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CALCULUS ALGORITHM FOR EVALUATION OF GRAVITATIONAL AND INERTIAL LOADS ACTING ON A SCARA INDUSTRIAL ROBOT IN PICK AND PLACE APPLICATIONS

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Abstract: *In this paper a calculus methodology for optimal structure design of numerically controlled (NC) axes of SCARA industrial robot (IR) is presented. Besides identification of optimal structure, also a performance check is presented correlated with performances desired to be achieved by the overall assembly of the robot. Such calculus methodology can be applied in conceptual design and optimization of mechanical structure for new robot prototypes (but with similar structure) or for correct identification of constructive robot variant or right constructive and functional parameters of a robot for a particular robotic application.*

Key words: *industrial robot, SCARA, calculus methodology, optimal structure.*

1. INTRODUCTION

Unlike other technical systems (machine tools, welding technology machinery, other industrial systems) for which there are "standardized" design methodologies for each type of kinematic chain (main, feed / positioning, auxiliary, etc.) in the design of industrial robots for each general architecture and the constructive variant respectively, the stages of designing the general assembly and the partial assemblies of the IR have a particular mathematical formalization. The main reason for such particularities in the design methodology is the variability of the articulated mechanical structure of the IR and the diversity of the particular constructive solutions usable for the assemblies of numerical controlled axes for the translation / rotation movements (T / R) of the mobile elements of the IR.

From this point of view, the general algorithm presented in the paper and the stages under it can be used are appropriates for two purposes:

- in conceptual development activities and optimal design of new prototypes for similarly / different IR having the same general architecture;
- for identification of necessary functional / constructive parameters and the opportunity to use a specific IR's model / size in current operation exploitation correlative with its specific integration and functionality into a certain robotic application, (for this case, by applying the present algorithm being possible to select the right type and the optimal constructive variant of some existing IR model in relation to the level of performance desired to be obtained in its exploitation).

The general objectives to be achieved by going through the major computational steps remain the same regardless the formalization of the particular method of calculus relations used for designing a T / R axis of any IR. The final goal of the entire design methodology is to identify the optimal complete structure of all IR NC axes and to verify their performances correlated with the desired performance to be achieved at the level of the IR general assembly. To achieve this final goal, three calculation steps should be followed: a first set of calculation steps specific to each general architecture and IR constructive variants; a second set of calculation steps for preliminary dimensioning, selection and final checking of each type of component integrated into the partial assemblies of the NC axes of the designed IR; a third set of calculation steps specific for the selection of the electric drive systems and the control systems of the NC axes, the overall performance evaluation and the final validation of the complete design of the NC axes of the IR.

2. ESTABLISHING THE DESIGN REFERENCE MODEL. PRELIMINARY DATA SET OF INITIAL CALCULATION

At this stage, for the beginning only the specificity of the application in which the RI will be integrated is considered to be known. For the application where the robot is targeted to be integrated, based on the comparative study of similar applications existing in the literature, the reference model (general architecture and constructive version of IR) similar to the one designed is identified first. The set of initial calculations for the RI to be designed is established on the basis of the basic

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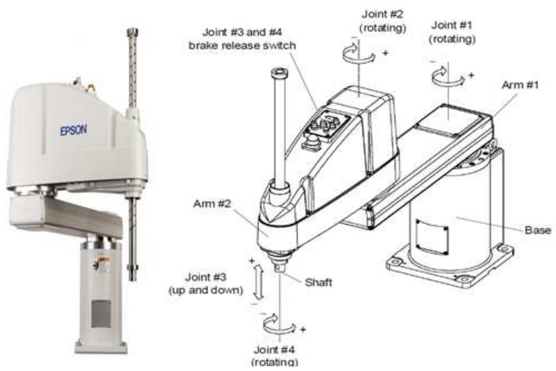
functional and constructive characteristics of the reference model similarly to the design, identified as being integrated into the respective application, and refers to: the specificities of the IR’s work tasks within the respective application; general architecture and constructive design of IR; the number and type of degrees of freedom of the IR; the constructive-functional specificity of the IR end effector; specific constructive parameters of IR (the number, type, the order of association and reciprocal position of movement axis corresponding to the major (active) joints of the robot; the specific shape and dimensions in the longitudinal and cross-section directions of the segments of the articulated mechanical structure of the IR; eccentricities and rotation angles defining the relative position and mutual orientation of the axis of motion of the major couplings of the IR in relation to the specific shape and dimensions of the segments of the articulated mechanical structure of the IR); maximum strokes and speeds of the IR's mobile elements on the numerically controlled axes; shape and dimensions (amplitude) of the IR workspace; maximum trajectory speed / minimum cycle time; maximum payload of IR; IR work accuracy, (for all of these, being necessary to take into account the basic design features of the IR's reference model, already usually specified in the technical data sheets / product specifications / product manuals developed by the IR manufacturer). To illustrate the previous-mentioned aspects regarding the identification of the reference model for the IR to be designed and its basic functional construction features respectively, the following are presented:



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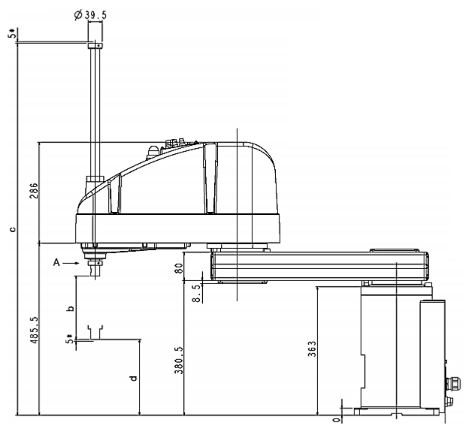

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Fig. 1. EPSON G10/G20 reference model for SCARA IR type with R1R2T3R4 kinematic [1]: *a* – general architecture; *b* – constructive parameters of IR reference model from technical datasheet [1].

- Figure 1,*a* shows general architecture and constructive variant of a reference model (EPSON G10/G20) in case of designing of a SCARA IR with R1R2T3R4 kinematics [1];
- Figure1,*b* presents the constructive and kinematic parameters for the RI considered as the reference model for RI to be designed;
- Table 1gives specifications from the IR's datasheet.

