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GRAPHICAL MODELING OF GEAR TOOTH GENERATING PROCESS WITH RACK-SHAPED TOOL, AIMING CONSTANT AREA OF DETACHED CHIPS

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Abstract: *The process of machining with rack shaped tool the cylindrical involute gears is commonly performed on MAAG machine tools, having wide applications in practice. Despite this process is fundamentally rigorous, at the same time it is characterized by a large variation of detached chip area, which negatively influences the toothing process productivity. This paper addresses the case of gear teeth generating with a multi-tooth rack-shaped tool, approached on the base of CATIA graphical modeling. It is proposed a modification of the rolling process, taking part between the centrodes attached to both workpiece and tool, such as the sum of the chips areas that are simultaneously detached by tool teeth at each stroke becomes quasi-constant. Thereby, a variation law for workpiece circular feed is determined, without altering the rolling condition, which would have as consequence the unwanted modification of generated teeth profile. This type of feed variation law can be implemented on NC machine tools of newest generation, provided with the possibility of driving the rolling kinematical chain with non-uniform, adjustable speed.*

Key words: *gear tooth machining, rack shaped tool, uniform area of detached chip, circular feed variation law, graphical modeling.*

1. INTRODUCTION

The principle of generating the gear teeth with a multi-tooth rack shaped cutter is well known and developed, in analytical form, no matter if speaking of involute or other type of tooth profile [1, 2]. Furthermore, the development of soft products with graphical applications enabled the graphical modeling of gear teeth generating, by representing the successive positions of the profiles during the rolling process between the two conjugated centrodes (rectilinear for the rack shaped tool, respective circular for the gear). Thus, the shape and the area of chips successively detached by tool teeth can be graphically determined [3, 4].

The above mentioned results underpinned the idea of finding the energetic load of tool tooth cutting edges. Existing results are presented in the cases of rack shaped cutter, hob mill, pinion cutter, hob mill with detachable teeth [5], as well as hob mill with entering tapered surface [6].

Later developments led to the design of rack shaped tools with reconfigurable geometry, which enable to obtain, in successive passes, substantial modifications of the detached chip area [7, 8], aiming the smoothing of tool energetic load during the toothing process. On this base, new solutions of toothing tools have been developed [9, 10]. They have the particularity that the positions of tool successive teeth can be adjusted relative to the rolling line – respective rolling circle, leading to a new, reconfigured tool profile, with positive impact in what concerns diminishing the variation of removed chip area. Such a scientific research is described in [11].

This paper presents an analysis, developed in the graphical design environment CATIA, of the shape and size of the chip detached by the successive teeth of rack shaped cutter, during the rolling process between the two conjugated centrodes (straight line for tool, respective circle for workpiece).

It should be noticed that removed chip shape, at each position of the tool relative to the workpiece, could have a significant influence on the magnitude of the cutting force needed to detach the chip, because the chip thickness substantially affects the value of specific cutting force. Thus, at very thin detached chips, this effect can prevail in main cutting force establishment.

The problem of generating gear teeth with a mono-tooth rack shaped tool has already been addressed [12], the variation law of the circular feed leading to uniform area of the detached chip being found in condition of strictly respecting the rolling condition. Feed variation law has been expressed depending on the current number of the tool double stroke – meaning, in fact, the space run along the workpiece rolling circle. The problem of using the multi-tooth tool involves to solute a similar problem, with the obvious difference that several tool teeth are simultaneously in contact with the workpiece, which leads to a substantially changed variation of total, cumulated area of removed chips.

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In what concerns paper structure, the next section defines the specific kinematics of toothing process performed with a rack shaped tool. The third section deals with toothing process graphical modeling. The fourth section addresses a numerical simulation aiming to find the circular feed variation law for cutting process smoothing, while the last one is for conclusion.

2. KINEMATICS OF THE TOOTHING PROCESS PERFORMED WITH RACK SHAPED TOOL

In Fig. 1, there are presented the couple of rolling centrodes, the workpiece exterior radius, R_e, the profile of the rack shaped tool and the reference systems needed for defining the teeth generating process. Because here is addressed the toothing process when machining straight teeth gears, the problem may be approached in a plane normal to gear axis so these reference systems are:

- *xy*, meaning the global system, which is fix
- *XY* – local system, rotating together with the workpiece, around its axis
- *ξη* – local system, moving together with the tool, along C_2 centrode.

