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ASPECTES REGARDING OPTIMAL DESIGN OF MACHINE TOOL FEED DRIVES

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Abstract: *The paper presents some of the main aspects concerning the design of feed drives of machine tools in optimum conditions. Some general considerations about the optimization vector including a set of parameters are presented. Also the optimization vector is analyzed and purpose function discussed, revealing the optimization vector as a favourable one having a limited number of parameters. Starting from the dynamic equation of the feed drives having as final mechanism the screw nut and pinion rack, respectively, the relation of the reduced moment of inertia to the electric motor rotor axis together with the expression of acceleration of the speed are presented. For four cases of transmission between motor and final mechanism in the feed drive, the relations of the optimum values for the optimization parameters (gear ratios, belt transmission ration, screw pitch, pinion diameter) are given. The problem of optimization is treated also by considering the expression of rotor acceleration as a function of two variables (where possible) and by finding its maximum together with the optimum parameters. Finally, the CAD models for the studied cases are used to obtain other information, such as the maximum torque supplied by the electric motor in optimum conditions. All applications are considered for an industrial project of refabrication of a machine for processing wheel set running profiles by turning, especially the feed drives that become specific for numerical control.*

Key words: *feed drive, dynamic equation, moment of inertia, acceleration, parameters, optimization.*

1. INTRODUCTION

To use the optimum technological potential created by processing with high speed cutting, machine tools are imposed new requirements regarding the construction, kinematics, driving and control, requirements that define a new conception of the machine tool. These requirements have as response on bringing on the market milling machining centers, machining centers by roto-milling, multifunctional machine tools and machine tools of hexapod type.

After choosing an F/P KC structure, in the design practice it goes to the next step consisting of optimizing the prototype. Consequently, the design of the mechanical structure of F/P KC in which the lead screw is supported by bearings the technical parameters indicated by the beneficiary must be considered, i.e. the length of travel l of the mobile element, mass-mass m_s of the assembly table-part; external load $F_{\max}$, speed n_{sc} of the lead screw, the torque M_t, temperature increase Δt, and other factors that influence the behavior of static, dynamic and thermal behavior.

Regarding the total stiffness k_{tot}, life in hours $L_{H,}$ natural frequency f_s, amplitude of external vibrations q_s, stability in buckling, and critical speed of the lead screw, the diameter d_{sc} should be chosen as high as possible. Instead, considering the acceleration time t_a, braking length l_f, thermal rigidity, and especially low purchasing costs, it is required the choice of a screw with a diameter as small as possible.

These contradictory trends require optimization of screw-nut-bearings assembly to satisfy part or all of the previously mentioned goals required by the customer. Consequently, it must optimize the construction depending on the diameter lead screw d_{sc}, experience and designers' "feel" being proven unsatisfactory.

The optimization of a mechanical structure of the F/P KC, in which the mechanism for converting the rotary motion into linear one is of ball-screw nut type, requires grounding of specific terms.

1.1. Optimization vector $p = [p_i]$

This vector contains all the parameters p_i of free factors or factors with a limited share on the dynamic behavior of F/P KC. According to studies recently achieved [1], the vector optimization p is defined as:

$$p = [p_i] = \left[d_{sc}, p_{sc}, z_{sp}, C_{din\ ax}, k_2, D_j\right]^T; i = 1, 2, ..., n_p, \quad (1)$$

where $C_{din\ ax}$ is the dynamic capacity of the axial bearings, k_z – stiffness of the balancing system of the masses with translation motion; D_j – pitch diameter of the gears from reducer structure (j = 1, 2,…, N; $N \geq 0$); z_{sp} – number of turns of the nut.

Depending on the purpose, the vector [p_i] is defined by all parameters n_p or only by some of them ((n^*_p).

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1.2. The whole constraints $q_k(p) \le 0$; $k = 1; 2;\ldots; n_B$, $n_B \ge 0$

It consists of all the important characteristics and properties of the mechanisms of the structure of F/P KC, which depend on the optimization parameters p_i. In general, one uses the characteristics that are required for certain limitations (e.g. total static stiffness, life, natural frequencies, etc.) to ensure the technological accuracy of the machine tools and quality of the machined surfaces. Using the conditions terms q_k (p), the internal forces caused by vibrations can be limited (i.e. the cutting force acting on the movable element of mass m_s). The user can choose which of the factors n_B^* of conditions q_k (p) will be used to achieve the optimization. This choice and simple boundary conditions imposed on the optimization parameters of the form $[p_{i\,\min} \le p_{i\,\max}; i = 1,2 \ldots n^*]$ define the accepted range P in which the optimum $\hat{p}$ must be searched.

