CALCULUS ALGORITHM FOR EVALUATION OF GRAVITATIONAL AND INERTIAL LOADS ACTING ON A 6 DOF ARTICULATED ARM TYPE INDUSTRIAL ROBOT

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Abstract: This paper is presenting a calculus methodology that may be used in conceptual design and mechanical structure optimization for developing new articulated arm industrial robots (IR) with 6 degrees of freedom (DOF). In the mean time this methodology offers the possibility to identify an optimal structure for the robot having the ability to do a performance check correlated with performances intended to be achieved by the robot within the application that it needs to be integrated.

Key words: industrial robot, articulated arm, calculus methodology, optimal structure.

1. INTRODUCTION

Nowadays, industrial robots are implemented in a wide range of application. Each application has its own particularities in terms of: what kind of activity should the robot perform (pick and place, assembly, welding, machining etc.), space available, weight of the operating tools or of the object that needs to be manipulated by the robot, complexity of movement and trajectories, performances needed to be achieved in the application (speed, precision, repeatability, etc.) and many other criteria that usually are different from an application to another. Existing robots are coming in a wide range of general architectures (constructive variants) and in a wide range of typo-dimensions with different constructive and functional parameters and performances.

In order to identify the robot that better fits in the application that it needs to be integrated or in order of optimal design the structure of the numerically controlled axis (NC) of some new IRs, the methodology presented in this paper may be applied. As stated in [1], where a calculus algorithm was presented for a SCARA robot model, due to the variability of the articulated mechanical structure of robots and the diversity of the particular constructive solutions used for realization of assemblies of subassemblies of robots, particular mathematical formalizations was also needed. Following a similar workflow as presented in [1], this time a particular calculus methodology is presented for optimal structure design of NC axes of an articulated arm type robot with 6 DOF. The mathematical algorithm further to be presented may be used for: conceptual development activities and optimal design of new prototypes of IRs as well as for identification of necessary functional constructive parameters and the opportunity to use an existing IR model / constructive variant in some specific application correlative with the specificity of their integration and operation in targeted robotic application.

The calculus algorithm is structured in three different stages: first, a set of calculation steps specific to each general architecture and IRs constructive variant analyzed; a second set of calculation steps for pre-dimensioning, preliminary selection and final verification of each type of component integrated in the partial assemblies of the NC axes of the designed IR; a third set of calculation steps specific to the selection of the electric driving system and the control system of the NC axes, as well as the overall performance evaluation and the final validation of the complete design of the NC axes of the IR.

2. INDUSTRIAL ROBOT REFERENCE MODEL. PRELIMINARY DATA SET OF INITIAL CALCULATION

When selecting or designing a robot, first, the application in which it will operate must be known. The characteristics of the application are the first information that needs to be known in order to further proceed on the calculus algorithm. By knowing these, some essential data can be extracted and used further. By means of a comparative study of similar existing applications, a reference model (general architecture and constructive version of the IR) can be identified [2, 3].

Initial calculations are need to be performed by taking into account the basic functional and constructive characteristics, thus for the reference model of the IR / the IR to be designed it is necessary to refer on: the specificities of the IR’s work tasks within the respective application; the general architecture and constructive design of IR; the number and type of degrees of freedom of the IR; the constructive-functional specificity of the IR end effector; the specific constructive parameters of IR (the number, type, the order of reciprocally disposal of movement axis
corresponding to major / active joints of the robot); the specific shape and dimensions in the longitudinal and cross-section directions of the segments of the articulated mechanical structure of the IR; the eccentricities and rotation angles defining the relative position and mutual orientation of the axis of motion of the major joints of the IR in relation to the specific shape and dimensions of the segments of the articulated mechanical structure of the IR; maximum ranges and speeds on the numerically controlled axes of the IR; shape and dimensions (amplitude) of the IR workspace; the maximum trajectory speed / minimum cycle time; the maximum payload of IR; IR working accuracy, (for all of these, taking into account the basic construction elements of the IR's reference model specified in the technical data sheets/product specifications/product manuals developed by the IR manufacturer [3, 4]. To illustrate the previous-mentioned aspects regarding the identification of the reference model for the IR to be designed (Fig. 1) and its basic functional construction features respectively, further the following aspects are presented [2].