The parameters of generating motions are: *φ*, meaning the workpiece rotation angle and *λ* – the parameter of tool translation motion. The rolling condition requires the existence of relation:

$$\lambda = R_r \cdot \varphi . \qquad (1)$$

The equations of the motions involved in teeth generating process are, in matricial form:

$$x = \omega_3^T(\varphi) \cdot X \qquad (2)$$

and

$$x = \xi + a , \qquad (3)$$

meaning gear rotation and tool translation, respectively. In relation (2), ω_3 means the coordinates transform matrix at the rotation around an axis normal to *xy* plane, while a vector gives the coordinates of *ξη* system origin expressed in *xy* system,

$$a = \begin{pmatrix} -R_r \\ -\lambda \end{pmatrix} . \qquad (4)$$

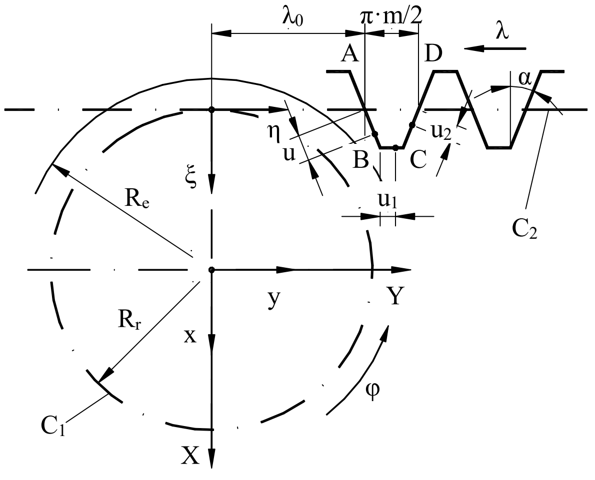


Fig. 1. The kinematics of teeth generating process.

In relation (4), λ_0 parameter gives the initial position of tool first tooth relative to *x*-axis.

The tool tooth profile, formed by three straight segments *AB*, *BC* and *CD* is defined in *ξη* reference system (see Fig. 1). The equations of first segment are:

$$\overline{AB}\left|\begin{array}{l} \xi = u\cos\alpha ; \\ \eta = \lambda_0 + u\sin\alpha , \end{array}\right. \qquad (5)$$

where α means the inclination angle of tool tooth flank (usually, α = 20°) and *u* is a linear parameter, having the variation limits:

$$u_{\min} = -1.2 \cdot m\cos\alpha,\ u_{\max} = 1.2 \cdot m\cos\alpha , \qquad (6)$$

with *m* representing the gear module. The other two segments equations are:

$$\overline{BC}\left|\begin{array}{l} \xi = u_{\max}\cos\alpha ; \\ \eta = \lambda_0 + u_1 , \end{array}\right. \qquad (7)$$

with

$$u_{1\min} = 0,\ u_{1\max} = \frac{\pi \cdot m}{2} - 2 \cdot u_{\max}\sin\alpha , \qquad (8)$$

and

$$\overline{CD}\left|\begin{array}{l} \xi = u_2\cos\alpha ; \\ \eta = \lambda_0 + \dfrac{\pi \cdot m}{2} - u_2\sin\alpha , \end{array}\right. \qquad (9)$$

with

$$u_{2\min} = -1.2 \cdot m\cos\alpha,\ u_{2\max} = 1.2 \cdot m\cos\alpha . \qquad (10)$$

The point where the contact between the rack shaped tool and the workpiece begins can be found by intersecting the circle:

$$X^2 + Y^2 = R_e^{\ 2} \qquad (11)$$

with tool tooth flank (*B* point), Fig. 1.