1.3. Purpose function $\Phi(p)$

Defined mathematically what is sought through optimization, the purpose function is expressed as the sum of three partial functions:

$$\Phi(p) = w_1(k_1 \cdot t_A)^2 + w_2(k_2 \cdot \sum J_i)^2 + w_3(k_3 \cdot m_k)^2, \quad (2)$$

where t_A is the start up time (acceleration time) of feed drive; $\sum J_i$ – reduced moment of inertia; m_k – lead screw mass; $k_{1,2,3}$ – coefficients-norm (introduced by computer); $w_{1,2,3}$ – weight coefficients selected by the user.

Choosing mathematical optimization method should be done rigorously, because it is not a *conditional* optimization. On the other hand, function-purpose $\Phi(p)$ and the restrictive conditions $q_k(p)$ are not linear. By solving the optimization problem it is defined

$$\min[\Phi(p)] = \Phi(\hat{p})$$
$$\hat{p}\begin{cases} p_{i\,\min} \le p_i \le p_{i\,\max}\ ;\ i = 1,2 \ldots n_p^* \\ q_k(p) \le 0\ ,\ k = 1,2 \ldots n_B^* \end{cases}, \quad (3)$$

To solve this problem of optimization one should use methods for numerical calculation of Penalty-Method type, based on which a computer program is achieved. Together with other programs designed for static and dynamic behavior study of feed drive it constitutes a software package to optimize the mechanical structure of these KC. Achieving these software packages is greatly facilitated if the design is conceived based on catalogues, as all the variables introduced already meet some special requirements on stiffness, durability, temperature rise.

Other parameters p_i to be determined can be established through the design catalogue (e.g. screw pitch p_{sc}, number of turns z_{sp} of the nut, preload forces in thrust bearings and nuts, dynamic capacity $C_{din\ ax}$). For other optimization parameters there are other recommendations on allowable range, which simplifies the optimization problem.

The dependence of *optimization quality* $\hat{p}$ judged by the value of the objective function $\Phi(\hat{p})$ and the optimum time t_{opt} needed to resolve through a computer program based on the number n^*_p of active parameters used for optimization, reveals that the vector optimization can be considered as the favourable one if:

$$p = [d_{sc}, p_{sc}, D_N, k_z]^T,\ (n_p^* = 4). \quad (4)$$

In the absence of the balancing system (screw drives are horizontally arranged), $m_z = 0$, so that $n^*_p = 3$..

After checking into practice of the results on optimizing of mechanical structures of feed drive type, the proposed methodology may be approved, the design time being reduced and the quality of these structures increasing.

2. DYNAMIC EQUATION OF THE FEED KINEMATIC CHAIN WITH SCREW NUT AS FINAL MECHANISM

For the dynamic analysis of the feed drive shown in Fig. 1, one starts from the dynamic equation of motion at the level of the rotation axis of the electric motor. In the equations is involved the reduced moment of inertia J_r at the rotor axis, angular velocity ω, as well the torques in the system:

$$J_r \cdot \frac{\mathrm{d}\omega}{\mathrm{d}t} + k \cdot \omega = M_m - M_0 - M_l\ , \quad (5)$$

where J_r is the moment of inertia reduced to the rotor axis;
k – viscous damping constant (for the driving electric motor it is supplied by the company catalogue);
ω – angular velocity of the element for reducing (electric motor rotor);
M_m – motor torque created by the electric motor;
M_0 – torque of the static friction in the motor bearings;
$M_{l\ red}$ – load torque generated by transmission utility reduced to the rotor motor axis (including friction in system others than those previously presented).