After the identification of an IR reference model and study of its characteristics an important aspect is to identify the specific location of the calculus centres. The calculus centres (Figs. 2a and b) are specific points localized on the robot structure where the loads acting on the overall robot will be reduced on each NC axis level in order to evaluate their effect on each IR’s partially assembled structure and performances [4, 5].

The loads that need to be identified and located on the calculus scheme are forces and torques. These will be reduced and applied in the calculus centres of each IR’s partial assemblies/subsystem. In this case it is presented an unequipped robot but in the IR’s real operation the robot may be equipped with different types of effectors / tools or has to perform different operations or manipulate different objects which should be not forgotten to be considered because these are also generating loads action on IRs structure.

In the mean time, by applying this calculus algorithm, loads acting on the overall IR’s structure, may be reduces as a specific set of forces and momentums acting on the IR’s base level, these information being usually very important for correctly selecting / checking the fixtures of the robot on the ground / the specific support to be

<table>
<thead>
<tr>
<th>Table 1. Main characteristics of the RI</th>
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<tr>
<td><strong>Weight</strong></td>
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<td><strong>Controlled axes</strong></td>
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<td><strong>Maximum speed</strong></td>
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<td>Joints 1, 2</td>
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<td><strong>Max load at wrist</strong></td>
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<td><strong>Repeatability</strong></td>
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<td><strong>H-reach</strong></td>
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<td><strong>Structure</strong></td>
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<td><strong>Applications (welding)</strong></td>
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Fig. 1. Reference model FANUC ARC Mate 120iB [2]:

(a) general architecture and constructive variant; (b) constructive parameters of IR extracted from product’s manual.

Fig. 2. Calculus centres for load reduction on each IR subassembly [5]: (a) front view; (b) side view.
ment into an application. An example about specific resultant loads necessary to be evaluated and specific values of these loads for the IR reference model are presented in Fig. 3 and Table 2 [2].

3. CALCULATION STEPS

In order to carry out the design calculations of the IRs general assembly next steps should be followed as stated also in [1]:

- for the determination of input data in the preliminary calculation steps, account shall be taken of the specificity of the application in which the IR is to be integrated and the basic functional design characteristics of the IR reference model previously established (the most relevant of which being IR’s specific constructive parameters and maximum working ranges, maximum payload and maximum speeds on the NC axes of IR);
- in order to begin the design of a \( k \) translation / rotation axis within a \( n \) degrees of freedom robot (DOF), all the constructive elements should be determined by previous design and calculations for all partial assemblies corresponding to \( k + 1, k + 2, \ldots n \) NC axes (in this case, \( n = 6 \)).

Taking into consideration all the above-mentioned aspects, the main stages of calculation for the design of the general ensemble of RI are carried out in the following sequence [3, 4, and 5]:

a) calculation steps that provide partial results, being used only as input data for other subsequent calculation steps such as:
1. elaboration of calculus scheme for IR to be designed;
2. centre of mass localization for each major partial assembly of the IR;
3. determining the overall distribution of gravitational and inertial loads applied on the overall structure of the robot (using the most unfavourable IR load configurations);
4. identification of calculation centres for each partial IR assembly;
5. placement in the mass centres the spatially distributed loads acting across the entire structure of the robot and determination of resultant reduced forces and torques \( (F_{i\text{red}}, M_{i\text{red}}) \) on each specific calculus centres of IR’s partially assemblies;
6. distribution of the previous resulting load components \( (F_{i\text{red}}, M_{i\text{red}}) \) on sets of components such as bearings or guidance components allowing movement of each IR’s mobile element and respectively on the components that included in the driving systems responsible with rotational / translational motion of each moving element on each NC axis.

b) calculation steps for final results usable for preliminary/final selection of the IR’s structure components/partial assemblies such as:
1. pre-dimensioning, preliminary selection and final verification of the assembly components used for materializing of bearings / guidance and respectively components for driving in motion of the movable elements on each NC axis;
2. preliminary selection and verification of servo-motors and position/speed encoders used on each NC axis;
3. final checking of selected servo-motors and servo-drive systems used for continuous adjustment of functional parameters of servo-motors for each NC axis;
4. performance evaluation on each robot NC axis,
5. performance evaluation for the overall robot general assembly.

