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CALCULUS ALGORITHM FOR EVALUATION OF GRAVITATIONAL AND INERTIAL LOADS ACTING ON A GANTRY INDUSTRIAL ROBOT IN PICK AND PLACE APPLICATIONS

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Abstract: *The paper presents the first stage of calculus methodology for optimal structure design of numerically controlled (NC) axes of gantry industrial robot (IR) type. Such calculus methodology can be applied in conceptual design and optimization of mechanical structure for new robot prototypes (with similar general architecture and NC axis structure), or for a correct identification of constructive robot variants of robots having the ability to do a performance check correlated with performances intended to be achieved by the robot within the application that it needs to be integrated (to identify right constructive and functional parameters of a robot for a particular robotic application).*

Key words: *industrial robot behavior, gantry robot, calculus methodology, NC axis, optimal structure.*

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1. INTRODUCTION

In the design of industrial robots for each general architecture and respectively constructive variant, the stages of designing the general assembly and the partial assemblies of the IR have a particular mathematical formalization. The main reason for such particularities in the design methodology is the variability of the articulated mechanical structure of the IR and the diversity of the particular constructive solutions usable for the assemblies of numerical controlled axes for the translation / rotation movements (T/R) of the mobile elements of the IR [1, 2].

From this point of view, the general algorithm presented in the paper and the stages under it can be used are appropriates for two purposes:

- in conceptual development activities and optimal design of new prototypes for similarly / different IR having the same general architecture;
- for identification of necessary functional / constructive parameters and the opportunity to use a specific IR's model / size in current operation exploitation correlative with its specific integration and functionality into a certain robotic application, (for this case, by applying the present algorithm being possible to select the right type and the optimal constructive variant of some existing IR model in relation to the level of performance desired to be obtained in its exploitation).

The general objectives to be achieved by going through the major computational steps remain the same regardless the formalization of the particular method of calculus relations used for designing a T / R axis of any IR. The final goal of the entire design methodology is to identify the optimal complete structure of all IR NC axes and to verify their performances correlated with the desired performance to be achieved at the level of the IR general assembly. To achieve this final goal, three calculation steps should be followed: a first set of calculation steps specific to each general architecture and IR constructive variants; a second set of calculation steps for preliminary dimensioning, selection and final checking of each type of component integrated into the partial assemblies of the NC axes of the designed IR; a third set of calculation steps specific for the selection of the electric drive systems and the control systems of the NC axes, the overall performance evaluation and the final validation of the complete design of the NC axes of the IR [1, 2, 3].

As stated in [1], where a calculus algorithm was presented for a SCARA robot model and [2] where a second calculus algorithm was presented for an articulated arm robot model, due to the variability of the mechanical structure of robots and the diversity of the particular constructive solutions used for realization of assemblies of subassemblies of robots, particular mathematical formalizations are needed to be developed for each IR type. Following a similar workflow as presented in [1 and 2], this time a specific calculus methodology is presented for optimal structure design of NC axes of a gantry type robot GUDEL FP 4 with 4 DOF.

2. THE INDUSTRIAL ROBOT REFERENCE MODEL. PRELIMINARY DATA SET OF INITIAL CALCULATION

When selecting or designing a robot, first, the application in which it will operate must be known. The characteristics of the application are the first information that needs to be known in order to further proceed on the calculus algorithm. By knowing these, some essential data can be extracted and used further. By means of a

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comparative study of similar existing applications, a reference model (general architecture and constructive version of the IR) can be identified [1, 2, 3].

Initial calculations are need to be performed by taking into account the basic functional and constructive characteristics, thus for the reference model of the IR / the IR to be designed it is necessary to refer on: the specificities of the IR's work tasks within the respective application; the general architecture and constructive design of IR; the number and type of degrees of freedom of the IR; the constructive-functional specificity of the IR end effector; the specific constructive parameters of IR (the number, type, the order of reciprocally disposal of movement axis corresponding to major / active joints of the robot); the specific shape and dimensions in the longitudinal and cross-section directions of the segments of the articulated mechanical structure of the IR; the eccentricities and rotation angles defining the relative position and mutual orientation of the axis of motion of the major joints of the IR in relation to the specific shape and dimensions of the segments of the articulated mechanical structure of the IR; maximum ranges and speeds on the numerically controlled axes of the IR; shape and dimensions (amplitude) of the IR workspace; the maximum trajectory speed / minimum cycle time; the maximum payload of IR; IR working accuracy, (for all of these, taking into account the basic construction elements of the IR's reference model specified in the technical data sheets / product specifications / product manuals developed by the IR manufacturer [1, 2, 3].

To illustrate the previous-mentioned aspects regarding the identification of the reference model for the IR to be designed (GUDEL FP 4) and its basic functional construction features respectively, following information are presented [4, 5, 6] (Figs. 1 and 2, Table 1).


a

b

Fig. 1. General architecture and constructive variant of the reference model GUDEL FP 4,
a – general view of 4 NC axis IR; *b* – C, B, A numerically controlled axis alternatives [4, 5, 6].

