

## MODELING AND SIMULATION OF HIGH SPEED SPINDLE, CURRENT PROBLEMS AND OPTIMIZATIONS – A SURVEY

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**Abstract:** Rotating machineries play an important role in industry and medicine. The main purpose of this study is to review in a systematic mode the main papers published in the area of high speed shafts, the main problematics and modeling and simulation. The studies about dynamics of spindle bearing assemblies are concentrated on two parts: one part refers to bearings and another one to shaft. Some paper deals with optimization of good functionality of assemblies that means deformations, temperature and vibration are in normal limits due to different design aspect as location of bearings or shaft design.

**Key words:** Spindle-bearing system, finite element method, stiffness matrix, dynamic models.

### 1. INTRODUCTION

Machines having rotating spindles like lathes, grinding machines, and milling machines are very important and useful for modern manufacturing processes. The assessment of a good functionality and a performance issue involves the deformation, vibrations and temperature basically due to unbalance of the main spindle, friction effects in bearings, cutting forces, chatter, and cooling systems. The effects can be a poor quality of the manufacturing product but also the wear or even destruction of machine as effect of a long time stress reflected by unfit functionality.

Spindle bearing systems belong to rotating machinery class and the operations of cutting material involves an analysis of dynamical model of the spindle bearing system. Many of the research articles use the approaches that are proposed for rotor dynamic analysis [1–6], however in the high speed spindle case the approach being more complex.

The dynamic behavior of spindle-bearing system is essential to be understood in order to design a performant machine that uses this system. The design can be less or more complex and the models used in simulation can vary from case to case depending on the tradeoff between accuracy of result and complexity of model and calculus.

There are many types of bearings used for rotors in rotating machinery and one of the most frequently type is angular contact ball bearings. The angular contact ball bearing offers some advantages in rotating machinery worth to be mentioned: good stiffness properties, low friction, long lifetime, and a good ratio between cost and effectiveness especially at high speed [7, 8]. The critical subsystem in the rotating machinery is made by ball

bearings that transfer the contact force, heat due to friction and vibration that is transferring through the other elements to the housing. The modeling of bearing dynamics influences the results of dynamic analysis for shaft bearing system. The spindle bearing system has at least two sets of bearings mounted in different positions depending of design condition and optimization objectives.

Most of modern machines have as component a motorized spindle [7]. The motorized spindle has the motor built-in the housing, no transmission elements (gears or coupling) being necessary, meanwhile the externally driven spindles have usually a belt transmission providing rotation from an electrical engine.

The ratio of power and volume can be especially high in the case of motorized spindle and as sequel a cooling system is necessary. The air or cooling liquid flows through a muff to stator of the motor and often the flow is present on the outer ring of the rolling bearings. An accurate dynamic model of spindle bearing systems must take into account the cooling parameters of this subsystem.

The clamping system for grinding wheel can play also a role in dynamic modeling of shaft deflections when cutting forces are taken into account. The cutting conditions and the spindle speed have the influence in dynamic modelling. Usually, two speed regimes are taken into account for dynamic modelling: low speed and high speed. The main differences can be seen in rolling bearing in angular contact calculus.

There are many studies that analyze theoretically and experimentally the dynamic behavior of machines that use spindles, housings, rolling bearings, and spindle-bearing systems. There are also few review papers that deal with this subject (e.g. [7, 9]). The purpose of this paper is to present structured view of the main approaches in dynamic modeling of spindle-bearing systems along with main possible optimizations.