The family of tool tooth successive positions relative to the workpiece results from equations (2) and (3) as:

$$X = \omega_3(\varphi)[\xi + a]. \qquad (12)$$

The coordinates of *B* point are:

$$B\left|\begin{array}{l} \xi_B = 1.2 \cdot m ; \\ \eta_B = \lambda_0 + 1.2 \cdot m\tan\alpha . \end{array}\right. \qquad (13)$$

The family (12) particularized in *B* point case has the equations:

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\gamma \end{pmatrix} \begin{pmatrix} 1.2 \cdot m - R_r \\ \lambda_0 + 1.2 \cdot m\tan\alpha - R_r \cdot \varphi \end{pmatrix} , \qquad (14)$$

or, after developing:

$$\left|\begin{array}{l} X = (1.2 \cdot m - R_r)\cos\varphi + (\lambda_0 + 1.2 \cdot m\tan\alpha - R_r \cdot \varphi)\sin\varphi ; \\ Y = -(1.2 \cdot m - R_r)\sin\varphi + (\lambda_0 + 1.2 \cdot m\tan\alpha - R_r \cdot \varphi)\cos\varphi . \end{array}\right. \qquad (15)$$

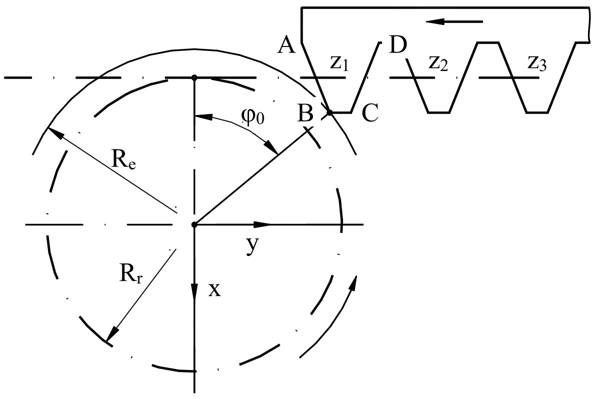


Fig. 2. The beginning of tool – workpiece contact.

From condition (11) and equations (15) one can find the value of angle φ_0, corresponding to the position of first contact between tool and workpiece (Fig. 2). The positions of all points belonging to the composite profile of rack shaped tool, corresponding φ_0 angle, can be further determined relative to the workpiece. Hereby, the characteristic positions of z_1, z_2, z_3 ... teeth can be also specified.

The successive cuts (i and $i + 1$) of the tool can be found for two successive values of φ angle (φ_i and φ_{i+1}, respectively), along the rolling process. Starting from here, the area of the chip detached by the tooth can be calculated. Thus, a representation of detached chip area can be drawn, in principle, versus the increment of the rolling motion

$$\Delta\lambda = R_r \cdot \Delta\varphi\,, \tag{16}$$

with

$$\Delta\varphi = \varphi_{i+1} - \varphi_i\,. \tag{17}$$

Note: There are, obviously, situations when two or more successive teeth of the tool are simultaneously in contact with the workpiece. As consequence, in this case the area of detached chip should be considered as sum of areas of the chips detached by all these teeth.

As the analytical solution for finding the detached chip area on the base of generating process kinematics shows to be low productive, we further present another approach to solve the same problem, by graphical modeling the toothing process.

3. CATIA GRAPHICAL METHOD FOR MODELING THE TOOTHING PROCESS

The following steps need to be made in order to simulate in CATIA (Computer Aided Three dimensional Interactive Applications) software:

- A cylindrical disc is generated starting from workpiece dimensions by using *Pad* modeling tool from *Sketch Based Features* toolbar, after applying geometrical and dimensional constrains. For a better understanding, we further address the actual case of a disc having 320 mm in diameter, corresponding to a gear with z = 30 teeth, of 10 mm module. The disc thickness is not relevant for finding detached chip area, so its value is not specified.