4. CALCULATION SCHEME ELABORATION FOR ROBOT REFERENCE MODEL

Elaboration of a calculus scheme corresponding to IR’s associated design model is a needed because a correct elaboration of this calculus scheme is the fundamental element for following calculation steps. The calculus scheme is a simplified representation of the overall kinematic structure of the IR, elaborated in accordance with the general architecture and specific constructive variant of the reference model of IR to be designed. As result, it should be realized in a form of a symbolic (but as realistic as it can) representation of the IR design model as a structural kinematic schema with the inclusion of all the major IR joints and links between them, by respect for the dimensional proportions between partial assemblies/components and by highlighting of all constructive and functional parameters (parameters that must be also included in the geometric and kinematic
model of the IR to be designed) with respect and marking also of the eccentric disposition of the IR’s components. In the representation of the calculus scheme the IR’s mobile elements must be brought into the positions / orientations leading to the most unfavourable loading configuration (for which the inertial and gravitational load lever arms acting on IR are maximum). If it is not possible to identify a single calculation scheme corresponding to the most unfavourable load configuration of the IR, alternative calculation configurations can be defined for which the calculations for the next steps will be performed in parallel up to the level of certain identification of the configuration that leads to the peak loads on IR [3, 4, and 5].

The results of the calculations made at this stage aim to determine the volume, the mass and the coordinates (x_Gi, y_Gi, z_Gi) of each mass centre specific to the structural elements/partial assemblies of the IR as well as the correct identification and representation on the calculation scheme of all the constructive and functional parameters specific to reference model of the IR to be designed. Correct evaluation of these elements is decisively influencing the correctness of the calculations that will be carried out in subsequent stages, since all the gravitational and inertial loads to be included will be applied exclusively on the mass centres of the partial assemblies of IR and will report directly to the masses of the partial assemblies / structural elements evaluated at this stage. Masses of each partial assembly are considered to be concentrated in their related mass centre. These must be evaluated and located in the calculation scheme by material points identified as location by specific constructive parameters (dimensions). Concentrated masses in the calculation scheme must allow identifying actual distribution of gravitational loads generated by all major partial assemblies/structural elements of the RI to be considered [3, 4].

Fig. 4 [5] illustrates an example of how to set up the calculation scheme of an articulated arm robot with 6 DOF (reference model FANUC ARC Mate 120iB).

5. INERTIAL AND GRAVITATIONAL LOAD DISTRIBUTION IN CALCULATION CENTERS

In this stage, gravitational loads (G_i), inertial forces and the central inertial momentums (relative to the centre axes passing through gravity centres) and the centrifugal inertia momentums (around some eccentric axes relative to the central ones) respectively must be determined.

Determining the spatial distribution of the gravitational and inertial loads taken into account in the design of the NC axes of the IR involves numerical evaluation and graphical representation on the previously elaborated calculation scheme for two major categories of loads: gravitational forces and inertial forces acting in mass centres characteristic for all partially assemblies / structural elements of the RI. For graphical representation on the calculation scheme gravitational and inertial force loads will be applied only in the mass centres, and the inertial momentums only around the rotation axes that generate them. In addition, for each type of inertial forces/inertia momentum, the direction and direction of the specific action must be determined in accordance with the movements made by the RI movable elements and the moment of occurrence of the inertial load taken into account (start/end time of movement).