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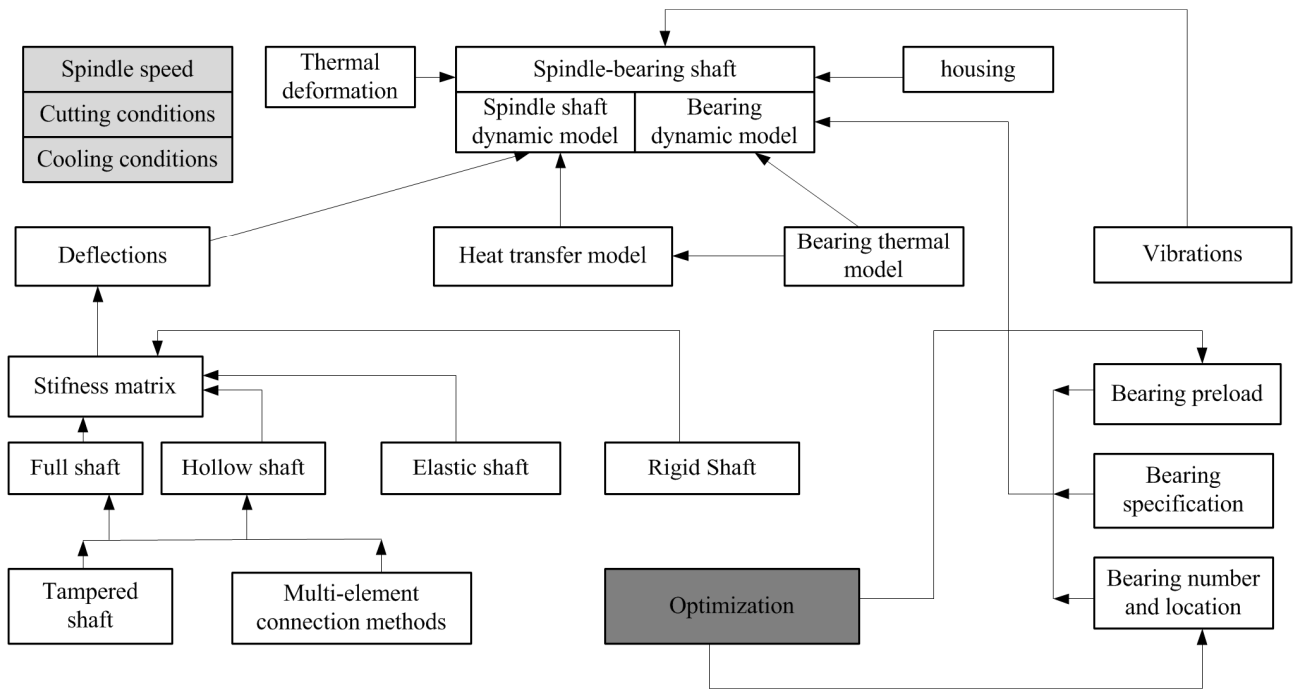


Fig. 1. The main approach subjects in spindle bearing dynamics and optimizations.

The main approach subjects in spindle bearing dynamics and optimizations along with interactions marked by connection lines with arrows are presented in Fig. 1. The schema is not an exhaustive. There are many papers that deal with particularly subjects connected with spindle-bearing subject that can be partially included in blocks of Fig. 1. The stability of shaft during exploitation is essential to prevent damages in rolling bearings and the spindle itself. The misalignment and inherent difference between geometric center of mass and real center of mass of spindle can generate and amplify the vibrations that finally can conduct to improperly function of system.

As methodology, the most used technique is based on finite element method. Even CAD/FEM tools exist, the models developed in this way cannot cover explicitly the dynamic phenomenon used by propose mathematical models, so focused developed application mainly based on Matlab simulation tools are propose by almost all authors.

2. DYNAMIC MODELS OF BEARINGS

In study of dynamic properties, the mass of bearing is usually neglected but in some situations this is included in the shaft mass.

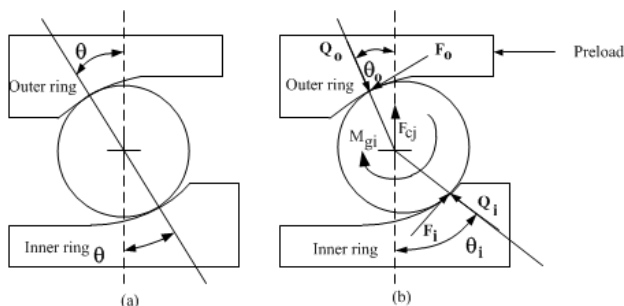


Fig. 2. Contact angles and contact forces of bearing: a – static load or low speed rotation; b – high speed rotation.