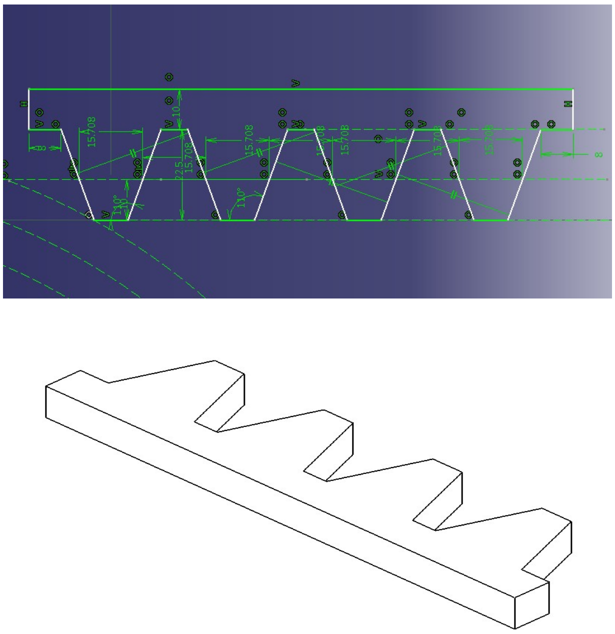
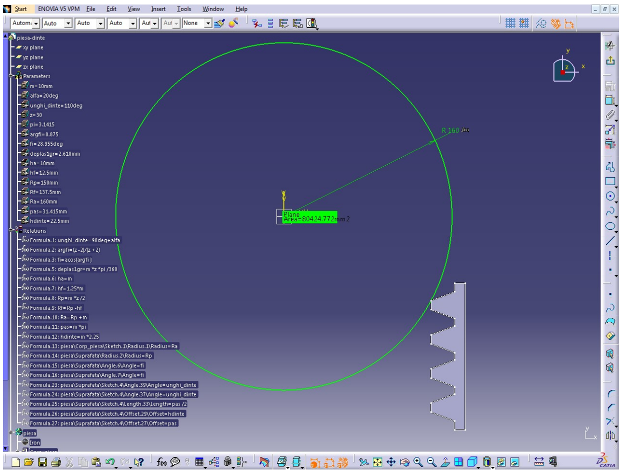


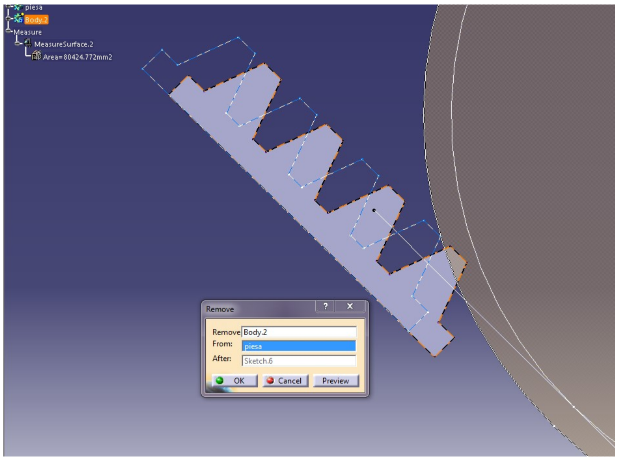
Fig. 3. The rack shaped tool model.



piesa-dinte
xy plane
yz plane
zx plane
Parameters
m=10mm
alfa=20deg
unghi_dinte=110deg
z=30
pi=3.1415
argfi=0.875
fi=28.955deg
deplas1gr=2.618mm
ha=10mm
hf=12.5mm
Rp=150mm
Rf=137.5mm
Ra=160mm
pas=31.415mm
hdinte=22.5mm
Relations
Formula.1: unghi_dinte=90deg+alfa
Formula.2: argfi=(z -2)/(z +2)
Formula.3: fi=acos(argfi)
Formula.5: deplas1gr=m *z *pi /360
Formula.6: ha=m
Formula.7: hf=1.25*m
Formula.8: Rp=m *z /2
Formula.9: Rf=Rp -hf
Formula.10: Ra=Rp +m
Formula.11: pas=m *pi
Formula.12: hdinte=m *2.25
Formula.13: piesa\Corp_piesa\Sketch.1\Radius.1\Radius=Ra
Formula.14: piesa\Suprafata\Radius.2\Radius=Rp
Formula.15: piesa\Suprafata\Angle.6\Angle=fi
Formula.16: piesa\Suprafata\Angle.7\Angle=fi
Formula.23: piesa\Suprafata\Sketch.4\Angle.39\Angle=unghi_dinte
Formula.24: piesa\Suprafata\Sketch.4\Angle.37\Angle=unghi_dinte
Formula.25: piesa\Suprafata\Sketch.4\Length.33\Length=pas /2
Formula.26: piesa\Suprafata\Sketch.4\Offset.29\Offset=hdinte
Formula.27: piesa\Suprafata\Sketch.4\Offset.27\Offset=pas
piesa
Iron

Fig. 4. Disc / tool model relative positioning.

