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GRAPHICAL & ANALYTICAL MODELING OF THE CUTTING SCHEME USED TO GENERATE SQUARE HOLES WITH SLOTTING TOOLS HAVING PERIODIC PROFILE

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Abstract: *The machining of interior surfaces having non-involute profile, by rolling method, with slotting tools generating by enwrapping, leads to an important variation of the detached chips area. This further leads to the uneven loading of tool cutting edges, from energetic point of view, with negative consequences in what concerns the tool wear or the elastic deformation of both tool and machined part, which, at its turn, causes machining errors. At the same time, the uneven loading of the machine tool mobile parts worsen the manufacturing system functionality. This paper addresses the case of generating square holes with slotting tools having periodic profile. It presents an analytical method for tool profiling and the graphical modeling in CATIA of the cutting scheme, when machining with constant circular feed. On this base, the machining process smoothening can be obtained by subsequently adopting a variable circular feed of the rolling motion, such as the detached chips have constant area in frontal section. A numerical application concerning an actual case is also included.*

Key words: *square hole machining, slotting tools with periodic profile, graphical modeling, constant area of detached chips, circular feed variation law.*

1. INTRODUCTION

The holes with non-involute profile (e.g. triangular, square or hexagonal, in transversal section) are commonly used at the parts used for transmitting torques, in association with corresponding profiled shafts [1], as alternative solution to the flutes or wedge assemblies. The torque transmission by profiled shafts and sleeves is convenient due to their high loading capacity, as well as to the simplicity of the tools needed for their machining [2]. The generation of hole profile by rolling method supposes to find the profile of the required slotting tool, and to apply the appropriate settings of the slotting machine [3].

In this paper, an analytical method for slotting tool profiling [4, 5], in the particular case concerning the machining of a square hole, is further suggested. The method enables to find numerically the tooth profile [6] and it is further implemented in CATIA graphical environment, in order to obtain the cutting scheme modeling in different manner relative to the already existing ones (e.g. [7]). The graphical modeling of the cutting scheme enables to find the variation law of the detached chips area, which is needed if intending to smoothen the machining process by replacing the uniform circular feed by a variable one, which is our final target.

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In what concerns paper structure, the next section defines the specific kinematics of square hole generating process performed with the slotting tool. The third section deals with the analytically finding of tool profile. The fourth section addresses the graphical modeling of the cutting scheme in a concrete machining case, while the last one is for conclusion.

2. KINEMATICS OF THE SQUARE HOLE GENERATING PROCESS, PERFORMED WITH THE SLOTTING TOOL

The Fig. 1 presents the transversal profile of the machined sleeve, the centrodes attached to the sleeve and to the slotting tool, as well as the rolling motions performed during the generating process.

The circle of radius R_{rp} having the center in the symmetry center of the square hole, circumscribed to it, is considered as centrode of the machined surface. The requirement of generating the entire length of square hole side, *BC*, leads to the necessity that both ending points, *B* and *C* retrieve on this centrode [1].

Note: The value of the rolling radius should be adopted after imposing the condition that the normal drawn in any point belonging to the generated profile intersects the rolling circle (the centrode of R_{rp} radius). For this reason, one can speak about a minimum value of the rolling radius, starting from which higher or equal values could be accepted. Obviously, in the case of the singular points from the reciprocal enwrapped profiles (as *B'* and *C'* from tool profile), the occurrence of so-called "interference trajectories" becomes possible, with all their known inconveniences.

Three motions are needed in order to generate the square hole profile:

- I – workpiece rotation of Φ_1 angular parameter, around its symmetry axis passing by *O* point,
- II – slotting tool rotation of Φ_2 angular parameter, around its symmetry center passing by O_t point, and
- III – rectilinear double stroke motion of the tool, parallel with worked piece axis.