The basic models for bearings are linear and nonlinear. The linear models are composed from spring or rotation spring meanwhile the other are more complex [10–11]. A rheological model for bearing that use  $K_{ij}$  stiffness for spring and  $C_{ij}$  for damping in parallel connection is given by equation:

$$\begin{bmatrix} m_h & 0 \\ 0 & m_h \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} C_{xx} & 0 \\ 0 & C_{yy} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} K_{xx} & 0 \\ 0 & K_{yy} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \end{bmatrix} \quad (1)$$

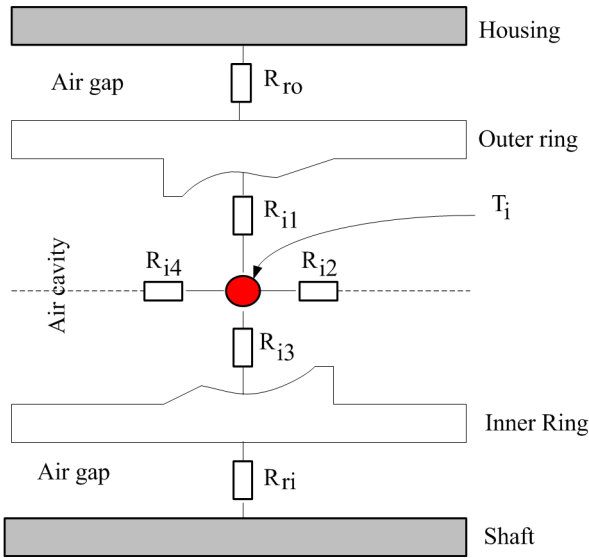
The damping coefficient is usually very small and as sequel, in many applications this factor can be neglected. The oil film bearing is a complicated sum of physic phenomenon but in many cases the same spring-damper model can be used.

The nonlinear models are based on contact point, the most used being EHL point contact (Elasto-Hydrodynamic Lubrication model) and Hertz contact. EHL theory is used in many papers that deal with rolling bearings.

The static load or low speed and high speed are different models of angular contact for ball bearings, Fig. 2. In Fig. 2,  $\theta_i, \theta_o$  are the contact angles,  $Q_i, Q_o$  – contact forces,  $F_{cj}$  – centrifugal force,  $M_{gi}$  – gyroscopic moment and  $F_i, F_o$  – forces due to gyroscopic moment [12–17]. The coefficient of friction is taken into account for equilibrium calculation between gyroscopic axial torque force and rotation speed in very few papers [18].

The stiffness contact between the balls and cage are complex and depend on load and the layout of bearing. The Hertz contact theory is the most used modeling the contact force between the inner and outer rings. If we denote by  $\delta$  the deflection and  $Q$  the contact force [9, 13, 19–20], it is obtained:

$$Q_i = k_i \delta_i^{3/2}, \quad Q_o = k_o \delta_o^{3/2} \quad (2)$$



**Fig. 3.** Heat transfer model in bearing and further to shaft and housing.

The stiffness coefficients  $k_i$  and  $k_o$  is a function of materials used for balls and ring, geometrical dimensions of bearing and loads. The contact angle is a function of forces, so an iterative method is required to calculate the stiffness coefficients. An empirical formula is proposed in [21] and used in [22]:

$$k_{as} = c_a P_a^{1/3} N_b^{1/3} (\sin(\theta))^{5/3} D_b^{1/3}; \quad (3)$$

$$k_{rs} = 0.64 c_a P_a^{1/3} N_b^{1/3} (\sin(\theta))^{2/3} \cos(\theta) D_b^{1/3}. \quad (4)$$

where  $c_a$  is an empirical data, based on experimental results,  $k_{as}$  static axial stiffness,  $k_{rs}$  – radial stiffness,  $P_a$  – axial preload,  $D_b$  – ball diameter,  $N_b$  – number of balls, and  $\theta$  – contact angle.

The contact angle is a function of forces, and iterative techniques are used to solve the nonlinear equations. The bearing preload mechanism affects the properties of spindle bearing systems and if a rigid preload is applied to bearing, a thermally preload occurs in most of the cases. If the preload is non-rigid, with springs, this thermally effect can be neglected.

At high speed, the radial stiffness decreases for angular ball bearing following a nonlinear curve [13]. An empirical formula is  $k_r = k_{rs} [1 - 6.52 \times 10^{-11} n^2]$ , where  $n$  is the number of revolutions per minute (rpm). At 30000 rpm, the stiffness is  $k_r \approx k_{rs} \cdot 0.94$ , which leads to a decrease with almost 6% of stiffness.

The theory from [12] includes the centrifugal and gyroscopic loading for the balls and the inner ring has five degree of freedom: three translation deflections and two rotations. There are also other approaches that improve this approach, e.g. a matrix form proposed in [23], even this model neglects the gyroscopic moments.