From the motion equation it is of interest to extract the angular acceleration for maximizing it. It becomes:

$$\varepsilon = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{(M_m - M_0 - M_{l\ red}) - k \cdot \omega}{J_r} \quad (6)$$

2.1. Different types of feed drive kinematic structures

Considering the four cases shown in Fig. 1 [3, 4], one can write the reduced moment of inertia for each case as follows:

- **Case 1** (Fig. 1,*a*)

$$J_{red} = J_{rot} + J_S + m_T \cdot \left(\frac{p}{2\pi}\right)^2; \quad (7)$$

The angular acceleration is:

$$\varepsilon = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{(M_m - M_0 - M_{l\ red}) - k \cdot \omega}{J_{rot} + J_S + m_T \cdot \left(\frac{p}{2\pi}\right)^2}\ , \quad (8)$$

Or by simplifying the equation, considering $M_0 = 0$ and $k = 0$

$$\varepsilon = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{M_m}{J_{rot} + J_S + m_T \cdot \left(\frac{p}{2\pi}\right)^2}. \qquad (9)$$

In terms of the spindle pitch p influence, the acceleration is greater when the pitch is smaller. Therefore, from a range of values of p, the smallest possible one should be chosen. A small value involves a small diameter and a smaller moment of inertia J_S, which has as effect also the possibility of increasing acceleration.

- **Case 2** (Fig. 1,*b*)

In this case the expression of the reduced moment of inertia is:

$$J_{red} = J_{rot} + J_{P1} + \left[J_{P2} + J_S + m_T \cdot \left(\frac{p}{2\pi}\right)^2\right] \cdot \frac{1}{i^2} \qquad (10)$$

and the acceleration of the motor rotor has the same form as Eq. (9) with denominator given by (10).

Due to difficulties in calculation, the final element acceleration will be considered:

$$a = \frac{M_m}{i_1 \cdot 2\pi} \frac{p}{J_{red} = J_{rot} + J_{P1} + \left[J_{P2} + J_S + m_T \cdot \left(\frac{p}{2\pi}\right)^2\right] \cdot \frac{1}{i^2}}$$

(11)

For maximization of acceleration from the point i of view, the derivative of acceleration versus i should be zero [3]:

$$\frac{\mathrm{d}a}{\mathrm{d}i} = 0 \Rightarrow$$

$$J_{P2} + J_S + m_T \cdot \frac{p^2}{4\pi^2} - (J_{rot} + J_{P1}) \cdot i^2 = 0; \qquad (12)$$

One obtains the optimum value of i:

$$i_{opt} = \sqrt{\frac{J_{P2} + J_S + \frac{m_T \cdot p^2}{4\pi^2}}{J_{rot} + J_{P1}}}. \qquad (13)$$

Using the same procedure, it is obtained the optimum value of p considering it as variable and i constant:

$$p_{opt} = \sqrt{\frac{\left[J_{P2} + J_S + (J_{rot} + J_{P1}) \cdot i^2\right]}{\frac{m_T}{4\pi^2}}}. \qquad (14)$$