For gravitational forces, the direction of action is vertical top down. The numerical evaluation of the gravitational forces is done by the relation:

\[ G_i = m_i g. \]  

The numerical evaluation of an inertial force \( F_j \) generated by a mass \( m_j \) in translational movement (joint \( j \)) with the maximum speed \( v_j \) is made with the relation:

\[ F_j = m_j a_{\tau j}, \]  

where \( F_j \) is the value of the inertial force, \( m_j \) – mass, and \( a_{\tau j} \) – acceleration generated in transition translation motion. Acceleration is determined by relation:

\[ a_{\tau j} = \frac{\Delta v_j}{t_{\tau j/acc}} = \frac{v_{j,max}}{t_{\tau j/acc}}, \]  

where \( \Delta v_j \) is speed variation:

\[ \Delta v_j = v_{j,max} - v_{j,min} = v_{j,max} - 0 = v_{j,max} \]  

Inertial force \( F_j \) will be represented in parallel directions to the direction of the movement axis of the movable element \( k \). Inertial forces generated by the rotational movement of a joint \( k \) of an IR can be of two types: centrifugal forces \( F_{cfk} \) and tangential forces \( F_{tgk} \). Centrifugal forces are oriented along the direction of the kinematic radius (gyration) which can be obtained by joining the rotation axis with the gravity centre of a considered material point. Cinematic radius is measured perpendicularly on the rotation axis of the joint, from axis to the mass centre for which the centrifugal force is calculated. Tangential forces are oriented perpendicular on direction of centrifugal forces (tangent to the circular trajectory). Numerical evaluation of inertial centrifugal force \( F_{cfj}^{(i)} \) and inertial tangential force \( F_{tgj}^{(i)} \) can be done with relations:
where $\omega_j$ is the maximum rotation speed in joint $j$, $R_{ij}$ – kinematic radius, and $a_{tg_{ij}}$ – tangential acceleration calculated with:

$$a_{tg_{ij}} = \varepsilon_j R_{ij},$$

where $\varepsilon_j$ is the angular acceleration determined by:

$$\varepsilon_j = \frac{\Delta \omega_j}{\Delta t} = \frac{\omega_{j_{\max}}}{f_{\text{rev/acc}}},$$

where variation of angular speed $\Delta \omega_j$ is:

$$\Delta \omega_j = \omega_{j_{\max}} - \omega_{j_{\min}} = \omega_{j_{\max}} - 0 = \omega_{j_{\max}}.$$

For graphical representation of all inertial and gravitational loads acting on the IR’s structure following steps can be followed: first in the mass centres all the gravitational forces acting on the IR are represented; successively representation of inertial forces for each active joint (one joint by one) may be made. E.g. for a 6 DOF robot first loads are determined for joint 6 (5, 4, 3, 2, 1 being considered locked), then the loads are determined for joint 5 (6, 4, 3, 2, 1 being locked) and so on. Final resulted loads are being considerate to be applied simultaneously so that the IR’s structure will be loaded in maximum loading conditions (corresponding to simultaneous movement of IR from all joints).

For better exemplification of scrolling down this calculation stage further are presented a series of calculation schemas completed with the representation and distribution of the loads based on which the design calculations of IR’s NC axes will be done for an articulated arm robot that has to be designed having as reference the FANUC AM 120i B20 robot model [2, 5].

In this paper the robot is considered to be placed on a supplementary translation axis (joint 0 on the figures) used usually for extension of the robots working space volume. The first figure in this series (Fig. 5) represents the calculation scheme with exclusively representation of gravitational forces distribution acting on the IR.

Numerical evaluation of gravitational forces presented previously in Fig. 6 is done:

- for mass of manipulated object;
- for masses of partial assemblies supplementary equipping the robot (effectors, sensors, coupling systems etc.);
- for masses of partially assemblies composing the robot.

IR partially assemblies design, their masses evaluation and load calculations should be done gradually starting from last elements of IR. In the situation that a preliminary approximately evaluation is needed masses for partial assemblies of the robot can be determined (estimated) by relations:

\[
m_{\text{TOT RI}} = (m_0 + m_1 + m_2 + m_3 + \ldots + m_8);
\]

\[
G_{\text{TOT RI}} = (m_0 + m_1 + m_2 + m_3 + \ldots + m_8) \times g;
\]

\[
m_0 = \rho_0 f_0 V_0; m_0 = \frac{m_{\text{TOT RI}}}{V_{\text{TOT RI}}};
\]

\[
m_1 = \rho_1 f_1 V_1; m_1 = \frac{m_{\text{TOT RI}}}{V_{\text{TOT RI}}};
\]