### 3. HEAT GENERATION IN BALL BEARINGS

In thermal model of spindle bearing system, the main sources of heat are the bearings and for motorized spindle the motor and possible viscous shear of air or oil in gaps between rotor and stator [24]. In most of the

papers, the heat source in the bearing is due to friction (friction due to lubrication viscosity and friction due to load – frictional torque and speed).

Two problems are taken into account: the heat generation (usually given by empiric or approximated formulas) and heat propagation to outer ring [13, 15, 25–28]. The main solution for heat propagation is in our opinion the heat transfer mesh by resistive rectangles, a discretization of continuous heat transfer in 2D space, Fig. 3. The detailed values of resistances from Fig. 3 are given in [29]. Based on work [13], the heat generated by a bearing is given by:

$$W_b = 1.047 \times 10^{-4} n M. \quad (5)$$

where  $n$  is the speed [rpm] and  $M$  – total frictional torque (the torque due to load –  $M_l$ , the torque due to spinning –  $M_s$  and torque due to viscous friction –  $M_v$ ).

$$M_l = f_1 F_\beta d_m; \quad (6)$$

$$F_\beta = F_a - 0.1 F_r \cdot f_1; \quad (7)$$

$$f_1 = s(P_0 / C_0)^h. \quad (8)$$

The value  $f_1$  depends on bearing design and load,  $s$  and  $h$  are taken from tables (e.g. for angular contact,  $s = 10^{-4}$ ,  $h = 0.4$ ),  $P_0$  – static load, and  $C_0$  is defined in ISO-76-2006.

$$M_v = \begin{cases} 10^{-7} f_0 (vn)^{2/3} d_m^3, & vn \geq 2000 \\ 160 \times 10^{-7} f d_m^3, & vn < 2000 \end{cases}; \quad (9)$$

$$d_m = (1 + \alpha \Delta T)(D_i + D_o) / 2. \quad (10)$$

In most of the cases,  $M_s$  is neglected, however in [15] a formula based on [13] is proposed:

$$M_s = \frac{3\mu Q a E}{8}. \quad (11)$$

where  $\mu$  is the friction coefficient,  $Q$  – normal contact force,  $a$  – semi-axis of ellipse (the major one) and  $E$  – the elliptical integral. A complete development of formula is given in [25, 30]. The formula is difficult to apply and in majority of cases being enough to consider only the rest of two moments.

### 4. SPINDLE SHAFT MODELS

Considerable work has been done in the area of bearing and spindle modeling but not much work has been done in coupling the spindle with bearings. The dynamic behavior of spindle is developed from elasticity theory for beams extended to rotors.

The beam theory has early started, before 1921. Three main theories developed for beams have been extended to rotor dynamics: Euler-Bernoulli beam theory, Rayleigh beam theory and Timoshenko beam theory.

Euler-Bernoulli is a simplification of elasticity theory that calculates the deflection of beam taking into account the bending phenomenon and translational inertia.

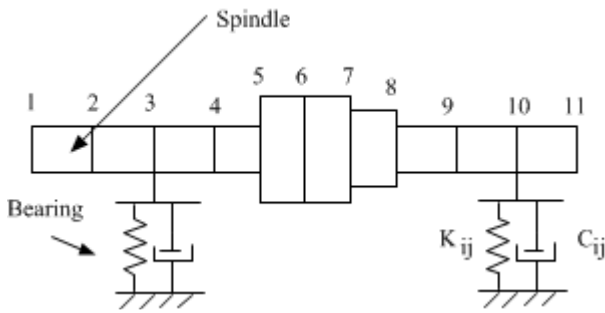


Fig. 4. Elements of spindle shaft.

The Bernoulli model is suitable for long thin beams. The Rayleigh beam theory extends the Bernoulli theory taking into account also the rotational inertia, meanwhile the Timoshenko model, the most complete one, takes into account the shear deformation. Centrifugal forces and gyroscopic moments are also included in Timoshenko rotor dynamic theory [31].

Most of the papers published in literature are related to Timoshenko beam. Two examples of papers using the Rayleigh beam theory are [32, 33]. Most of the papers use FEM (Finite Element Model) to calculate the deflection of spindle, as in [34]. The specific shapes can be cylinder, disk or tapered (or truncated cone). The shaft is in most of the cases a hollow one that is more resilient to shear forces than a solid one.