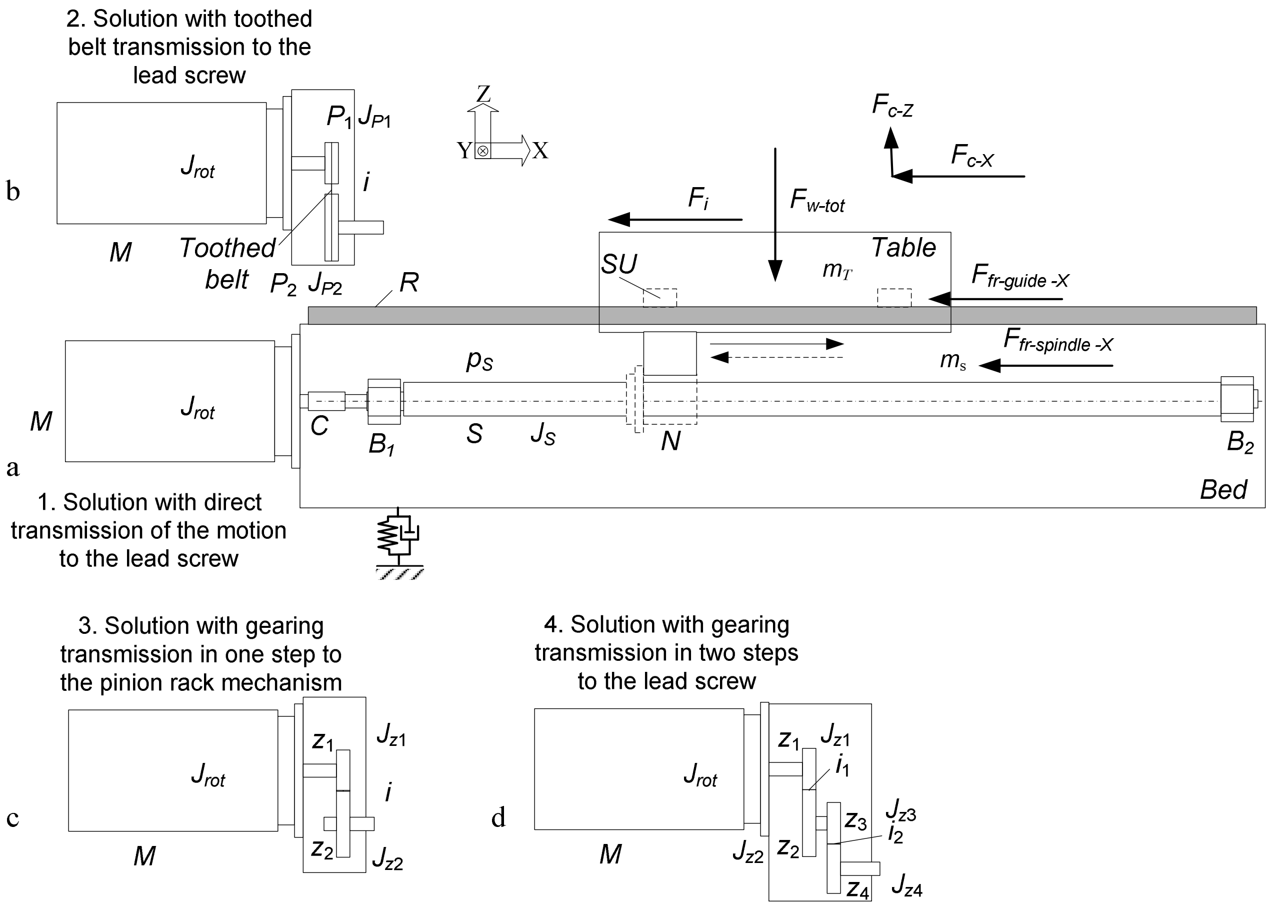

Fig. 1. Kinematic structure of a feed chain having the screw nut mechanism as final one with: *a* – direct transmission; *b* – belt transmission; *c* – gear; *d* – gearing in two steps.

- **Case 3** (Fig. 1,*c*)

The case 3 is similar to the case 2 with the difference that the two pulleys are replaced by two gears. Therefore, the relations for i_{opt} and p_{opt} are:

$$i_{opt} = \sqrt{\frac{J_{z2} + J_S + \frac{m \cdot p^2}{4\pi^2}}{J_{rot} + J_{z1}}}; \tag{15}$$

$$p_{opt} = \sqrt{\frac{\left[J_{z2} + J_S + \left(J_{rot} + J_{z1}\right) \cdot i^2\right]}{\frac{m}{4\pi^2}}}. \tag{16}$$

- **Case 4** (Fig. 1,*d*)

The case 4 is characterized by two gear transmissions z_1 / z_2 and z_3 / z_4 and also by the reduces moment of inertia given by

$$J_{red} = J_{rot} + J_{z1} + \left(J_{z2} + J_{z3}\right) \cdot \frac{1}{i_1^2} + \left[J_{z4} + J_S + m_T \cdot \left(\frac{p}{2\pi}\right)^2\right] \cdot \frac{1}{i_1^2 \cdot i_2^2}. \tag{17}$$

The optimization problem is more complex in this case due to the variable number, which is three (i_1, i_2, p).

The expression of the optimum pitch p is similar to the previous ones but specific to the kinematic chain configuration (for linear acceleration a):

$$p_{opt} = \sqrt{\frac{\left(J_{rot} + J_{z1}\right) \cdot i_1^2 \cdot i_2^2 + \left(J_{z2} + J_{z3}\right) \cdot i_2^2 + J_{z4} + J_S}{\frac{m_T}{4\pi^2}}}. \tag{18}$$

The acceleration dependency on the transfer ratios is given by a function of two variables i_1 and i_2 of the second order.