Between the angular parameters Φ_1, Φ_2, on one side, and the worked part and tool rolling radii, on the other side, the following condition must be obeyed:

$$R_{rp} \cdot \phi_1 = R_{rt} \cdot \phi_2 , \qquad (1)$$

The meaning of condition (1) is that the centrodes associated to the worked part and to the tool are rolling without sliding. Because the motion III has intermittent character, the motion of rolling between the two centrodes should not be considered continuous, but incremental, the relation (1) becoming:

$$R_{rp} \cdot \Delta\phi_1 = R_{rt} \cdot \Delta\phi_2 . \qquad (2)$$

In relation (2), $\Delta\Phi_1$ and $\Delta\Phi_2$ mean incremental values of the circular feed motions, measured in mm/dbl. stroke.

The tool (the slotting cutter) may have the profile composed by z_s = 1, 2 or 3 lobes – if z_s = 4, the tool would become a punch with the shape identical to the generated hole profile. The case of the slotting tool with three lobes is further considered. As consequence, the gearing ratio between the motion I and II is:

$$i = \frac{\phi_2}{\phi_1} = \frac{R_{rp}}{R_{rt}} = \frac{4}{3} = \frac{z_p}{z_t} . \qquad (3)$$

The definition of the following reference systems is needed in order to describe the kinematics of the generating scheme (meaning the relative motion between worked part and tool), Fig.1:

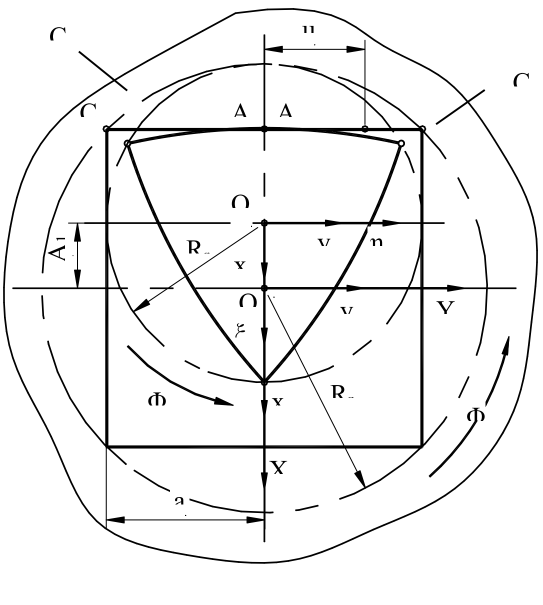


Fig. 1. The kinematics of teeth generating process.

- *xy* – the global system having the origin into *O* point
- *XY* – relative system, rotating together with the workpiece, around its axis, initially overlapped to *xy* system
- x_0y_0 – global system having the origin into O_t point
- $\xi\eta$ – relative system, rotating with the tool around O_t, initially overlapped to x_0y_0 system.

3. THE SLOTTING TOOL PROFILE

The first problem to be solved for modeling the detached chips area is to find the slotting tool profile (*C'B'* arc, see Fig. 1). In fact, because the hole profile is symmetric, it is sufficient to find *A'B'* arc profile.

The parametric equations of *AB* segment, written into *XY* reference system, are:

$$\Sigma_{AB}\left| \begin{array}{l} X = -a; \\ Y = u, \end{array} \right. \qquad (4)$$

with *a* meaning half of square hole side and *u* – variable parameter along *AB*.

The rotation motions I and II are described, respectively, by the transforms:

$$x = \omega_3^T(\phi_1) \cdot X , \qquad (5)$$

and

$$x_0 = \omega_3^T(\phi_2) \cdot \xi , \qquad (6)$$

In relations (5) and (6) ω_3 is the matrix for the coordinates transform associated to rotation around the third axis of the reference system (usually denoted by *Z*).