The pulley, when it exists, is usually modeled as a rigid disk. The main sources of chatter vibrations of high speed machines are the spindle and tool holder along with tool (e.g. grinding wheel).

#### 4.1. Dynamic analysis

To construct a FEM for spindle shaft, the beam is first discretized, and each node has assigned a number of degree of freedom, Fig. 4. There are two methods that are used for writing the equations of motion of the spindle:

- (1) based on elasticity theory;
- (2) calculation of kinetic and potential energy followed by determination of the Lagrangian in order to deduce the equations of motion. In matrix form, the equations of motion are given by [20]:

$$[M^b]\{\ddot{q}\} - \omega[G^b]\{\dot{q}\} + ([K^b] + [K^a] - \omega^2[M^c])\{q\} = \{F^b\}, \quad (12)$$

where  $[M^b]$  is the mass matrix for spindle,  $[M^c]$  – mass matrix for centrifugal forces,  $[G^b]$  – gyroscopic matrix,  $[K^b]$  – stiffness matrix,  $[K^a]$  – stiffness matrix for axial forces,  $\{F^b\}$  – forces (distributed and concentrated forces), and  $q$  – variables (including deflections).

The matrix stiffness calculation generated a number of important researches. The stiffness matrix is calculated easily for cylinders (hollow or not) but the problem is more complicated in case of tapered elements [35, 36].

The truncated cone is approximated with a number of cylinders equally spaced and the matrix is concatenated [36]. The precision depends on the number of cylinders (usually between 10 and 20). This implies many

calculations. The solution is computationally expensive if we want a high precision (e.g. 100 stepped cylinder).

Other solutions have been proposed valid for arbitrary shapes but the calculus algorithm is complex and the matrix form should be improved [37, 38].

A turning point in modeling using FEM of spindle split in many basic elements (usually cylinders of different radius) is multi-element connection of stiffness matrices in a single matrix of reasonable dimensions [39]. Even the classic methods exist, the most known being the superposition method (direct stiffness method) [39], there are methods that can offer higher precision, integral calculation function being achieved with difficulty for exact solution [37, 38].

The integration of spindle and bearing dynamics leads to spindle-bearing dynamic system. If we denote by  $x$  the deflection, the integrated equation becomes [20]:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} + \{R(x)\} = \{F\{t\}\}. \quad (13)$$

The matrices  $M$ , and  $C$  can have different forms, depending on the complexity level and simplifications proposed by different authors (structural damping matrix included or not, gyroscopic matrix included or not and so on). The form itself can be slightly different depending on application, so a supplementary coefficient or a different sign can be included [9, 22].

An analytical solution for eq. (12) is hard to be found and in fact it is possible practical only for simplified and simple cases. Numerical solution is the common method used to solve equation (12), usually based on finite difference method, and FEM approach. The most used method is Newton-Raphson method [20]:

$$[M]\{\ddot{x}\}_i^{t+\Delta t} + [C]\{\dot{x}\}_i^{t+\Delta t} + [K]\{\Delta x\}_i = \{F(t)\}_i^{t+\Delta t} - \{R(x)\}_{i-1}^{t+\Delta t}. \quad (14)$$

The method is quadratic convergent but other iterative methods can be used to solve numerically the equation of motion on the condition to verify the convergence of solution.

The calculation of stiffness matrix for multi-element spindle using the classic direct method can lead to very big matrices that are difficult to manipulate. In [41, 42] the authors proposed a method that can be used for asymmetric multi-segment bearings rotors by using receptance coupling method. The receptance technique is proposed to be used in an earlier work as an algorithm that can interconnect the matrices used in beam dynamics to predict the frequency response and deflection response to beam [43]. The receptance technique is based on combining two matrices in one by considering the points of receptance and function receptance at end of each segment.

#### 4.2. Heat Transfer in Spindle-Bearing Systems

In the spindle bearing systems, the major source of heat are the bearings. The heat transfer model has as scope the calculation of temperature distribution in the spindle-bearing system based on rolling bearing heat source [40].