The transfer ratios are influencing also the moments of inertia of the gears ($J_{z1} - J_{z4}$). Considering the same modulus m and gear widths B for the two gearings, the gear diameters are:

$$D_{zi} = m \cdot z_i. \tag{19}$$

The moments of inertia are given by

$$J_{zi} = \frac{MR^2}{2} = \frac{\rho \cdot V \cdot \left(\frac{D_{zi}}{2}\right)^2}{2} = \frac{\rho \cdot \pi \cdot \left(\frac{D_{zi}}{2}\right)^2 \cdot B \cdot \left(\frac{D_{zi}}{2}\right)^2}{2} = \frac{\rho \cdot \pi \cdot B \cdot \left(\frac{D_{zi}}{2}\right)^4}{2} = \frac{\rho \cdot \pi \cdot B \cdot \left(\frac{m \cdot z_i}{2}\right)^4}{2} \tag{20}$$

The transfer ratios and the corresponding tooth numbers, considering known z_1 and z_2 are:

$$i_1 = \frac{z_2}{z_1}, \quad i_2 = \frac{z_4}{z_3} \Rightarrow \quad z_2 = z_1 \cdot i_1, \quad z_4 = z_3 \cdot i_2 \tag{21}$$

In practice, they are in the range 1/5 – 1/1. Therefore, for gears z_2 and z_4 the moments of inertia are depending on i_1 and i_2, respectively:

$$J_{z1} = \frac{\rho \cdot \pi \cdot B \cdot \left(\frac{m \cdot z_1 \cdot i_1}{2}\right)^4}{2}; \quad J_{z2} = \frac{\rho \cdot \pi \cdot B \cdot \left(\frac{m \cdot z_3 \cdot i_2}{2}\right)^4}{2}. \tag{22}$$

Having three variables (i_1, i_2, p), the problem of optimization could be solved by representing the acceleration as surface by a function of two variables in condition of making one (p) constant.

- **Case 5** (Fig. 2)

The feed kinematic chain structure shown in Fig. 2 has the particularity that the final mechanism is of rack pinion type. There is also a gear transmission z_1 / z_2 having the ratio i. This kinematic chain type is used for strokes of the final elements (tables, slides) long and very long used in heavy duty machine tools. In this case, the mechanism screw nut cannot be used, the spindle length being limited. Following the same reasoning as in previous cases, the moment of inertia reduced to the motor rotor axis is:

$$J_{red} = J_{rot} + J_{z1} + \left[J_{z2} + J_S + J_P + m_T \cdot \left(\frac{d}{2}\right)^2\right] \cdot \frac{1}{i^2}. \tag{23}$$

There are two parameters i and d that influence the acceleration relation:

$$\varepsilon = \frac{M_m}{\left(J_{rot} + J_{z1}\right) \cdot i^2 + J_{z2} + J_S + J_P + m_T \cdot \frac{d^2}{4}}. \tag{24}$$

The acceleration becomes maxim when

$$\frac{d\varepsilon}{di} = 0; \quad \frac{d\varepsilon}{dd} = 0. \tag{25}$$

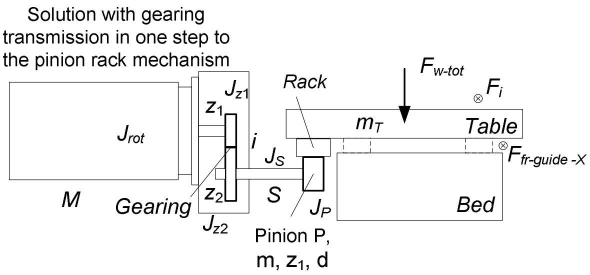

Fig. 2. Kinematic structure of a feed chain having the rack pinion mechanism as final one.

Therefore, the optimum values i_{opt} and d_{opt} are obtained as:

$$i_{opt} = \sqrt{\frac{J_{z2} + J_S + J_P + \dfrac{m_T \cdot d^2}{4}}{J_{rot} + J_{z1}}};$$

$$d_{opt} = \sqrt{\frac{\left[J_{z2} + J_S + J_P + \left(J_{rot} + J_{z1}\right) \cdot i^2\right]}{\dfrac{m_T}{4}}}. \quad (26)$$

2.2. Problem of maximum solved by surface representation

For the case in which two parameters p_1 and p_2 are involved in angular acceleration expression,

$$\varepsilon = f(p_1, p_2), \quad (27)$$

one can use methods of representation of functions of two variables as surfaces together with maximization procedures. Software MATLAB supplies instruments for plotting surfaces by *mesh* function and extracting the maximum of a function by *max*. The two parameters are given values for representation in ranges by using *meshgrid*.