In Fig. 2 and Table 2 we can observe the maximum acceleration applicable to the characteristic TCP for the reference IR model to be designed. Of course this is just one case because the effector could be mounted eccentric and then the maximum loads values would be others.

At this point another important aspect is the localization of the calculus centres to reduce the loads applied to the robot at the level of each partial assembly (Fig. 3).

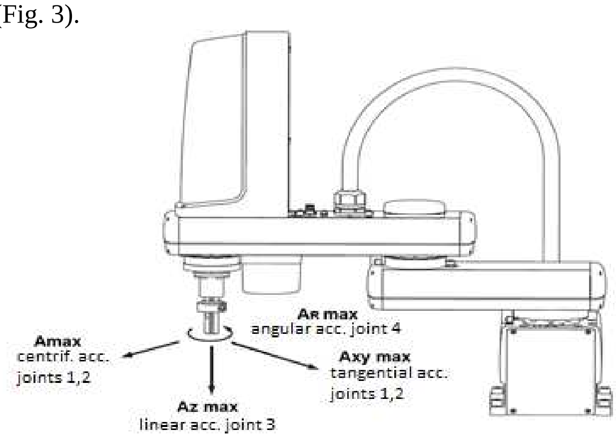


Fig. 2 Maximum inertial loads [1].

Table 1

Robot EPSON G10/G20 technical parameters

Weight	
G10/G20 85	48 kg
Driving method	AC servo (all joints)
Operating speed	
Joints 1, 2	11000 mm/s
Joint 3	1100 mm/s
Joint 4 G10, G20	2400, 1700 deg/s
Repeatability	
Joints 1, 2	0.025 mm
Joint 3	0.01 mm
Joint 4	0.005 deg
Payload rated/max	
G10	5/10 kg
G20	10/20 kg

Table 2

Maximum acceleration values

Robot model	Amax [m/s^2]	Axymax [m/s^2]	Azmax [ms^2]	Armax [rad/s^2]
R6YXGL250	88	47	**23**	110
R6YXGL350	108	35	**23**	110
R6YXGS300	108	35	23	110
R6YXGL400	98	24	23	110
R6YXGS400	98	24	23	110
R6YXGL500	62	22	23	110
R6YXGL600	54	1	23	110

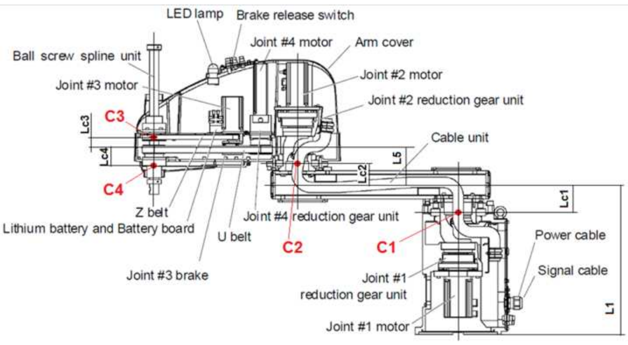


Fig. 3. Calculus centres (C1, C2, C3, C4 for applied loads reducing) [1].

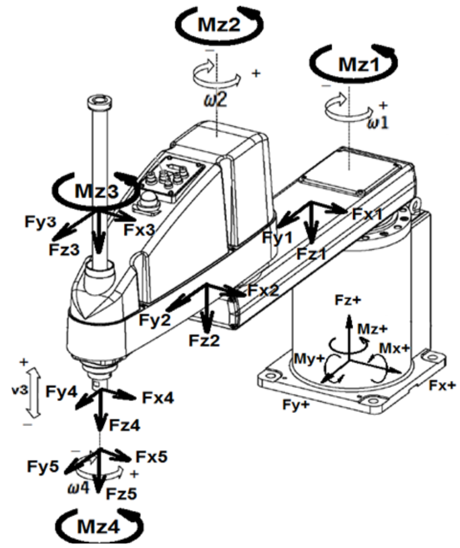


Fig. 4. Forces and torques applied in partially assembly's mass centres [3 and 4].

Then, loads need to be identified and located on the calculation scheme (as both specific forces and torques). These loads will be afterwards reduced and applied in the mass centre of each for each IR's partial assemblies / subsystems, (that are different than the calculus centres, previously identified). Generally speaking, for the considered SCARA IR these loads are shown in Fig. 4.

3. SUCCESSION OF CALCULATION STAGES

As regards the succession of major calculation steps for the design of the IR general assembly in addition to the above-mentioned aspects, the following fundamental rules must be considered for carrying out the calculations the IR's general assembly [2, 3, and 4]:

- for the determination of input data in the preliminary calculation steps, it should be taken in account the specificity of the application in which the IR is integrated and the functional design characteristics of the IR reference model previously selected (the most relevant of which being IR's specific constructive parameters and, maximum ranges and speeds on the numerically controlled axes of IR).
- in order to begin the design of a *k* translation / rotation axis within a *n* degrees of freedom (DOF) robot, all the constructive elements for all partial assemblies corresponding to $k + 1$, $k + 2…n$ NC axes should be determined by previous design and calculations stages.