The relative position between the global systems x_0y_0 and *xy* is defined through the transform

$$x_0 = x + A_{12} \cdot \vec{i} , \qquad (7)$$

with

$$A_{12} = R_{rp} - R_{rt} , \qquad (8)$$

The relative motion between the mobile systems *XY* and $\xi\eta$ results from relations (5), (6) and (7) as:

$$\xi = \omega_3(\phi_2)\left[\omega_3^T(\phi_1) \cdot X + \begin{pmatrix} A_{12} \\ 0 \end{pmatrix}\right], \qquad (9)$$

or, in expanded form, after also replacing *X* from (4):

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{pmatrix} \cos\phi_2 & \sin\phi_2 \\ -\sin\phi_2 & \cos\phi_2 \end{pmatrix}\left[\begin{pmatrix} \cos\phi_1 & -\sin\phi_1 \\ \sin\phi_1 & \cos\phi_1 \end{pmatrix}\begin{pmatrix} -a \\ u \end{pmatrix} + \begin{pmatrix} A_{12} \\ 0 \end{pmatrix}\right]. \qquad (10)$$

The equations of the family of part flanks *AB*, written in 3-D, into $\xi\eta\zeta$ system, are:

$$(\Sigma)_{\phi_1}\left| \begin{array}{l} \xi = -a\cos(\phi_2 - \phi_1) + u\sin(\phi_2 - \phi_1) + A_{12}\cos\phi_2 ; \\ \eta = a\sin(\phi_2 - \phi_1) + u\cos(\phi_2 - \phi_1) - A_{12}\sin\phi_2 ; \\ \zeta = t . \end{array} \right. \qquad (11)$$

In the system (11), *t* means a variable parameter taking arbitrary values.

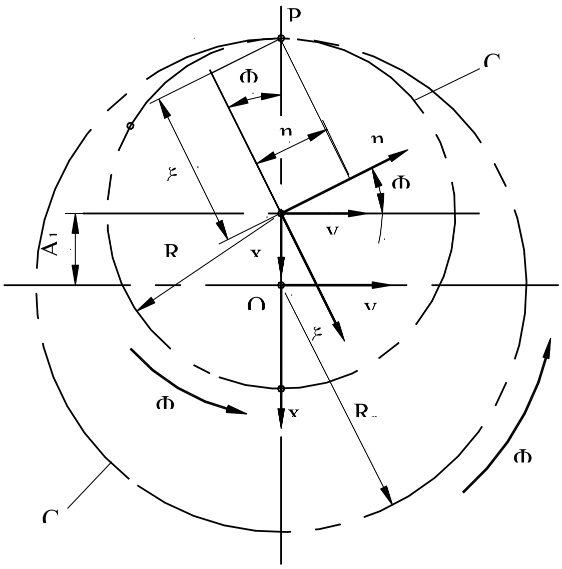


Fig. 2. The rolling centrodes & the gearing pole.

The envelop of $(\Sigma)_{\Phi1}$ family of profiles gives the shape of the cutting edge of the slotting cutter and represents, in principle, a cylindrical surface having the generators normal to ξη plan.

The enveloping condition associated to the family $(\Sigma)_{\Phi1}$ can be enounced according to the Minimum distance theorem [6]. Hereby, the envelop of part flanks family (11) generated in the motion relative to the tool reference system ξη, when $t = 0$, is the locus of the points belonging to profiles family that, in the different rolling positions (corresponding to values of Φ_1 parameter), have minimum distance to gearing pole P, Fig. 2.

The gearing pole coordinates corresponding to a certain position of ξη system, determined by the value of Φ_2 parameter are:

$$P\left|\begin{array}{l}\xi_P = -R_{rt}\cos\phi_2;\\ \eta_P = R_{rt}\cdot\sin\phi_2.\end{array}\right. \quad (12)$$

The distance from gearing pole to the current point from AB profile can be calculated in ξη system with:

$$d = \sqrt{\left(\xi + R_{rt}\cos\phi_2\right)^2 + \left(\eta - R_{rt}\sin\phi_2\right)^2}\,. \quad (13)$$