- In a separate file, the solid model of the tool is sketched and extruded (Fig. 3).
- The disc and the tool model are positioned such as tool rolling line is tangent to gear rolling circle and the first tooth of the tool touches the disc (see Fig. 4). Starting from this position, the specific motions are given to both tool and disc, obeying to rolling condition (1).
- The disc rotates in counterclockwise sense with the angular increment Δφ = 0.4°, corresponding to which the tool model advances with Δλ = 1.0472 mm. In resulting position, the tool model partially overlaps to the disc, the intersection domain representing the shape and having the area of the chip whose removal is simulated.
- The intersection domain is extracted from disc surface by applying *Remove* command from *Operations* modeling tool (Fig. 5).



Area=80424.772mm2

Remove
Remove Body.2
From: piesa
After: Sketch.6

Fig. 5. Disc / tool model intersection.

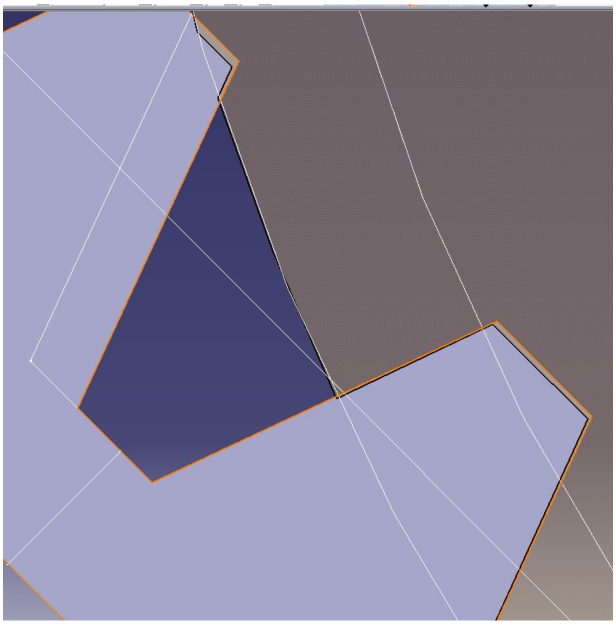


Fig. 6. Traces of two successive cuts.

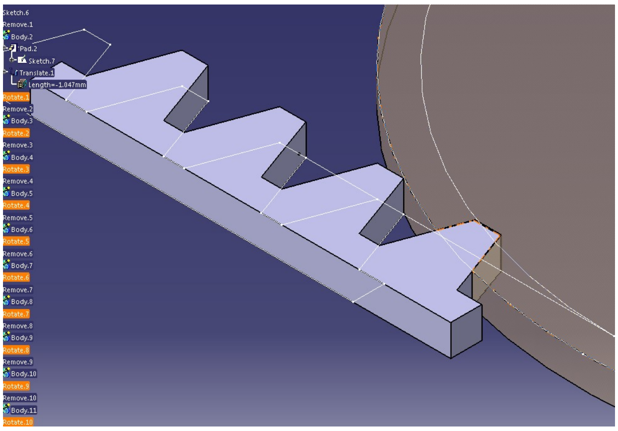


Fig. 7. Disc / second tool tooth contact.

- The disc remaining surface is measured with the help of *Measure item* command from *Measure* toolbar. The difference between previous and current surfaces means the removed chip surface (see Fig. 6).
- The last three steps are iteratively repeated until, when cumulated disc rotations reaches the value

$$\varphi_t = 360° / z \,, \tag{18}$$

  the second tool tooth touches the disc, Fig. 7 (in the addressed case, φ_t = 12°).
- The third tool tooth comes in contact with the disc after another 12° rotation, while when disc rotation arrives to 36°, all four tool teeth are simultaneously in contact with the disc (Fig. 8).
- After a certain number of cutting cycles (here, 111 cycles), the first tooth of the tool model exits from the contact with the disc, after fully generating both flanks of a tooth space.
- To continue the simulation, the rack shaped tool shifts back (Fig. 9) with a dividing pitch

$$p = 2\pi \cdot R_r / z \,, \tag{19}$$

  where p = 31.416 mm. The precision of tool model repositioning can be checked by finding that at the intersection of the two items there are no surface differences.
- The successive cutting cycles are simulated again, until the next tooth space is obtained, and so on.