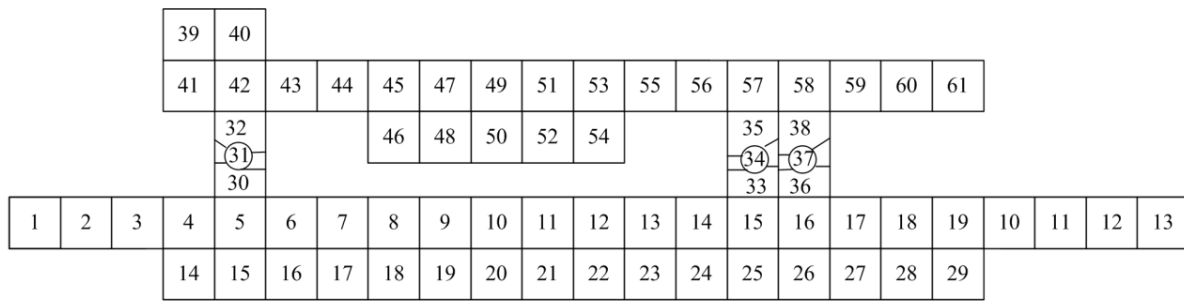


Fig. 5. Discretization of spindle bearing system (pulley belt transmission).

Few assumptions are made to calculate the distribution of temperature and thermal expansion from bearings through spindle to cage:

- the heat transfer is symmetric in rapport of longitudinal axis;
- shaft and the housing are approximated by circular cylinders;
- lubrication temperature is constant at one step time;
- all the ball bearing work at the same temperature;
- the only sources of heat in spindle are the bearings.

Related to the last assumption, there are a number of papers that consider the cutting process as source of heat, but this approach has not been found in literature in conjunction with heat source of bearing.

In motorized spindle, the heat transfer is more complicated [22, 24, 45] than in simple spindle bearing system as it is developed in [44]. The main heat transfer is made by convection, diffusion and radiation.

Some simplifications are usually made in considering the heat distribution, in order to propose a feasible model, e.g. the temperature in spindle is moderate and as consequence the radiation is neglected, small holes are neglected, etc. [15].

The models that are based on thermal networks of resistances assume that the thermal resistance is linear [46]:

$$R = \frac{L}{KA}, \quad (15)$$

where  $L$  is the length,  $A$  – cross sectional area and  $K$  – thermal conductivity.

A discretization in rectangular area of resistances has a density high enough in order to approximate correctly the heat diagram of temperature in all assembly. This approach involves a number of elements that can be up to 70 resistances for only one bearing [15].

The discretization method by finite difference (FD) can be applied to the whole structure and the balance energy method applied to each node gives the equation (conservation energy for node in a finite surrounded volume) [44]:

$$\sum_{i=1}^4 \frac{T_n - T_{ni}}{R_{ni}} + q_n = m_n c_n \frac{\partial T_n}{\partial t}, \quad (16)$$

where  $T_n$  and  $T_{ni}$  are the temperatures of node  $n$  and the related node  $ni$ ,  $R_{ni}$  – thermal resistance between node  $n$  and  $i$ ,  $q_n$ ,  $m_n$ , and  $C_n$  – heat transfer rate, mass, and

thermal capacity of node  $n$ , respectively. The Runge-Kutta method can be applied to solve the system.

Even if it seems to be simple, in eq. (16) the heat transfer coefficients need to be determined [15]. The operation involves the calculation of heat convection coefficients using Nusselt number for laminar and turbulent flow and results from [47] for free convection.

There are other approaches about heat dissipation analysis, few of them worth to be mentioned. The work [48] takes into account empirical formulas for air flow (and pressure of the air from a ventilation system). In article [27], the authors propose the use of more precise mathematical equations incorporating no formulas excepting the consideration for thin oil lubrication film in bearings. The results seem to be obtained only from experiments. An extensive approach is done in [49].

The axial thermal expansion is caused in most of the cases by the shaft and the spacer [22], this approach being present in few papers, probably due to the fact that the most important faults appear in bearings. The difference in expansion is linearly dependent on temperature [22].

### 3.3. Thermal deformations

Thermal deformations are mechanic deformations due to heat generation and propagation through the system. There are very few papers that deal with thermal deformation in a comprehensive approach [50, 51]. Transient thermal analysis is taken into account by both papers and thermic equilibrium is used taking into account the heat loads, convective heat transfer, thermal contact resistance (TCR) and bearing stiffness. Each paper presents a different method the accuracy of modeling being comparable with real measurement values.

The main difference between papers is the type of spindle bearing system: motorized in [50] and belt driven in [51]. In [50] the authors proposed a sophisticated method to calculate the total deformation of shaft using asperity quantification in contact mechanical model.