\[
m_2 = \rho_2 f_2 V_2; m_2 = \frac{m_{\text{TOT RI}}}{V_{\text{TOT RI}}};
\]

\[
m_8 = \rho_8 f_8 V_8; m_8 = \frac{m_{\text{TOT RI}}}{V_{\text{TOT RI}}};
\]

where: $m_{\text{TOT RI}}$ and $V_{\text{TOT RI}}$ are representing total mass and total volume for the reference model of IR to be designed, $m_i$ and $V_i$ represent partial masses and volumes of IR’s subassemblies, $\rho_i$ are the average densities of the materials that the sub-assemblies are built and $f_i$ is a coefficient representing the degree of fulfillment of $V_i$ volumes. After determining of masses and localization of gravity centres for each IR subassembly it is possible to continue to determine the inertial loads generated by possibilities of movement in the active joints of IR.

As for the reference robot model case with 6 NC axes (rotational axes) plus an additional translation axis (on which the robot is placed in order to extend its working range) the calculation of the inertial loads applied to the IR’s overall structure begins with the determination of inertial loads generated by the possibility of movement on the supplementary translation axis (NC axis 0 on the figure). These loads are represented in Fig. 6.
Fig. 7. Representation of IRs parameters for inertial forces distribution generated by motion in joint 1 [5].

Fig. 8. Representation of IRs parameters for inertial forces distribution generated by motion in joint 2 [5].

Fig. 9. Representation of IRs parameters for inertial forces distribution generated by motion in joint 3 [5].
In order to correct evaluate and represent of all inertial forces generated by the successive motion in joint 1, 2 and 3 in each affected mass centre of the robot's partially assemblies, first of all IRe constructive parameters and their specific kinematic radius need to be evaluated (the acting direction and sense of each pair of the centrifugal and tangential inertial forces applied in a mass centre being along respectively perpendicularly on the kinematic radius characteristic for that mass point versus the rotation axis).

That is why first, Figs 7, 8 and 9 present in their left side the robot parameters in terms of link dimensions and elements eccentricities, joint positions, mass centres of subassemblies (m1, m2,..., m8) and calculation centres (C1, C2,..., C6) being also marked on the schemas. The right side of Fig. 7, 8 and 9 present for each i joint, the motion possibility of 1, 2 and 3 IRe's joints marked with m0i and for each affected mass centre the corresponding kinematic radius with reference to the moving joint.

Following these, Figs. 10, 11 and 12 show inertial loads (tangential and centrifugal) generated by the motion in IRe's first three joints (all rotational). The first rotational joint of them (having the corresponding calculation centre C1) represents the IRe's base rotation (1-st NC axis of the robot). The second rotational joint represents the rotation of the fist link of the articulated arm and the third rotational joint represent the rotation of the second link of the articulated arm.

For each mass point considered as affected in Fig. 10, 11 and 12, the centrifugal and tangential inertial loads generated by the movement of corresponding joints are figured with green and red arrows.

Numerical evaluation of these forces can be done with relations [5]:

\[ F_i^j = m_i a_{T_j}, \quad a_{Tj} = \frac{\Delta \omega_j}{\tau_{fr/acc}} = \frac{\omega_{i_{max}}}{\tau_{fr/acc}} \]

\[ \tau_{fr/acc} = 0.5 \text{ sec}. \]  

(16)

For each subassembly of the robot or element marked with its own mass centre the inertial forces generated by the translation on joint 0 can be determined individually with relations:

\[ F_0^0 = m_1 a_{T_0}, \quad a_{T_0} = \frac{\Delta \omega_0}{\tau_{fr/acc}} = \frac{\omega_{i_{max}}}{\tau_{fr/acc}} \]

\[ \tau_{fr/acc} = 0.5 \text{ sec}. \]  

(17)

For numerical evaluation of inertial loads previously presented in Fig. 10, 11 and 12 following mathematical relations may be used:

\[ F_{cf 1}^i = m_i \omega_i^2 R_{11}, \quad F_{tg 1}^i = m_i a_{tg 11}, \]

\[ a_{tg 11} = \epsilon_i R_{11}, \quad R_{11} = 0, \quad \epsilon_i = \frac{\Delta \omega_i}{\Delta t} = \frac{\omega_{i_{max}}}{\tau_{fr/acc}} \]