The calculated values of removed chips area (cumulated area, if more than one tool tooth is in contact with the disc) are recorded and further processed, in order to find the corresponding variation law.

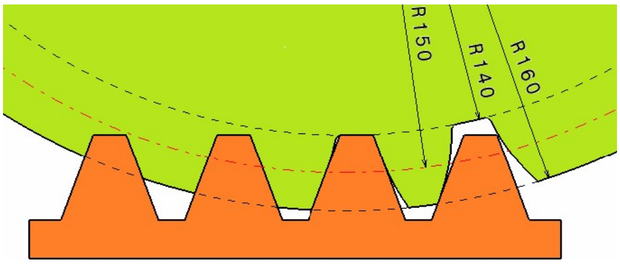


Fig. 8. Tool model / disc four teeth contact.

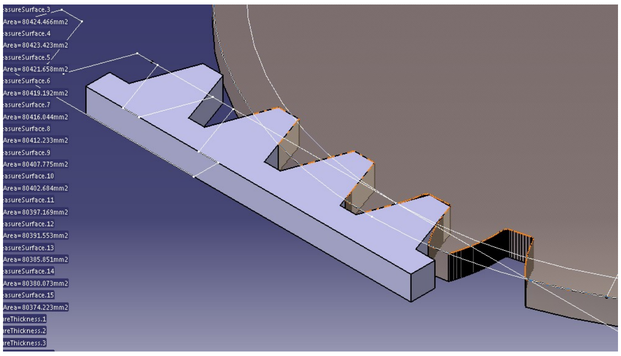

asureSurface.3
Area=80424.466mm2
asureSurface.4
Area=80423.423mm2
asureSurface.5
Area=80421.658mm2
asureSurface.6
Area=80419.192mm2
asureSurface.7
Area=80416.044mm2
asureSurface.8
Area=80412.233mm2
asureSurface.9
Area=80407.775mm2
asureSurface.10
Area=80402.684mm2
asureSurface.11
Area=80397.169mm2
asureSurface.12
Area=80391.553mm2
asureSurface.13
Area=80385.851mm2
asureSurface.14
Area=80380.073mm2
asureSurface.15
Area=80374.223mm2
reThickness.1
reThickness.2
reThickness.3

Fig. 9. Tool model repositioning.

4. NUMERICAL SIMULATION

Table 1 samples the numerical results obtained in the addressed toothing simulation, as follows: the first and the last ten cutting cycles corresponding to the situation when only the first tool tooth contacts the workpiece, followed by the cutting cycles from the interval between the first and the second repositioning of the tool.

In Fig. 10, there are presented the variation of chips cumulated area during the toothing process (in blue, thicker line) and the variation of area for the chip detached by each tool tooth (in red, thinner line). It should be noticed that, after the first tool repositioning, we have the same variation law of chips cumulated area between every two successive tool repositionings.