The structure of a kinematic chain with two gear transmissions given in Fig.1,*d* and the constants of the Eq. (18) shown in Table 1 are considered together with the motor torque M_m.

Using the values given in Table 1, and p = 10 mm, the acceleration surface is obtained (Fig. 3). The maximum value can be obtained together with the corresponding optimum values i_{1opt} and i_{2opt} by using function *max*. It can be easily observed from the graph that the values are 5 for both ratios for an angular acceleration of ε = 156.1 rad/s^2.

The case 2 (Fig. 1,*b*) can be treated also by surface plotting followed by the maximization procedure. For plotting, the corresponding values are taken from Table 1. The rotor acceleration relation is:

$$\varepsilon = f(i_1, p). \quad (28)$$

The surface plot (Fig. 4) shows that the maximum value of acceleration ε = 58.17 rad/s^2 is obtained for i_1 = 5 and p = 5 mm. As one can see on the chart, great values of the pitch p are not recommended in terms of maximum acceleration. The most important conclusion is that the ratios should be close to i = 5 in all cases.

Table 1

Constants resulted from the structure shown in Fig. 3

Quantity	Value
p	10 mm
J_{z1}	2.05E-05 kgm^2
J_{z2}	0.000368 kgm^2
J_{z3}	3.92E-05 kgm^2
J_{z4}	0.000627 kgm^2
J_{rot}	0.064 kgm^2
J_s	0.00023306 kgm^2
m_t	450 kg
M_m	10 Nm

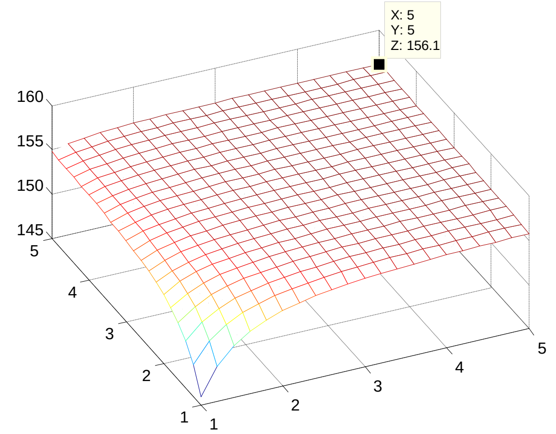

Fig. 3. Surface representation of acceleration $\varepsilon = f(i_1, i_2)$.

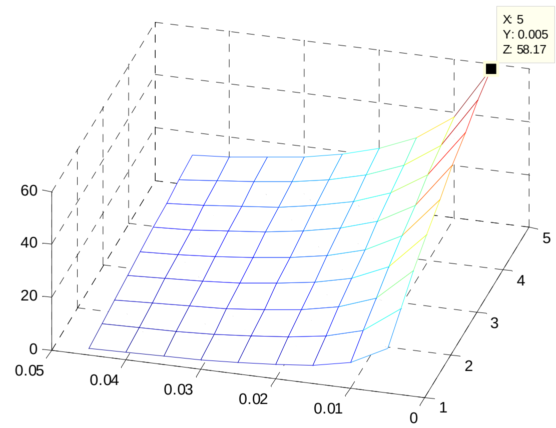

Fig. 4. Surface representation of acceleration $\varepsilon = f(i_1, p)$.

The meaning of maximum acceleration is not regarded as a limitation. It indicates only the optimum values of parameters i_1, and p.

The same results can be obtained for the case 3 (Fig. 1,*c*) when the gear is replaced by a toothed belt transmission P_1/P_2.

For the feed drive structure with rack pinion as final mechanism, the acceleration surface of the variables i_1 (gear ration) and d (pinion diameter) is shown in Fig. 5.

In this case, the maximum value of acceleration is obtained for i_1 = 5 and pinion diameter d = 50 mm. As in the previous cases, the recommended transfer ratio is 5.

