanced Off-line programming and simulation of a flexible robotic manufacturing cell using Kawasaki PC-ROSET* Proceedings in Manufacturing Systems, Vol. 6, Iss. 4, 2011, pp. 239−244.

[13] Y. Xu, M.C. Nechyba, *Fuzzy inverse kinematic mapping: Rule generation, efficiency, and implementation*, Proceedings of the 1993 IEEE/RSJ International Conference, Vol. 2, 1993, pp. 911−918.

[14] S. Gurjeet, K.B. Vijay, *ANFIS Implementation for Robotic Arm Manipulator*, International Journal of Engineering Research & Technology (IJERT), Vol. 1, Iss. 4, 2012.

[15] V.C. Moulianitis, E.M. Kokkinopoulos, N.A. Aspragathos, *A Method for the Approximation of the Multiple IK Solutions of Regular Manipulators Based on the Uniqueness Domains and Using MLP*, Robotics and Mechatronics, Vol. 37, 2016, pp. 273−281.

[16] H. Ananthanarayanan, R. Ordóñez, *Real-time Inverse Kinematics of (2n+ 1) DOF hyper-redundant manipulator arm via a combined numerical and analytical approach*, Mechanism and Machine Theory, Vol. 91, 2015, pp. 209−226.

[17] J.C. Craig, *Introduction to robotics: mechanics and control*, Addison-Wesley, 1990.