(18)

Numerical evaluation of inertial loads generated by motion in joint 1 in m1, m2, m3,..., m8 mass centres:

\[ F_{cf 1}^1 = m_1 \omega_1^2 R_{11}, F_{tg 1}^1 = m_1 a_{tg 11}, a_{tg 11} = \epsilon_1 R_{11}, R_{11} = 0, \quad \epsilon_1 = \frac{\Delta \omega_1}{\Delta t} = \frac{\omega_{1_{max}}}{\tau_{fr/acc}} \]

(19)

Numerical evaluation of inertial loads generated by motion in joint 1 in m1, m2, m3,..., m8 mass centres:

\[ F_{cf 1}^3 = m_3 \omega_3^2 R_{23}, F_{tg 3}^3 = m_3 a_{tg 23}, a_{tg 23} = \epsilon_2 R_{23}, R_{23} \neq 0, \quad \epsilon_2 = \frac{\Delta \omega_2}{\Delta t} = \frac{\omega_{2_{max}}}{\tau_{fr/acc}} \]

(20)

Numerical evaluation of inertial loads generated by motion in joint 2 in m3, m4, m5, m6, m7, m8 mass centres:

\[ F_{cf 2}^4 = m_4 \omega_4^2 R_{24}, F_{tg 4}^2 = m_4 a_{tg 24}, a_{tg 24} = \epsilon_2 R_{24}, R_{24} \neq 0, \quad \epsilon_2 = \frac{\Delta \omega_2}{\Delta t} = \frac{\omega_{2_{max}}}{\tau_{fr/acc}} \]

For numerical evaluation of inertial loads generated by motion in joint 3 in m3, m4, m5, m6, m7, m8 mass centres:
\[ F_{cf\,4}^3 = m_4 \rho^2 R_{34}, \quad F_{tg\,4}^3 = m_4 a_{tg\,34}, \quad a_{tg\,34} = \varepsilon_2 R_{34}, \quad R_{34} \neq 0, \quad \varepsilon_2 = \frac{\Delta a}{\Delta t} = \frac{\theta_a}{t_{fr/acc}} \]
\[ F_{cf\,5}^3 = m_5 \rho^2 R_{35}, \quad F_{tg\,5}^3 = m_5 a_{tg\,35}, \quad a_{tg\,35} = \varepsilon_3 R_{35}, \quad R_{35} \neq 0, \quad \varepsilon_3 = \frac{\Delta a}{\Delta t} = \frac{\theta_a}{t_{fr/acc}} \]
\[ F_{cf\,8}^3 = m_8 \rho^2 R_{38}, \quad F_{tg\,8}^3 = m_8 a_{tg\,38}, \quad a_{tg\,38} = \varepsilon_8 R_{38}, \quad R_{38} \neq 0, \quad \varepsilon_8 = \frac{\Delta a}{\Delta t} = \frac{\theta_a}{t_{fr/acc}} \]

For getting an overall total distribution of the gravitational and inertial forces acting in the worst case of loading of the IR the superposition method may be now applied.

The final calculation stages will result in taking into account all acting loads and reduce them in each calculus centre specifically for each IR joint in order to identify \((F_{i\,red}, M_{i\,red})\) loads.

Having the specific \((F_{i\,red}, M_{i\,red})\) forces and torques acting on each IR joint identified, their repartition on different specific sets of components for bearings/guideways, as well as driving kinematic chain of each IR’s mobile element may be performed and respectively afterwards, the optimum design / performance checking procedures for individual IR’s NC axis may be started.

6. CONCLUSIONS

The general algorithm presented in the may be used for two purposes:

- in conceptual development activities and optimal design of new prototypes of similar IRs to existing or different IR variants;
- for identification of necessary functional constructive parameters and the opportunity to use IR’s operation correlative with the specificity of their integration and operation in a certain robotic application.

For the last purpose by applying the present algorithm it is possible to select the type and the optimal constructive variant of IR in relation to the level of performance desired to be obtained in its exploitation.

REFERENCES