Taking into consideration all the above-mentioned aspects, the main stages of calculation for the design of the general assembly of RI are carried out in the following sequences:

a) Calculation steps that provide partial results, being used only as input data for other subsequent calculation steps such as:

1. elaboration of calculation scheme for the IR to be designed;
2. identifying the localization of the mass centres for each important partial assembly of the IR;
3. identifying the overall distribution of gravitational and inertial loads applied on the overall structure of the robot (using the most unfavourable IR loading configuration);
4. identification of the calculation centres for each partial IR assembly;
5. placement spatially distributed loads in the calculation centres across the entire structure of the robot
6. determination of resultant reduced forces and torques (F, M)
7. distribution of the previous reduced load components (F, M) applied on each IR joint, in specific loads applied on bearings or guideways components of the mobile elements and respectively on the components included in the kinematic chain / driving system responsible for rotational / translational movement of each IR mobile element.

b) Calculation steps for final results usable in preliminary / final selection of the IR's standardized components / partial assemblies such as:

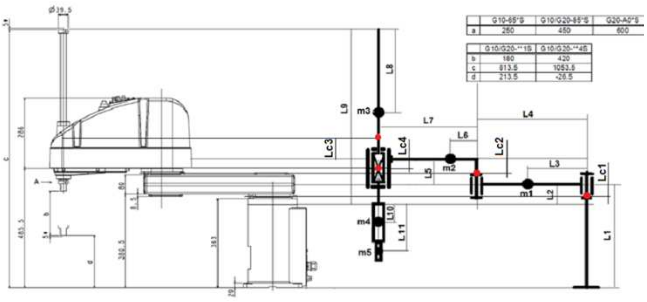
8. preliminary dimensioning, selection and final verification of the machine elements used for materializing of bearings / guidance for mobile element and respectively components responsible with driving of the mobile elements on each NC axis;
9. preliminary selection and verification of servomotors and position / speed encoders used on each NC axis;
10. final checking of selected servomotors and servo-systems used for continuous adjustment of functional parameters of servomotors for each NC axis;
11. performance evaluation on each robot subassembly;
12. performance evaluation on the overall robot assembly.

4. CALCULATION SCHEME ELABORATION FOR ROBOT REFERENCE MODEL

The correct elaboration of the calculation scheme corresponding to IR's associated design model is a fundamental element for following calculation steps. The calculation scheme is a simplified representation of the overall kinematic structure of the IR, elaborated in accordance with the general architecture and specific constructive variant of the reference model of IR to be designed, [3 and 4]. As result, it should be realized in a form of a symbolic (but as realistic as possible) representation of the IR design model as a structural kinematic schema with the inclusion of all major IR's joints and links between them, with respect for dimensional proportions between partial assemblies / components and with the highlighting of all constructive and functional parameters (that must be also included in the geometric and kinematic model of the IR to be designed) with respect of the eccentric disposition / reciprocally offsets of IR's partially assemblies.

In the representation of the calculation scheme the IR's mobile elements must be brought into the positions / orientations leading to the most unfavourable loading configuration (for which the levels of inertial and gravitational loads acting on IR arm are maximized). If it is not possible to identify a single calculation scheme corresponding to the most unfavourable loading configuration of the IR, alternative calculation configurations may be defined, the calculations for the next steps being performed in parallel up to the level of certain identification of the configuration that leads to the peak loads applied on the designed IR [3 and 4].

The results of the calculations made at this stage aim to determine the volume, the mass and the coordinates (x_{Gi}, y_{Gi}, z_{Gi}) of each mass centre specific to the structural elements / partial assemblies of the IR as well as the correct identification and representation on the calculation scheme of all the constructive and functional parameters specific to reference model of the IR to be designed. Correct evaluation of these elements is decisively influencing the correctness of the calculations that will be carried out in subsequent stages, since all the gravitational and inertial loads to be included will be applied exclusively on the mass centres of the partial assemblies of IR and will report directly to the masses of the partial assemblies / structural elements evaluated at this stage.



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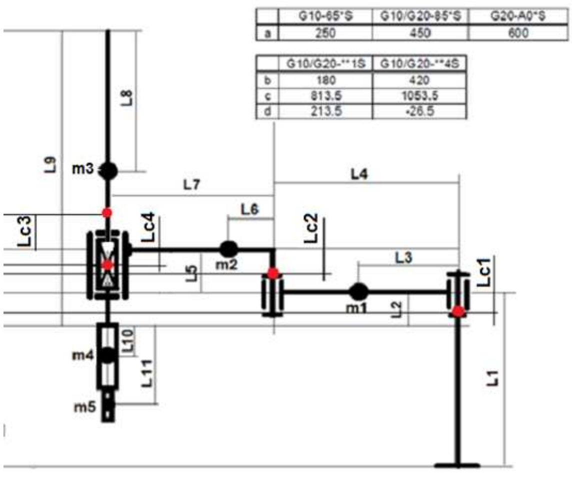

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Fig. 5. Calculation scheme for SCARA robot [4]:
a – correspondence with the reference model EPSON G10/G20;
b – localization of mass and calculus centres on the simplified scheme.

Masses of each partial assembly are considered to be concentrated in their related mass centre. These must be evaluated and located in the calculation scheme by material points identified as location by specific constructive parameters (dimensions). Concentrated masses in the calculation scheme must allow identifying actual distribution of gravitational loads generated by all major partial assemblies / structural elements of the RI to be considered. To illustrate an example of how to set up the calculation scheme for an articulated arm robot (SCARA) Figs. 5,*a* and *b* are presented [4].

5. DETERMINATION OF DISTRIBUTION FOR GRAVITATIONAL AND INERTIAL LOADS

Continuing after the previous step, in this calculation stage, using the achieved calculation scheme and the IR configuration leading to the most loading case for all IR partially assemblies to be designed, the following have to be determined [4]:

- gravitational loads (*Gi*) corresponding to each structural element of IR and each partial subassembly and their spatial distribution on the general IR full assembly;
- inertial forces generated in al mass centres as a result of the IR mobile elements movement in active major rotation / translation joint and their spatial distribution;
- the main inertial momentums (relative to the central axes passing through the mass centres) and the centrifugal inertia momentums (around some eccentric axes relative to the central ones) of the structural elements / partial assemblies of the IR, generated by the rotational movements in each major/active joint.

Determining the spatial distribution of the gravitational and inertial loads taken into account in the design of the NC axes of the IR involves numerical evaluation and graphical representation on the previously elaborated calculation scheme for two major categories of loads: gravitational forces and inertial forces acting in mass centres characteristic for all partially assemblies / structural elements of the RI. For graphical representation on the calculation scheme gravitational and inertial force loads will be applied only in the mass centres, and the inertial momentums only around the rotation axes that generate them. In addition, for each type of inertial forces / inertia momentum, the direction and direction of the specific action must be determined in accordance with the movements made by the RI movable elements and the moment of occurrence of the inertial load taken into account (start / end time of movement).