Table 1

Variation of removed chips area

Cutting cycle no.	φ [deg]	λ [mm]	A_{disc} [mm^2]	A_{chip} [mm^2]
0	0	0	80424.772	0.000
1	0.4	1.047	80424.396	0.376
2	0.8	2.094	80423.281	1.115
3	1.2	3.142	80421.445	1.836
4	1.6	4.189	80418.911	2.534
5	2	5.236	80415.697	3.214
6	2.4	6.283	80411.820	3.877
7	2.8	7.330	80407.300	4.520
8	3.2	8.378	80402.147	5.153
9	3.6	9.425	80396.618	5.529
10	4	10.472	80390.993	5.625
...	...	...	...	...
21	8.4	21.991	80325.462	6.083
22	8.8	23.038	80319.381	6.081
23	9.2	24.086	80313.305	6.076
24	9.6	25.133	80307.250	6.055
25	10	26.180	80301.217	6.033
26	10.4	27.227	80295.211	6.006
27	10.8	28.274	80289.239	5.972
28	11.2	29.322	80283.300	5.939
29	11.6	30.369	80277.410	5.890
30	12	31.416	80271.569	5.841
...	...	...	...	...
112	32.8	85.870	79406.292	10.553
113	33.2	86.918	79395.906	10.386
114	33.6	87.965	79385.694	10.212
115	34	89.012	79375.661	10.033
116	34.4	90.059	79365.810	9.851
117	34.8	91.106	79356.149	9.661
118	35.2	92.154	79346.681	9.468
119	35.6	93.201	79337.408	9.273
120	36	94.248	79328.335	9.073
121	36.4	95.295	79319.089	9.246
122	36.8	96.342	79309.309	9.780
123	37.2	97.390	79299.020	10.289
124	37.6	98.437	79288.240	10.780
125	38	99.484	79276.993	11.247
126	38.4	100.531	79265.299	11.694
127	38.8	101.578	79253.179	12.120
128	39.2	102.626	79240.647	12.532
129	39.6	103.673	79227.955	12.692
130	40	104.720	79215.388	12.567
131	40.4	105.767	79202.958	12.430
132	40.8	106.814	79190.678	12.280
133	41.2	107.862	79178.553	12.125
134	41.6	108.909	79166.598	11.955
135	42	109.956	79154.822	11.776
136	42.4	111.003	79143.224	11.598
137	42.8	112.050	79131.806	11.418
138	43.2	113.098	79120.564	11.242
139	43.6	114.145	79109.499	11.065
140	44	115.192	79098.607	10.892
141	44.4	116.239	79087.887	10.720

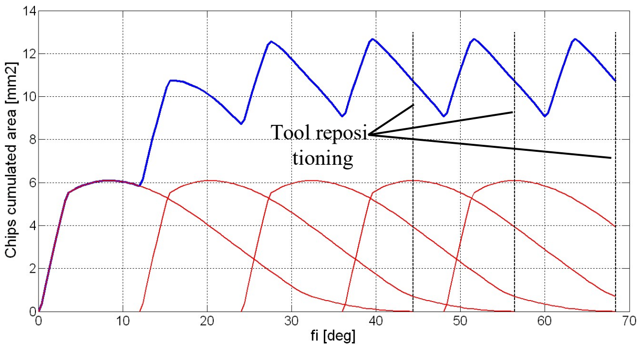


Fig. 10. Variation of removed chips cumulated area versus workpiece rotation angle.

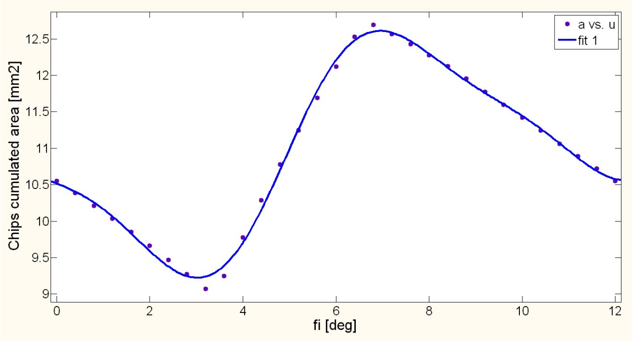


Fig. 11. Approximation function fitting.

As consequence, the cutting process smoothing can be reached by turning the corresponding curve in another one, as close as possible to a horizontal line, by finding the appropriate variation law for the circular feed of the machined workpiece.

In this purpose, an analytical expression for the function approximating the variation of chips cumulated area is needed. This can be done with the help of MatLab soft, by using *Curve Fitting Tool*. For the addressed numerical simulation, we have chosen a *Sum of sinuses* model of approximation function:

$$f(x) = a_1 \sin(b_1 \cdot x + c_1) + a_2 \sin(b_2 \cdot x + c_2) + \\ + a_3 \sin(b_3 \cdot x + c_3). \tag{20}$$

After creating an approximation dataset by the rows 112 ... 141 from Table 1 (the values of φ being translated, for easier operating, such as the first point of variation curve corresponds to φ = 0), the results of searching *f* function parameters were:

$$a1 = 12.41;\ b1 = 0.02742;\ c1 = 0.9511; \\ a2 = 1.273;\ b2 = 0.6995;\ c2 = 2.744; \\ a3 = 0.2865;\ b3 = 1.344;\ c3 = -0.2542. \tag{21}$$

The approximation curve is depicted in Fig. 11.