The rough surface morphology is described by fractal approach [52], and TCR is calculated taking into account the deformation of asperities of type elastic, plastic and fully plastic, based on approaches proposed in [53].

The approach in [51] is different, the authors proposing the use of a network to model the transient response of spindle. The thermal network, as general method supposed to split the assembly (in our case the

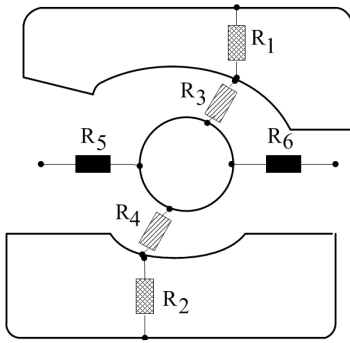


Fig. 6. The seven node model [based on 51].

spindle, bearings, balls, rings, and housing or cage) in a FEM manner in blocks that are modeled by thermal resistance, Fig. 5. The main purpose of the paper is to calculate the radial deformation of spindle under combined effect of stress and thermal deformation along with centrifugal stress. Also, the authors investigated the assembly tolerance effects an extended the approach to axial deformation [51].

The authors proved that the seven node model (Fig. 6) gives the curve closest to points collected from experimental data. The transient mode considers, different from steady state, that the net thermal flow in one node is equal to the internal energy status leading to an equation with derivative approach that suggests evolution [51], somewhat like equation (16). By discretization,

$$\frac{T_0^{k+1} - T_1^{k+1}}{R_{01}} + \frac{T_0^{k+1} - T_2^{k+1}}{R_{02}} + \frac{T_0^{k+1} - T_3^{k+1}}{R_{03}} + \frac{T_0^{k+1} - T_4^{k+1}}{R_{04}} = Q_{f0} - C_0 \rho_0 \frac{T_0^{k+1} - T_0^k}{\Delta \tau_k}, \quad (17)$$

where  $Q_{f0}$  is frictional heat in node 0, and  $\Delta \tau$  step in time series discretization.

From graphic analysis it results that for given system running at 8000 rpm the calculated temperature on outer ring is around 38.5 °C, meanwhile the real measured value is around 34.5 °C, with a difference of  $\approx 15\%$  [51].

The radial deformation in system for 8000 rpm is found to be [51]: 9.5  $\mu\text{m}$  for transient method (proposed by authors), 14.5  $\mu\text{m}$  for steady state method, meanwhile the experimental value is 11.5  $\mu\text{m}$ , that is an error of 5  $\mu\text{m}$  for steady state and an error of 2  $\mu\text{m}$  for transient method.

### 3.4. Vibration Analysis

Shaft vibration can have as cause few factors: residual unbalance, axial asymmetry, fluid bearings, and nonlinear properties of machinery components.

Two main subjects are related to approaches in vibration analysis for spindle-bearing systems: free vibrations [54] and frequency response function (FRF) under dynamic conditions, practical a modal analysis of assembly [11, 19, 42, 55–57].

The first step in [54] is the calculation of sum of kinetic and potential energy of all elements in spindle shaft model. The Lagrange equation is used and the equation of motion is obtained of a form similar to

eq. (12). The free vibrations are obtained by set  $\{F^b\} = 0$  and applying the method of separation of variables ( $w(x, t) = W(x) \cdot T(t)$ ) in terms of  $\Omega = 2\pi f$ , where  $\Omega$  is the frequency of rotation to be determined. In the case of rotating Bernoulli beam, the solutions are simple, but in the case of rotating Timoshenko beam they are more difficult to be obtained. Some papers (e.g. [11]) propose methods to calculate the eigensolutions and as sequel the natural vibrations using an earlier work done by [58].