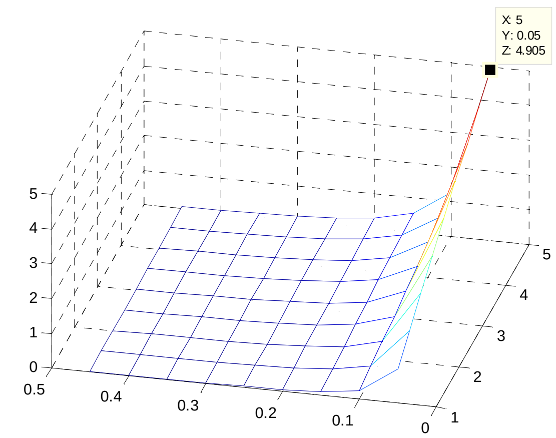

Fig. 5. Surface representation of acceleration $\varepsilon = f(i_1, d)$.

The pinion diameter should be as small as possible for an optimum solution. Its diameter depends on the end shaft diameter on which it is mounted.

3. MULTIBODY MODELS OF FEED DRIVES

Let us consider the same example of the feed drive of a machine for processing by turning the wheel set.

The kinematic chain was modelled in Inventor Professional (Fig. 6) [6]. The model is transferred in the Dynamic Simulation module [7]. The simulation is done for a variation of the speed set in a specific window. The rotor axis is speeded up from 0 to 3000 rpm = 314 rad/s.

The simulation of the radial feed drive is supplying the graph of the torque [4] about Z axis of the rotor (Fig. 7). The model is achieved in the same conditions as the previous one (J_{rot} = 0.064515, M_T = 415 kg, etc.). The maximum torque obtained is M_m = 10.5 Nm for an acceleration time t_{acc} = 0.3 s.

For the variant with gear transmission, the maximum torque is reduced according to the transfer ratio of the gear i = 3.846 with regard to the solution without transmission (Fig. 8). The maximum torque is M_m = 2.5 Nm (t_{acc} = 0.3 s) (Fig. 9). Obviously, by changing the speed and time, the maximum acceleration given by the optimization procedure can be obtained. Therefore, the acceleration time becomes another optimized parameter.

4. CONCLUSIONS

The paper brings in discussion a few possible solutions for a feed drive used in radial motion of the a turning machine for wheel set profile processing, considering the parameters that are involved in the dynamic equation of the kinematic chain. The parameters are connected with mechanisms used in the feed drive, namely gear ratios − i, toothed belt transmission − i, screw nut − p, rack pinion –d. The moment of inertia is considered with

Fig. 6. CAD functional medal of the radial feed drive of a turning machine for wheel sets.

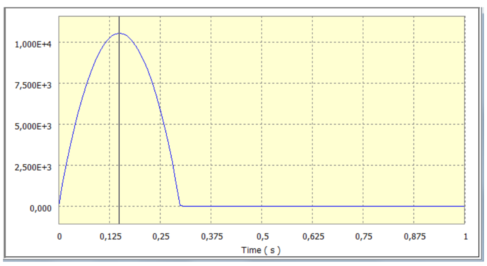

Fig. 7. Variation of torque in the acceleration phase.

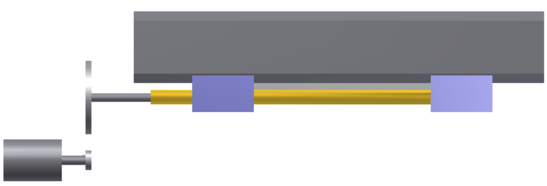

Fig. 8. CAD functional model of the radial feed drive of a turning machine for wheel sets.

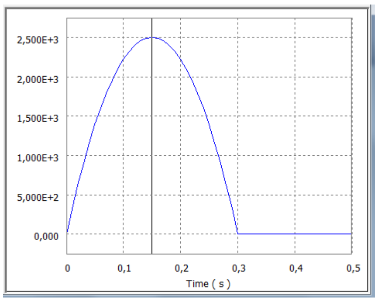

Fig. 9. Variation of torque in the acceleration phase (Case 2).

respect to the rotor motor axis. Some relations regarding the acceleration equation for the feed drive variants are presented at the level of the rotor (rotation), or at final element level (translation). The maximization of the acceleration equation led to the known optimum relations of the parameters involved. The method of representation the function acceleration of two variables and and maximization is presented. The conclusions are related to gear ratios that are recommended to be 5, the screw pitch to be 5 mm or 10 mm depending on the screw diameter. The diameter of the rack is also recommended to be as small as the construction allows. The paper proposes also the use of multibody models for making verification after parameters optimization, namely to obtain the maximum torque at the rotor axis level in different conditions, especially of acceleration time.

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