The imposition of minimum condition to $d = d(u)$ function means to annul its derivative against u. This leads to the following equation:

$$\left(\xi + R_{rt}\cos\phi_2\right)\cdot\xi_u' + \left(\eta - R_{rt}\sin\phi_2\right)\cdot\eta_u' = 0\,. \quad (14)$$

By replacing in (14) ξ and η from (11), after calculus, the condition (14) reduces to:

$$u = R_{rp}\sin\phi_1\,. \quad (15)$$

The equations of profiles family (11) together with condition (15) lead to the parametric equations of the family envelop, representing, in fact, the profile of the slotting tool cutting edge. If the condition (3) is also considered, then these equations take the form:

$$S_{A'B'}\left|\begin{array}{l}\xi = -a\cos\left[(i-1)\phi_1\right] + R_{rp}\sin\left[(i-1)\phi_1\right] + A_{12}\cos\left(i\cdot\phi_1\right);\\ \eta = a\sin\left[(i-1)\phi_1\right] + R_{rp}\cos\left[(i-1)\phi_1\right] - A_{12}\sin\left(i\cdot\phi_1\right);\\ \zeta = t.\end{array}\right. \quad (16)$$

The successive positions of the tool relative to worked part model determine the shape and the area of the chips detached during the machining process. It is obvious that the analytical solving of the problem concerning the finding of the detached chips variation law during the rolling process is difficult. Hereby, the development of a graphical method for solving the problem is more than welcome.

Note: Before engaging the slotting tool in part material, for generating the square profile, a cylindrical hole should be machined by drilling.

The implementation of the analytical results from above were sampled in the case of machining a square hole having a = 20 mm. According to the presented considerations, the rolling radii were chosen as $R_{rp} = a\cdot\sqrt{2} = 20.284$ mm, while $R_{rt} = R_{rp}/1 = 7.071$ mm.

A dedicated MatLab application was used in order to calculate the coordinates of the points from tool profile. Due to problem symmetry, only half of this profile was actually determined for u = 0 ... 20 mm. The interval was meshed in n_u = 101 points, the distance between each

Table 1

The slotting tool profile (excerpt)

Crt. no.	ξ [mm]	η [mm]
1	−12.928	0
2	−12.928	0.180
3	−12.928	0.360
4	−12.927	0.541
5	−12.925	0.721
6	−12.923	0.902
7	−12.921	1.082
8	−12.918	1.263
9	−12.915	1.443
10	−12.911	1.624
....		
46	−12.491	8.143
47	−12.471	8.325
48	−12.451	8.507
49	−12.430	8.689
50	−12.409	8.871

Crt. no.	ξ [mm]	η [mm]
51	−12.387	9.054
52	−12.365	9.236
53	−12.342	9.419
54	−12.319	9.601
55	−12.295	9.784
....		
92	−11.041	16.649
93	−10.995	16.839
94	−10.949	17.029
95	−10.903	17.219
96		
97	−10.807	17.602
98	−10.758	17.793
99	−10.708	17.985
100	−10.658	18.178
101	−10.606	18.371

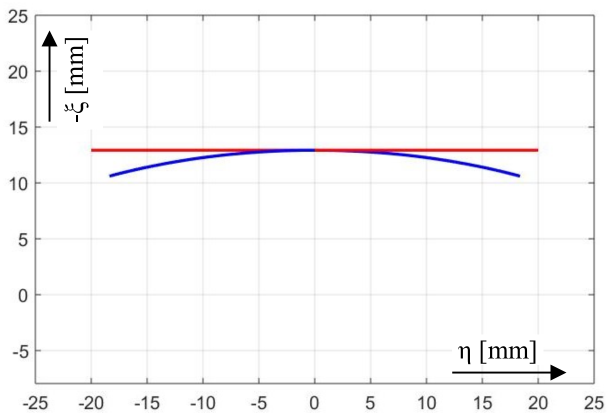


Fig. 3. The slotting tool & the square hole profiles.

two successive points being of 0.2 mm. The results are sampled in Table 1. Figure 3 presents the tool profile resulted by joining the points. Here the coordinates (in mm) are referred to tool symmetry axis.