The forced vibrations are usually made by an axial force applied axially to a spindle or transversal via bearings or rolling bearings (most frequently). The approaches are different mainly due to authors' concentration on different causes and estimate the influences leading to the apparition of vibration and sometimes of resonance in spindle shaft. In [55], the imbalance of rotating shaft is split in two parts: a static one (detected without rotation) and dynamic one, two equations being developed:

$$Q(r, f, \varepsilon, M, C_1, C_2, K_1, K_2) = U_S \cdot \omega^2 \cdot \sin(\omega \cdot t); \quad (18)$$

$$P(f, \varepsilon, M, \theta, I_\theta, I_o, J_1, J_2, C_1, C_2, K_1, K_2) = U_D \cdot \omega^2 \cdot \sin(\omega \cdot t). \quad (19)$$

where  $M$  is the spinning mass,  $f$  – the spindle speed [Hz],  $\varepsilon$  – eccentricity,  $C_{1,2}$  – the radial spinning damping, (eccentricity is taking into account),  $K_{1,2}$  – radial stiffness of bearings,  $I_\theta$  – traverse moment of inertia,  $U_S$  – static imbalance,  $U_D$  – dynamic imbalance,  $\omega$  – angular velocity of shaft, and  $J_{1,2}$  – distance between center of gravity and bearing.

In bearings, the dynamic loads have an harmonic character and for robust spindle dynamics the bearing must be isotropic,  $K_2(f, \varepsilon) = K_2(f, \varepsilon)$  and  $J_1 = J_2$  (if there are only two bearings). The graphic of resonance frequency ( $f_{RF}$ ) depending on spindle system with speed is practically linear and there is a difference of resonance frequency for cylindrical and conical shaft, e.g. for frequency of rotating shaft of 1000 Hz,  $f_{RF} \approx 2100$  Hz, and for conical shaft  $f_{RF} \approx 1980$  Hz [55].

A different approach is used in [19], where FRF is considered based on nonlinear model of bearings and simulating cutting force. The pulley is modeled as a rigid disk and the tool is connected rigidly to tool holder which is mathematically modeled by translational and rotational springs. The translational and rotational springs are used for modeling connection with the other parts of assembly, e.g. between spindle head and housing. The FRF method was used to find the natural frequencies in  $x$  direction (longitudinal) for tool holder that is of prismatic type [19]. In [42], the authors used multi-segment Timoshenko beams and in calculation of FRF complex stiffness of bearings was used.

## 4. OPTIMIZATION METHODS

Most of optimizations in spindle-bearing systems refer to the design of spindle, choice of bearings and tools (e.g. grinding wheel) and solution for components. There are relatively few optimizations that refer to two aspects: location of bearings [59] and preload optimizations [60].

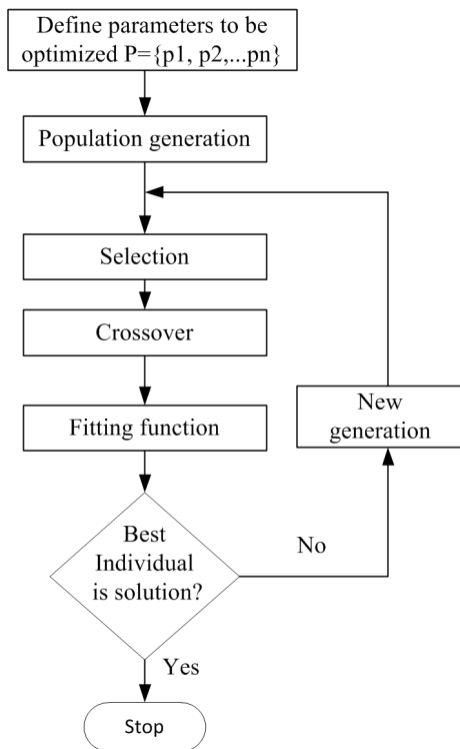


Fig. 7. Summary of GA suitable to be used in optimization problems for spindle-bearing systems.

The evolutionary methods (e.g. Genetic Algorithms – GA) are some of the most efficient methods of optimization. The position of bearing is coded in chromosomes and the schema from Fig. 7 is applied. Objective function is the minimization of the natural frequencies of spindle-bearing systems in order to be the smallest with regard to the working ones. In [60], the optimum preload is obtained from graphics selecting the lowest temperature of bearing function.

## 5. CONCLUSIONS

In this paper we reviewed succinctly the most studied aspects in spindle bearing system in order to have the possibility for choosing appropriate future research directions that would follow the trends in the field with the possibility of achieving new multi parameter integrating models capable to estimate the high speed spindle behaviour.

The optimization problems seem to be more promising ones due to various possibilities to find an optimal solution for a function that has many variables, in search space. The global minimum or maximum for fitness function can be obtained in an elegant manner in most of the cases when analytical solution is very hard to be found. These approaches will be investigated in the further research.

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