For gravitational forces, the direction of action is vertical and the way of action from the top to down. The numerical evaluation of the gravitational forces is done by the relations:

$$G_i = m_i\, g. \qquad (1)$$

The numerical evaluation of an inertial force F_{ij} generated for a mass m_i by a translational movement in the joint j when the mass is moving by maximum speed v_j is made with the relation:

$$F_i^j = m_i a_{Tj}, \qquad (2)$$

where F_i^j represents the value of the inertial force, m_i is the mass and a_{Tj} is the acceleration generated by starting / stopping / speeding up / slowing down of a moving element in a translation motion. Acceleration is determined by relation:

$$a_{Tj} = \frac{\Delta v_j}{t_{fr/acc}} = \frac{v_{j\,max}}{t_{fr/acc}}, \tag{3}$$

where Δv_j is speed variation along acceleration / breaking time $t_{fr/acc}$ (usually having a value of 0,5 seconds):

$$\Delta v_j = v_{j\,max} - v_{j\,min} = v_{j\,max} - 0 = v_{j\,max}. \tag{4}$$

Inertial force F_i^j will bere represented in parallel directions to the direction of the movement axis of the movable element *k*. Inertial forces generated by the rotational movement of a joint *k* of an IR can be of two types: centrifugal forces F_{cfi} and tangential forces F_{tgi}. Centrifugal forces are oriented along the direction of the kinematic radius (gyration) which can be obtained by joining the rotation axis with the mass centre of a considered material point. Cinematic radius is measured perpendicularly on the rotation axis of the joint, from axis to the mass centre for which the centrifugal force is calculated. Tangential forces are oriented perpendicularly on the direction of the centrifugal forces (tangent to the circular trajectory). Numerical evaluation of inertial centrifugal force ($F_{cf\,i}^j$) and inertial tangential force ($F_{tg\,i}^j$) be done with relations:

$$F_{cf\,i}^j = m_i \omega_j^2 R_{ij}, \tag{5}$$
$$F_{tg\,i}^j = m_i a_{tg\,ij}, \tag{6}$$

where: ω_j is the maximum rotation speed in joint *j*, R_{ij} – kinematic radius, and $a_{tg\,ij}$ – tangential acceleration calculated with:

$$a_{tg\,ij} = \varepsilon_j R_{ij}, \tag{7}$$

where ε_j is the angular acceleration determined by:

$$\varepsilon_j = \frac{\Delta\omega_j}{\Delta t} = \frac{\omega_{j\,max}}{t_{fr/acc}}, \tag{8}$$

where variation of angular speed $\Delta\omega_j$ is:

$$\Delta\omega_j = \omega_{j\,max} - \omega_{j\,min} = \omega_{j\,max} - 0 = \omega_{j\,max}. \tag{9}$$

For graphical representation of all inertial and gravitational loads acting on the IR's structure following steps can be followed: first in the mass centres all the gravitational the forces acting on the IR are represented; successively representation of inertial forces for each active joint (one joint by one) may be made. For example, for a 6 DOF robot first loads are determined for joint 6 (5, 4, 3, 2, 1 being considered *locked*), then the loads are determined for joint 5 (6, 4, 3, 2, 1 being *locked*) and so on. Final resulted loads are being considerate to be applied simultaneously so that the IR's structure will be loaded in maximum loading conditions (corresponding to simultaneous movement of IR from all joints). For better exemplification of previous steps

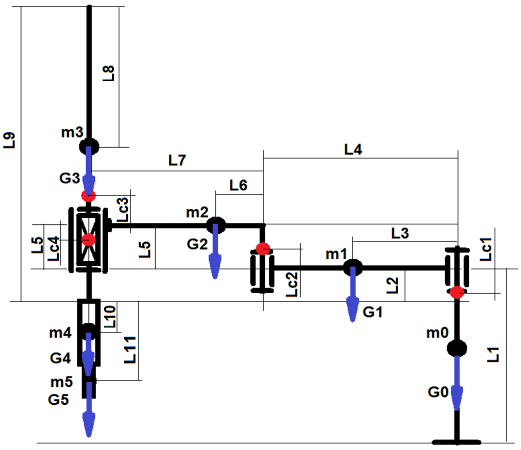

Fig. 6. Representation of gravitational loads [4].

following figures are representing successively some calculus scheme with representation of loads being considerate for a SCARA robot that has to be designed and having as reference the model EPSON G10 / G20.

Numerical evaluation of gravitational forces presented previously in Fig. 6 is done:

- for mass of manipulated object;
- for masses of partial assemblies supplementary equipping the robot (effectors, sensors, coupling systems etc.);
- for masses of partially assemblies composing the robot.

IR partially assemblies design, their masses evaluation and load calculations should be done gradually starting from last elements of IR. In the situation that a preliminary approximately evaluation is needed, masses for partial assemblies of the robot can be determined (estimative) by relations:

$$m_{TOT\,RI} = (m_0 + m_1 + m_2 + m_3); \tag{10}$$
$$G_{TOT\,RI} = (m_0 + m_1 + m_2 + m_3)\,g; \tag{11}$$
$$m_0 = \rho_0 f_0 V_0,\ m_0 = \frac{m_{TOT\,RI}}{V_{TOT\,RI}} f_0 V_0,\ G_0 = m_0\,g; \tag{12}$$
$$m_1 = \rho_1 f_1 V_1,\ m_1 = \frac{m_{TOT\,RI}}{V_{TOT\,RI}} f_1 V_1,\ G_1 = m_1\,g; \tag{13}$$
$$m_2 = \rho_2 f_2 V_2,\ m_2 = \frac{m_{TOT\,RI}}{V_{TOT\,RI}} f_2 V_2,\ G_2 = m_2\,g; \tag{14}$$
$$m_3 = \rho_3 f_3 V_3,\ m_3 = \frac{m_{TOT\,RI}}{V_{TOT\,RI}} f_3 V_3,\ G_3 = m_3\,g; \tag{15}$$