4. GRAPHICAL MODELING OF THE CUTTING SCHEME

A graphical solution has been developed, in CATIA environment, in order to obtain the cutting scheme modeling and to find the shape and the area of the detached chips at each double stroke of the slotting tool.

For a better understanding, the solution is further introduced by presenting its implementation in an actual case: the machining of a square hole having a = 20 mm.

At first, the cutting tool model is generated. In this purpose, the relation (16) is used for calculating the coordinates (ξ, η) of the points belonging to one of cutting tool edges. These coordinates are input in *CATIA Sketcher* module and joined in a spline curve represeting one side of the tool (Fig. 4). The closed profile of the tool results by mirroring (with *Mirror* tool from *Operation* toolbar) this side after conveniently chosen axes (Fig. 5).

Finally, the tool solid model is obtained by profile extruding (Fig. 6).

The worked piece model consists in a circular rim, created in a separate file with *Pad* modeling tool (Fig. 7). The diameter of the pre-existing circular hole was considered equal to the side of the square hole following to be generated. The rim exterior diameter is 100 mm.
The two models are then put in the relative position corresponding to the start of the machining process. Separate, incremental rotation motions, according to the analytical generating scheme, are given to both models, such as the rolling condition (2) is obeyed.

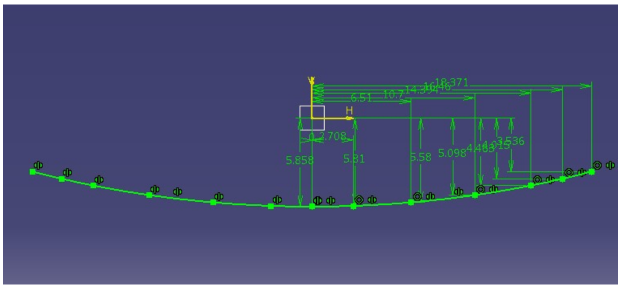


Fig. 4. Modeling of the slotting tool cutting edge.

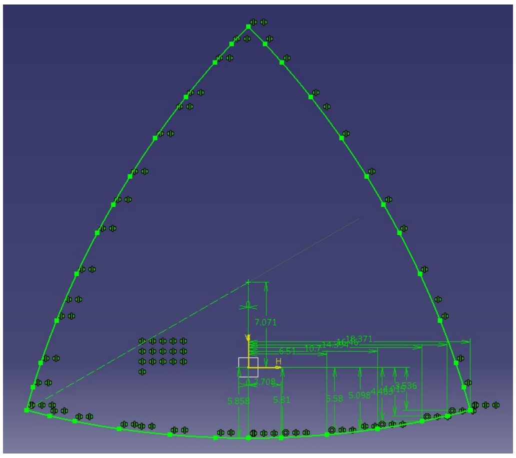


Fig. 5. Modeling of the slotting tool profile.

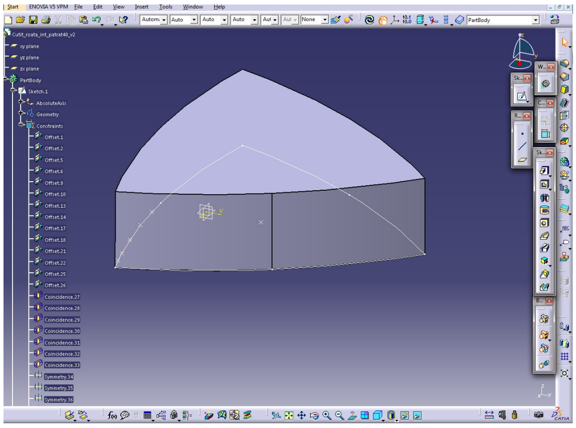


Fig. 6. The tool solid model.