The MatLab application already developed and introduced in (12), which finds the circular feed variation law such as the removed chip area becomes uniform, has been used in this case also.

After noticing the limit-values of chips cumulated area (Fig. 10), we have chosen a middle value (A_c = 11 mm^2) as target-value of the chips cumulated area after cutting process smoothing.

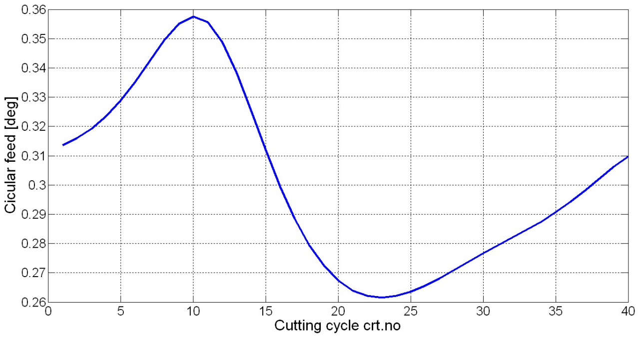


Fig. 12. Circular feed variation law if toothing process smoothing is intended.

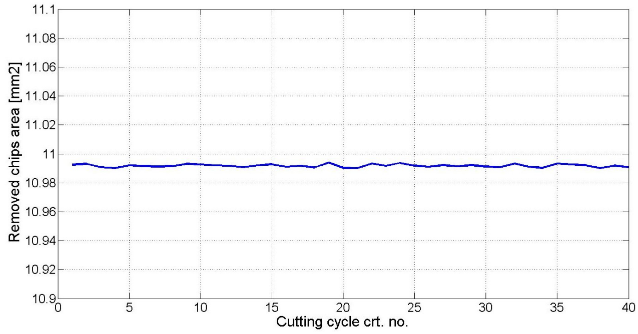


Fig. 13. Variation of removed chips cumulated area versus cutting cycle current number when the modified circular feed is applied.

The expression of chips area variation law, which is also input information for the application, has been obviously obtained from (20) and (21).

The result – the values of the workpiece circular feed for each double stroke of the rack shaped tool are presented, in graphical form, in Fig. 12. It should be noticed that, in this case, the total number of double strokes needed between two successive tool repositionings increases from 30 to 40. This is directly related to the set value of A_c. If the tooth machining process target is a higher productivity, then a bigger value should be set for A_c, while if a better quality of teeth flanks surface is aimed, than A_c value might be reduced even more.

The effect of applying the feed variation law that was found in what concerns the values of removed chips cumulated area is presented in Fig. 13. As it can be easily observed, these values remain quasi-constant.

6. CONCLUSIONS

This paper addresses the problem of smoothing, from energetically point of view, the teeth machining process, in the case of gear teeth generating with a multi-tooth rack-shaped tool. In this purpose, the specific kinematics of this process has been analytically modeled, at first. Then, the toothing process has been graphically modeled in CATIA and, on this base, an algorithm for measuring the cumulated area of the chips detached by all tool teeth being concomitantly in contact with the workpiece has been further developed. The algorithm feasibility has been tested by applying it in the actual case of a gear with 30 teeth of module 10 mm. The variation law of chips area has been found by using MatLab facilities and a trigonometric model. In the end, the values of the workpiece circular feed, needed in order to maintain constant, at each stroke, the cumulated area of detached chips, have been determined.

All presented results sustain the viability of the imagined solution, which could be implemented on NC machine tools enabling to perform the manufacturing process with variable cutting regime. As future challenge we can mention the finding of an unitary solution for toothing process smoothing, such as a parametric shape of the circular feed variation law, the only thing needing to be done by the operator being to adjust parameters values according to the number of teeth and module of the machined gear.

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