where: $m_{TOT\,RI}$ and $V_{TOT\,RI}$ are representing total mass and total volume for the reference model of IR to be designed, m_i and V_i – partial masses and volumes of IR's subassemblies, ρ_i – average densities of the materials from which the sub-assemblies are built, and f_i – coefficient representing the degree of fulfilment of V_i volumes. After determining of masses and localization of gravity centres for each IR subassembly it is possible to continue to determine the inertial loads generated by possibilities of movement in the active joints of IR. As for the SCARA robot case with 4 NC axes the calculation of the inertial loads applied to the overall structure begins with the determination of inertial loads generated by the possibility of movement on the 4th NC axis. The 4th NC axis of presented SCARA model is a rotational axis (for effectors roll motion), thus inertial loads are resumed to the sum of inertial momentum of manipulated object, effectors and adapter elements, ball-screw/ball spline and the final flange of IR (the total

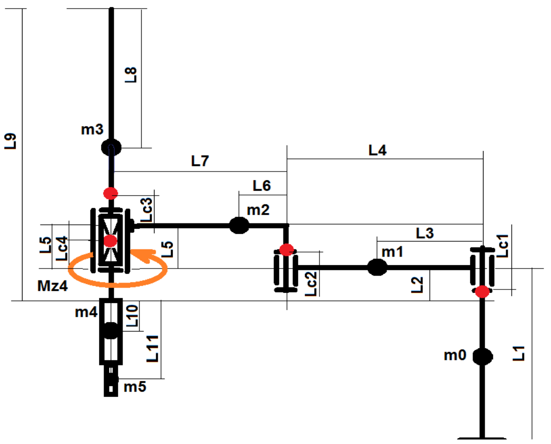

Fig. 7. Representation of inertial momentum generated by 4th rotation axis [4].

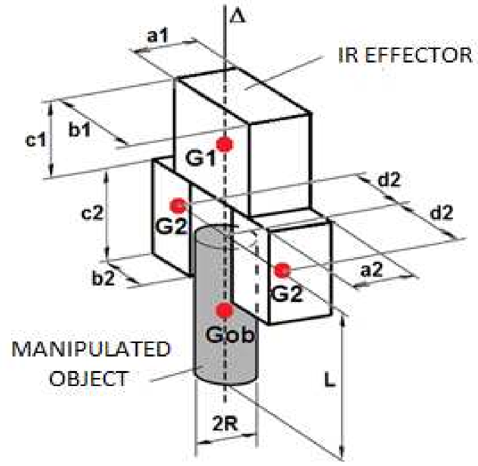

Fig. 8. Calculation scheme for determining inertial momentum generated by 4th axis rotation for a centric effector case [4].

value of the inertial momentum representing the momentum that needs to be exceeded by the 4th axis driving system). Related to inertial momentum acting on 4th NC axis Figs. 7 and 8 are presented.

As previously mentioned, for numerical evaluation of total inertial momentum in case of a centric effector the following relations can be used:

$$M_{z4} = J_{z4}\varepsilon_4 = (J_{z\,ob} + J_{z\,ef})\varepsilon_4 = $$
$$= (J_{z\,ob} + J_{z\,corp\,ef} + J_{z\,bacuri} + J_{tija\,RI} + J_{flansa\,RI})\varepsilon_4;$$
$$M_{z4} = J_{z4}\varepsilon_4 = (J_{z\,ob} + J_{z\,corp\,ef} + 2J_{z\,cf\,1bac} + J_{tija\,RI} + $$
$$+ J_{flansa\,RI})\varepsilon_4 \text{ [Nm]}; \qquad (16)$$

$$J_{z\,ob} = \frac{1}{2} m_{ob} R_{ob}^2 = \frac{1}{2}\rho_{ob}\pi R_{ob}^4 L_{ob} \text{ [Kg m}^2\text{]};$$
$$J_{tija\,RI} = \frac{1}{2} m_{tija\,RI} R_{tija\,RI}^2 \text{ [Kg m}^2\text{]};$$
$$J_{flansa\,RI} = m_{flansa\,RI} R_{flansa\,RI}^2 \text{ [Kg m}^2\text{]};$$
$$\varepsilon_4 = \frac{\Delta\omega_4}{\Delta t} = \frac{\omega_{4\,max}}{t_{fr/acc}} \text{] = [rad/sec}^2\text{]}; \qquad (17)$$

where: $\Delta\omega_4$ is angular speed variation for the 4th axis, $J_{z\,corp\,ef}$ – inertial momentum of effector body in Kg m^2, $J_{z\,bacuri}$ –inertial momentum of effector fingers in Kg m^2, $J_{tija\,RI}$ – inertial momentum of ball-screw/ball-spline in Kg m^2, $J_{flansa\,RI}$ – inertial momentum of the flange in Kg m^2.

In Fig. 9, an example of eccentric effector is presented. As long as rotation axis is not longer passing through gravity centres of the effector, the inertial moments of the manipulated object, of the effector body

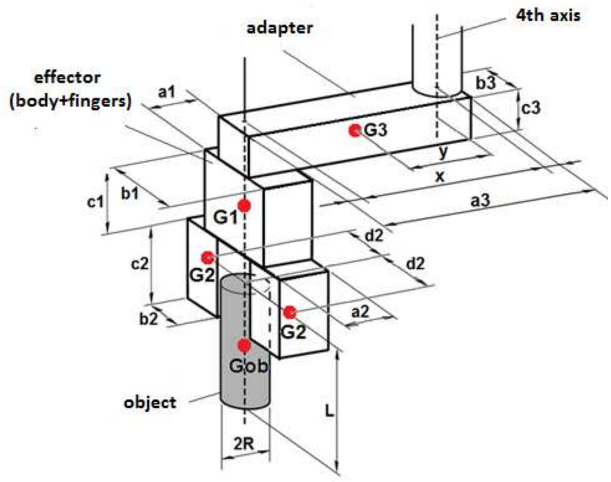

Fig. 9. Calculation scheme for determining inertial momentum generated by 4th rotation axis for an eccentric effector case [4].