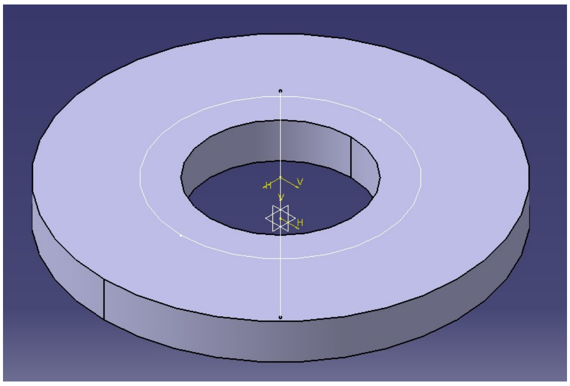


Fig. 7. The worked piece solid model.

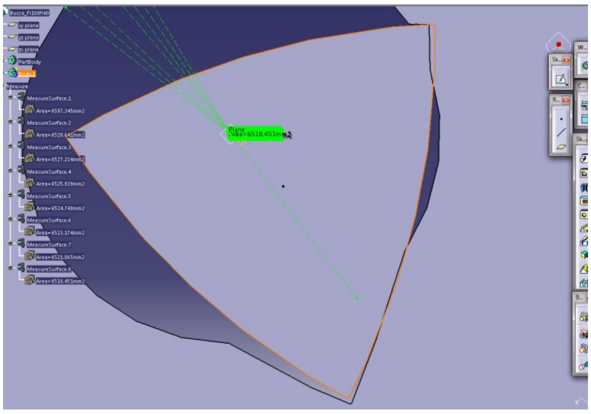


Fig. 8. Modeling of the generating process (side 1).

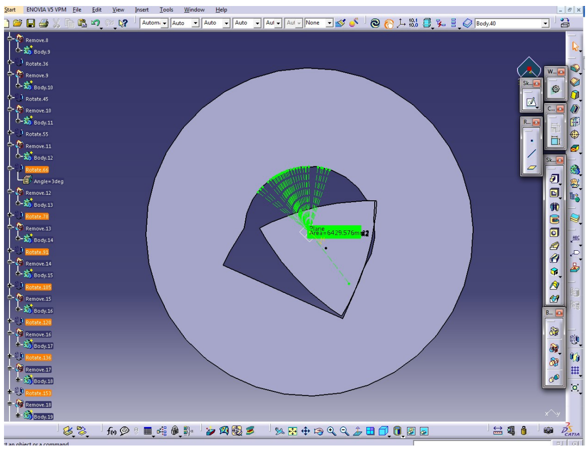


Fig. 9. Modeling of the generating process (side 2).

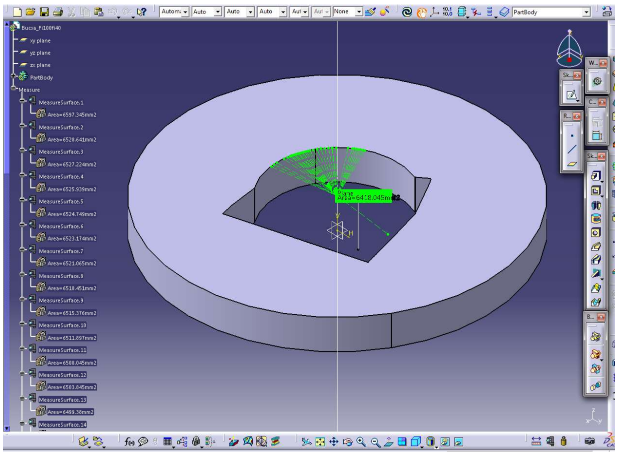


Fig. 10. Modeling of the generating process (side 2, 3D).