and of the adapter elements must be determined as centrifugal momentums with reference to the axis of movement of the 4th joint of IR, adding finally the centric inertial momentum for the flange and for the ball-screw or ball-spline. In this case numerical evaluation of total inertial centrifugal momentum can be calculated with following relations:

$$M_{z4} = J_{cf\,z4}\varepsilon_4 = $$
$$= (J_{z\,cf\,ob} + J_{z\,cf\,ef} + J_{z\,cf\,adaptor} + J_{tija\,RI} + J_{flansa\,RI})\varepsilon_4 = $$
$$= (J_{z\,cf\,ob} + J_{z\,cf\,corp\,ef} + J_{z\,cf\,2bacuri} + J_{z\,cf\,adaptor} + $$
$$J_{tija\,RI} + J_{flansa\,RI})\varepsilon_4 \text{ [N m]}; \qquad (18)$$

$$M_{z4} = J_{z4}\varepsilon_4 = $$
$$= (J_{z\,cf\,ob} + J_{z\,cf\,corp\,ef} + J_{z\,cf\,2bacuri} + J_{z\,cf\,adaptor} + $$
$$J_{tija\,RI} + J_{flansa\,RI})\varepsilon_4 \text{ [N m]} \qquad (19)$$

$$J_{z\,cf\,ob} = \frac{1}{2} m_{ob} R_{ob}^2 + m_{ob}x^2 = \frac{1}{2}\rho_{ob}\pi R_{ob}^4 L_{ob}$$
$$+ m_{ob}x^2 \text{ [Kg m}^2\text{]};$$
$$J_{tija\,RI} = \frac{1}{2} m_{tija\,RI} R_{tija\,RI}^2, J_{flansa\,RI} = $$
$$= m_{flansa\,RI} R_{flansa\,RI}^2 \text{ [Kg m}^2\text{]}; \quad \varepsilon_4 = \frac{\Delta\omega_4}{\Delta t} = $$
$$\frac{\omega_{4\,max}}{t_{fr/acc}} \text{ [rad/sec}^2\text{]}; \quad (20)$$

where $J_{z\,cf}$ represents the centrifugal inertial momentum, $J_{z\,cf\,ef}$ – centrifugal inertial momentum of the effector composed by centrifugal inertial momentum of the effector body $J_{z\,cf\,corp\,ef}$ and the centrifugal inertial momentum of the two effector fingers $J_{z\,cf\,2bacuri}$ (all in Kg m^2).

Supplementary to upper mentioned inertial centrifugal momentums, it must be taken into consideration the inertial centrifugal momentum of the adapter part/assembly $J_{z\,cf\,adaptor}$ in Kg m^2. For all of them the relations used to evaluate their inertial momentum are below detailed:

$$J_{z\,cf\,corp\,ef} = J_{z\,corp\,ef} + m_{ob}x^2 = $$
$$= \frac{1}{12} m_{corp\,ef}(a1^2 + b1^2) + m_{ob}x^2 = $$
$$= \frac{1}{12}\rho_{ef} V_{ef}(a1^2 + b1^2) + m_{ob}x^2 = $$
$$= \frac{1}{12}\rho_{ef}\, a1\, b1\, c1(a1^2 + b1^2) + m_{ob}x^2; \qquad (21)$$

$$J_{z\,cf\,2bacuri} = 2[\tfrac{1}{12}m_{1bac}(a2^2+b2^2)+m_{1bac}(x^2+d2^2)]=$$
$$= 2[\tfrac{1}{12}\rho_{bac}V_{1bac}(a2^2+b2^2)+\rho_{bac}V_{1bac}(x^2+d2^2)]=$$
$$= 2[\tfrac{1}{12}\rho_{bac}a2\,b2\,c2(a2^2+b2^2)+\rho_{bac}a2\,b2\,c2(x^2+d2^2)] \quad (22)$$

$$J_{z\,cf\,adaptor} = \tfrac{1}{12}m_{adaptor}(a3^2+b3^2)+m_{adaptor}y^2=$$
$$= \tfrac{1}{12}m_{adaptor}(a3^2+b3^2)+\rho_{adaptor}a3\,b3\,c3y^2. \quad (23)$$

After determining of inertial loads generated by the movement in the 4th axis calculations it may be continued by further 3rd axis calculation procedure.

As in this case the robot is a SCARA model (with a R1 R2 T3 R4 kinematic) the third NC axis of the IR is a translation axis and calculations of inertial loads applied on the IR's structure involves determination of inertial forces generated in al gravity centres of partial subassemblies/components driven in motion directly or indirectly by the translation on 3rd axis. From this point of view in Fig. 10 is presented the calculation scheme completed with exclusive consideration of inertial forces acting on IR generated by the possibility of motion in joint 3 (translation). Numerical evaluation of these forces can be done with relations:

$$F_i^j = m_i a_{Tj},\; a_{Tj} = \frac{\Delta v_j}{t_{fr/acc}} = \frac{v_{j\,max}}{t_{fr/acc}},$$
$$t_{fr/acc} = 0.5\,sec. \quad (24)$$

For the case presented in Fig. 10, due to the fact that translation motion on axis 3 is driving directly the ball-screw body/ ball-spline body coupled with effector mounting flange the inertial forces can be determined with relations:

$$F_5^3 = m_5 a_{T3} = m_5\frac{v_{3\,max}}{0.5};$$
$$F_4^3 = m_4 a_{T3} = m_4\frac{v_{3\,max}}{0.5};$$
$$F_3^3 = m_3 a_{T3} = m_3\frac{v_{3\,max}}{0.5}; \quad (25)$$

where: m_3 is total mass of ball screw/ball spline plus effector flange in kg, m_4 – effector mass in kg, and m_5 – mass of the manipulated object in kg.