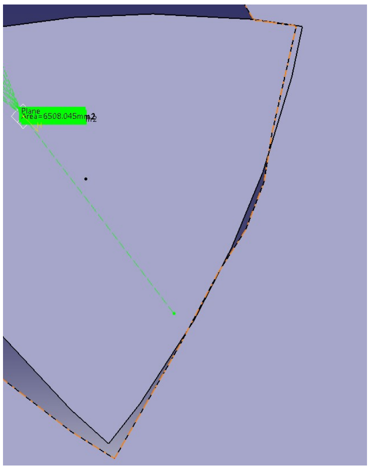


Fig. 11. Modeling of the detached chip shape.

In the addressed case, the rotation increments are $\Delta\Phi_1 = 3°$ for the worked piece model and $\Delta\Phi_2 = 4°$ for tool model. After each couple of correlated incremental rotations, the models profiles are intersected (Figs. 8−10), using *Remove* tool from *Boolean operations* toolbar. Then, the common part of the two entities is removed and the remaining area of the worked piece is measured with the help of *Measure item* command from *Measure* toolbar. The difference between previous and current surfaces means the removed chip surface (see Fig. 11).

Table 2 presents an excerpt from the numerical results obtained in the addressed numerical simulation, for generating all four sides of the square hole. The second and third columns contain the rotation angle of the worked part, respective of the tool, the fourth column – the remaining area of the rim after removing the part – tool intersection area at current iteration, while the fifth – the detached chips area.

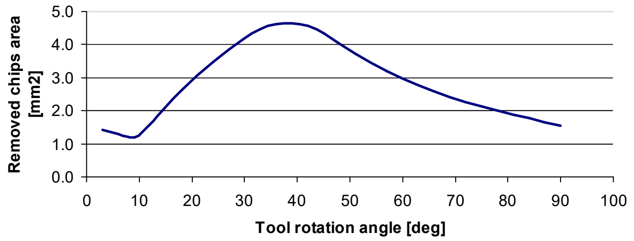
In Fig. 12, the variations of detached chips cumulated area during square hole generating process are presented. It should be noticed that, at the machining of each from the first three sides, two cutting edges of the tool are concomitantly engaged in the cutting process, while at the machining of the fourth side, only one cutting edge actually removes chips. For this reason, the variation of detached chips area is different in these two cases, as one can observe in Figs. 12,*a* and *b*.

Table 1

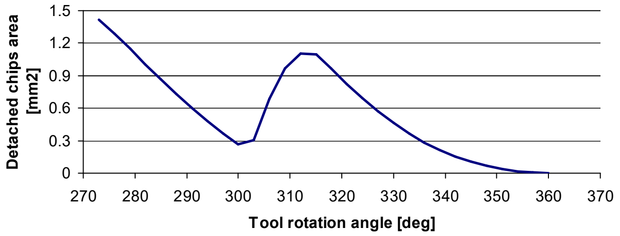
Variation of removed chips area

Stroke no.	φ_1 [deg]	φ_2 [deg]	A_{rim} [mm^2]	A_{chip} [mm^2]
0	0	0	6528.641	0.000
1	3	4	6527.224	1.417
2	6	8	6525.939	1.285
3	9	12	6524.749	1.190
4	12	16	6523.174	1.575
5	15	20	6521.065	2.109
6	18	24	6518.451	2.614
7	21	28	6515.376	3.075
8	24	32	6511.897	3.479
9	27	36	6508.045	3.852
10	30	40	6503.845	4.200
11	33	44	6499.380	4.465
12	36	48	6494.769	4.611
13	39	52	6490.126	4.643
14	42	56	6485.569	4.557
15	45	60	6481.223	4.346
16	48	64	6477.188	4.035
17	51	68	6473.463	3.725
18	54	72	6470.018	3.445
19	57	76	6466.826	3.192
20	60	80	6463.863	2.963
...	...	...	...	...
66	198	264	6346.853	2.614
67	201	268	6343.778	3.075
68	204	272	6340.299	3.479
69	207	276	6336.447	3.852
70	210	280	6332.247	4.200
71	213	284	6327.782	4.465
72	216	288	6323.171	4.611
73	219	292	6318.528	4.643
74	222	296	6313.971	4.557
75	225	300	6309.625	4.346
76	228	304	6305.590	4.035
77	231	308	6301.865	3.725
78	234	312	6298.420	3.445
79	237	316	6295.228	3.192
80	240	320	6292.265	2.963
...	...	...	...	...
106	318	424	6257.939	0.970
107	321	428	6257.113	0.826
108	324	432	6256.420	0.693
109	327	436	6255.849	0.571
110	330	440	6255.386	0.463
111	333	444	6255.020	0.366
112	336	448	6254.737	0.283
113	339	452	6254.526	0.211
114	342	456	6254.375	0.151
115	345	460	6254.272	0.103
116	348	464	6254.207	0.065
117	351	468	6254.170	0.037
118	354	472	6254.153	0.017
119	357	476	6254.147	0.006
120	360	480	6254.147	0