After determining of the inertial loads generated by movement on axis 3 calculus may be continued for determining of loads generated by motion of NC axis number 2. The second NC axis of the robot is the rotation

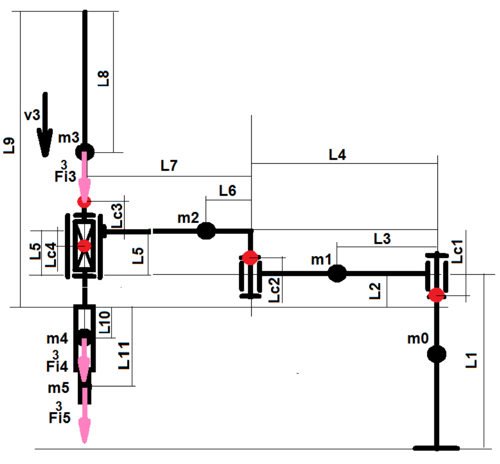

Fig. 10. Representation of distribution for inertial forces acting on IR's structure generated by motion of axis 3 [4].

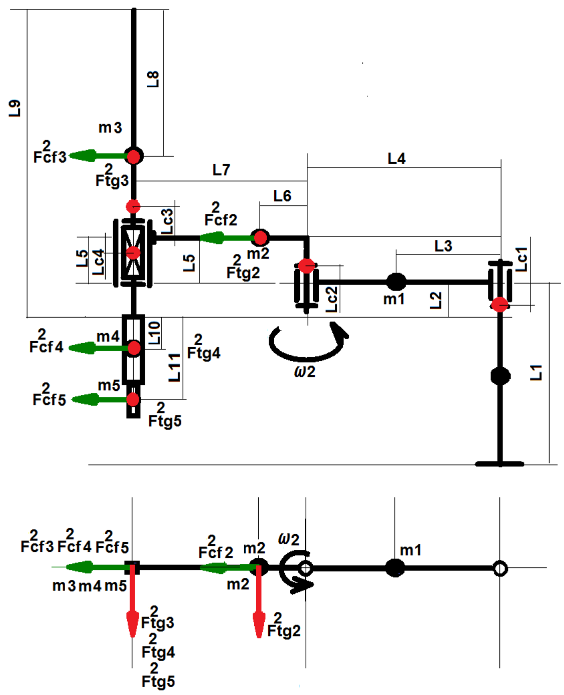

Fig. 11. Graphical representation of inertial forces distribution acting on IR generated by motion of axis 2 [4].

axis of the robot's 2nd segment, relatively to the first segment of the robot arm. In Fig. 11 is represented the appropriate calculation scheme completed with loads taken into consideration for spatial distribution of gravitational and inertial loads for NC axes of the IR, considering exclusively the inertial forces distribution acting on the IR generated by the motion allowed by joint 2 (rotation).

Numerical evaluation of these inertial forces can be done using following relations:

$$F_{cf\,i}^j = m_i\omega_j^2 R_{ij},\; F_{tg\,i}^j = m_i a_{tg\,ij},\; a_{tg\,ij} = \varepsilon_j R_{ij},\; \varepsilon_j = \frac{\Delta\omega_j}{\Delta t} = \frac{\omega_{j\,max}}{t_{fr/acc}}; \quad (26)$$

$$F_{cf\,2}^2 = m_2\omega_2^2 R_{22},\; F_{tg\,2}^2 = m_2 a_{tg\,22},\; a_{tg\,22} = \varepsilon_2 R_{22},$$
$$R_{22} = L_6,\; \varepsilon_2 = \frac{\Delta\omega_2}{\Delta t} = \frac{\omega_{2\,max}}{t_{fr/acc}};$$
$$F_{cf\,3}^2 = m_3\omega_2^2 R_{32},\; F_{tg\,3}^2 = m_3 a_{tg\,32},\; a_{tg\,32} = \varepsilon_2 R_{32},$$
$$R_{32} = L_7,\; \varepsilon_2 = \frac{\Delta\omega_2}{\Delta t} = \frac{\omega_{2\,max}}{t_{fr/acc}};$$
$$F_{cf\,4}^2 = m_4\omega_2^2 R_{42},\; F_{tg\,4}^2 = m_4 a_{tg\,42},\; a_{tg\,42} = \varepsilon_2 R_{42},$$
$$R_{42} = L_7,\; \varepsilon_2 = \frac{\Delta\omega_2}{\Delta t} = \frac{\omega_{2\,max}}{t_{fr/acc}};$$
$$F_{cf\,5}^2 = m_5\omega_2^2 R_{52},\; F_{tg\,5}^2 = m_5 a_{tg\,52},\; a_{tg\,52} = \varepsilon_2 R_{52},$$
$$R_{52} = L_7,\; \varepsilon_2 = \frac{\Delta\omega_2}{\Delta t} = \frac{\omega_{2\,max}}{t_{fr/acc}}. \quad (27)$$

After determining of inertial loads generated by the second axis motion, calculations can be continued for first IR axis. The first robot axis is also a rotation axis and inertial loads must be determined for inertial centrifugal and tangential forces generated in all gravity centres of partial assemblies/components that are driven directly (robot segment 1 with mass m_1) or indirectly (robot segment 2 with mass m_2 and previous specified elements with masses m_3, m_4, and m_5) by rotation motion on first NC axis of the robot. From this point of view, in the following Fig. 12 is presented the calculation scheme completed with inertial loads considering exclusively the distribution of inertial forces acting on IR structure and generated by the motion of joint 1. Similarly to those presented previously, numerical evaluation of these inertial forces can be determined using following relations:

$$F^j_{cf\,i} = m_i\omega_j^2 R_{ij},\ F^j_{tg\,i} = m_i a_{tg\,ij},\ a_{tg\,ij} = \varepsilon_j R_{ij},$$
$$\varepsilon_j = \frac{\Delta\omega_j}{\Delta t} = \frac{\omega_{j\,max}}{t_{fr/acc}}, \tag{28}$$

which in case of moving exclusively on joint 1 can be particularized as:

$$F^1_{cf\,1} = m_1\omega_1^2 R_{11}, F^1_{tg\,1} = m_1 a_{tg\,11}, a_{tg\,11} = \varepsilon_1 R_{11},$$
$$R_{11} = L_3,\ \varepsilon_1 = \frac{\Delta\omega_1}{\Delta t} = \frac{\omega_{1\,max}}{t_{fr/acc}},$$
$$F^1_{cf\,2} = m_2\omega_1^2 R_{21}, F^1_{tg\,2} = m_2 a_{tg\,21},$$
$$a_{tg\,21} = \varepsilon_1 R_{21},\ R_{21} = L_4 + L_6,\ \varepsilon_1 = \frac{\Delta\omega_1}{\Delta t} = \frac{\omega_{1\,max}}{t_{fr/acc}}$$
$$F^1_{cf\,3} = m_3\omega_1^2 R_{31}, F^1_{tg\,3} = m_3 a_{tg\,31},$$
$$a_{tg\,31} = \varepsilon_1 R_{31},\ R_{31} = L_4 + L_7,\ \varepsilon_1 = \frac{\Delta\omega_1}{\Delta t} = \frac{\omega_{1\,max}}{t_{fr/acc}},$$
$$F^1_{cf\,4} = m_4\omega_1^2 R_{41}, F^1_{tg\,4} = m_4 a_{tg\,41},$$
$$a_{tg\,41} = \varepsilon_1 R_{41},\ R_{41} = L_4 + L_7,\ \varepsilon_1 = \frac{\Delta\omega_1}{\Delta t} = \frac{\omega_{1\,max}}{t_{fr/acc}},$$

$$F^1_{cf\,5} = m_5\omega_1^2 R_{51},\ F^{1}_{tg\,5} = m_5 a_{tg\,51},$$
$$a_{tg\,51} = \varepsilon_1 R_{51},\ R_{51} = L_4 + L_7,\ \varepsilon_1 = \frac{\Delta\omega_1}{\Delta t} = \frac{\omega_{1\,max}}{t_{fr/acc}}. \tag{29}$$

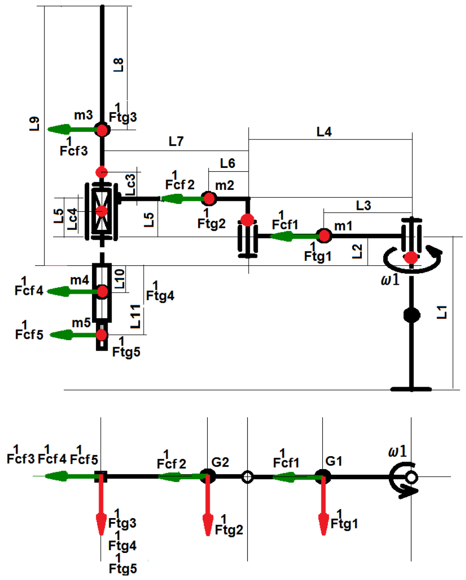

Fig. 12. Exclusively graphical representation for distribution of inertial forces generated by joint 1 motion [4].

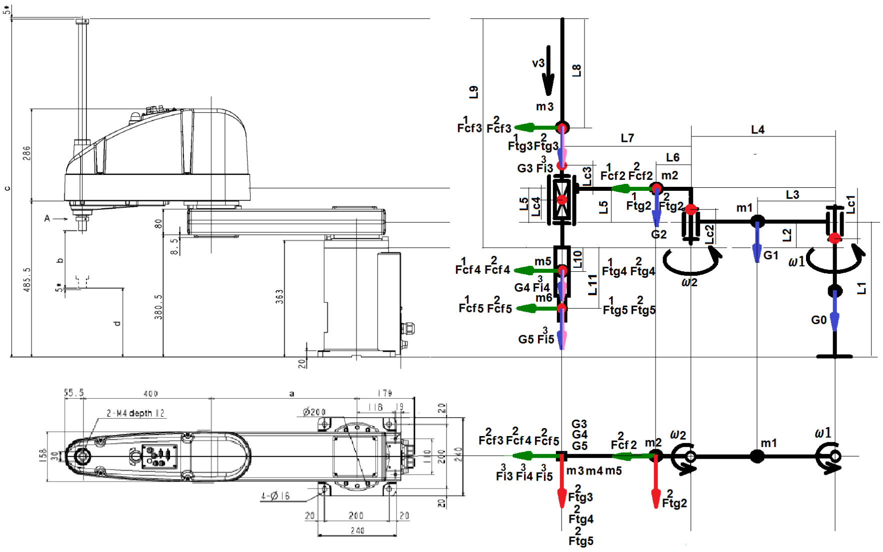

Fig. 13. Graphical representation on the calculus scheme of all gravitational and inertial loads acting on IR structure in correspondence with the reference IR model [4].

Having the gravitational and inertial loads determined individually the graphical representation of all loads acting on the overall IR's structure can now be presented. The case considered is that of simultaneous moves in all joints (taking into account the cumulative effect of all inertial and gravitational loads) as shown in Fig. 13.

6. CONCLUSIONS

The general algorithm presented in the may be used for two purposes:

- in conceptual development activities and optimal design of new prototypes of similar IRs to existing or different IR variants;
- for identification of necessary functional constructive parameters and the opportunity to use IR's operation correlative with the specificity of their integration and operation in a certain robotic application.

For the last purpose by applying the present algorithm it is possible to select the type and the optimal constructive variant of IR in relation to the level of performance desired to be obtained in its exploitation.

REFERENCES

[1] ***EPSON SCARA ROBOT G10/G20 series manipulator manual; Document ID: EM151R2859F; `http://robots.epson.com`.

[2] A.F. Nicolescu, M.D. Stanciu, D. Popescu, *Concepţia şi exploatarea roboţilor industriali* (Design and operation of industrial robots), Edit. Printech, Bucharest, 2004.

[3] A.F. Nicolescu, *Roboţi Industriali* (Industrial Robots), Edit. Didactică şi Pedagogică, Bucharest, 2005.

[4] A.F. Nicolescu, *Industrial Robot's Design and Operation*, University "Politehnica" of Bucharest, L-A3-S2-CERI2-RB, `ttp://imst.curs.pub.ro/2017/course/view.php?id=866`.