The curve depicted in Fig 12 resulted by points joining. Because the analytical law of the circular feed is needed for machining process smoothing, an analytical law should also be found for modeling the detached chips area. This can be done by fitting curves to the points generated by graphical modeling and meaning the detached chips area at each tool stroke. In this purpose, MatLab *Curve Fitting Tool* was used. Two different

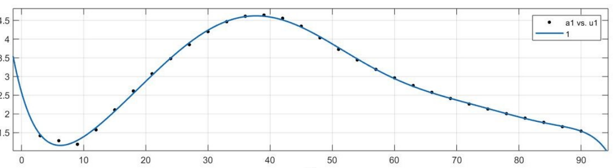


a

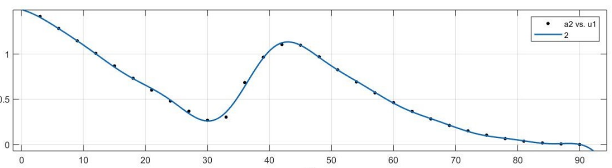


b

Fig. 12. The variation of detached chips area:
a − when machining each of the first three sides;
b − when machining the fourth side.



a



b

Fig. 13. The fitted curves: *a* − first pattern of area variation;
b − second pattern of area variation.

types of curves were found as the most suitable for the two types of area variation: a polynomial function of 9th degree (Fig. 13,*a*), respective a sum of sinuses:

$$f(x) = a_1 \sin(b_1 x + c_1) + \ldots + a_6 \sin(b_6 x + c_6). \quad (17)$$

The actually resulted curves are depicted in Fig. 13.

The Root Mean Square Error (RMSE) resulted at curves fitting was 0.0511 (in the first case), respective 0.0245 (in the second case).

5. CONCLUSION

This paper addresses the problem of smoothing, from energetically point of view, the machining process, in the case of generating square holes with slotting tools with periodic profile. The proposed solution consists in imposing a uniform area of the detached chips by working with a circular feed varying after a suitable law.

In this purpose, the specific kinematics of this process has been analytically modeled, at first. Then, the tool profile was determined as envelop of the family of surfaces described by the worked part in its motion relative to tool reference system. The enveloping condition has been written according to the Minimum distance method. Finally, the square hole generating process was graphically modeled in CATIA environment, this enabling to find the shape and the area of the detached chips.

The problem of machining a square hole having the side of 40 mm was actually addressed. In the case of the first three sides, the numerical results show an identical variation for the detached chips area, while at the fourth side machining there is a different variation. The explanation for this consists in the different number of cutting edges that are simultaneously engaged in cutting. Analytical expressions were determined for the variation law of the detached chips area by fitting curves to the points obtained through graphical modeling. For the first variation pattern, a 9th degree polynomial function was found as the best choice, while for the second – a trigonometric function is suitable.

The finding of the variation laws for the detached chips area is further enabling to determine the circular feed variation law such as the machining process becomes quasi-uniform.

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