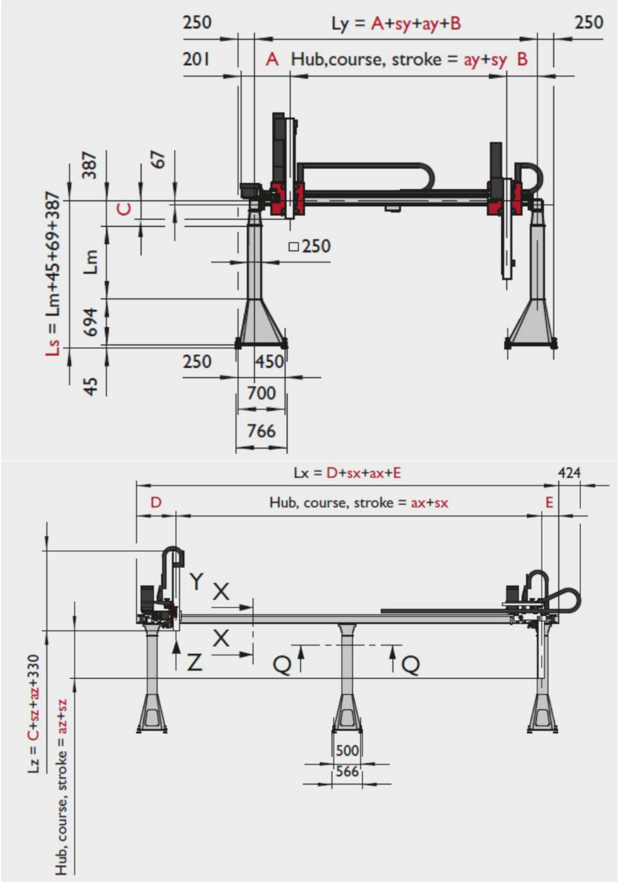


Fig. 2. Constructive parameters of IR reference model extracted from product's catalog [4].

Table 1

GUDEL FP 4 main functional characteristics [4]

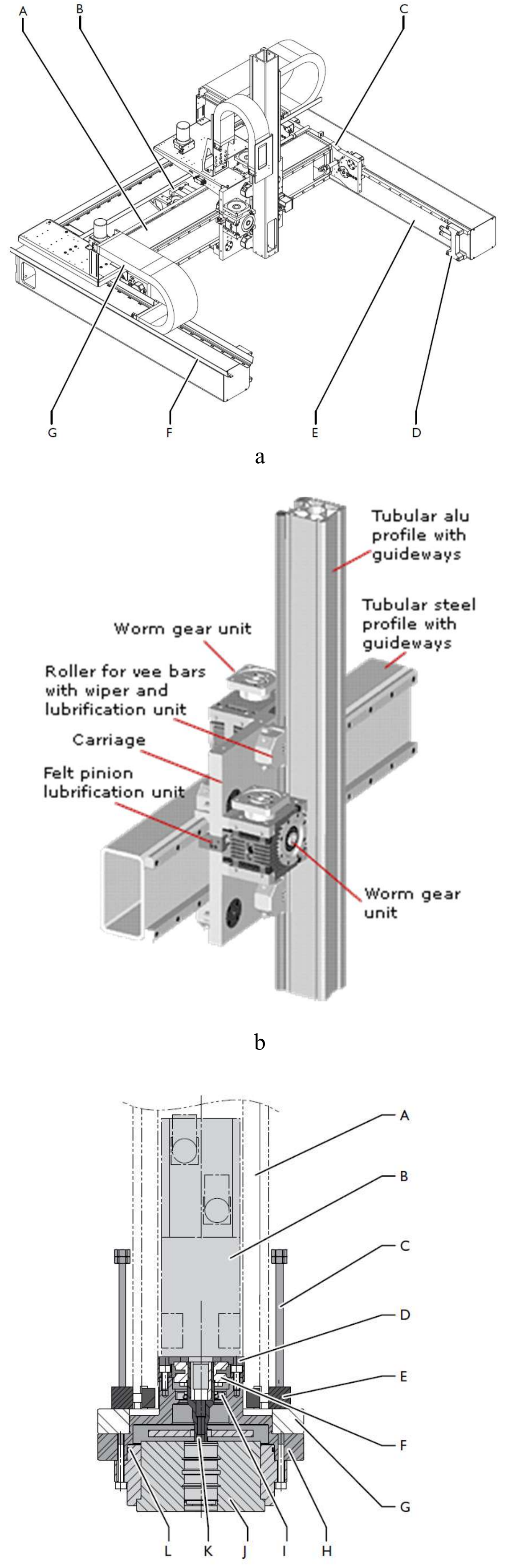
Axis movement	Working range	Maximum Velocity
Axis 1 / X Axis	Max 100000 mm	150 / 120 / 75 mm/s
Axis 2	Max .000 mm	200 / 120 / 75 mm/s
Axis 3	Max 1200 mm	120 / 75 / 45 mm/s
Axis 4	+360° to -360°	360°/s
Axis 5	+240° to -240°	240°/s
Axis 6	+360° to -360°	360°/s
Repeatability	High dynamic / optimal / high load	Maximum payload
± 0.02 mm		800 / 1250 / 2000 N

After the identification of an IR's reference model and its characteristics an important aspect is to identify IR's specific NC axis structures and the location of the calculus centers. The calculus centers are specific points localized on the robot structure where the loads acting on the overall robot will be reduced on each NC axis level in order to evaluate their effect on each IR's partially assembly structure and performances [3].

For this purpose, accordingly the NC axis type (for rotation or translation of a mobile IR's element) two important rules need to be considered [3]:

- for a rotation NC axis, the calculus center is located at the intersection of the middle plain of the bearing system of the mobile element by the rotation axis of considered IR's joint;
- for a translation NC axis the calculus center is located at the intersection of three plains: the middle plain for the length of mobile element, the middle plain for the wide of the mobile element and the middle plain for the high of the guiding system of the mobile element for the considered IR's joint.

For considered IR model (GUDEL FP 4), Figs. 3a, b, and c illustrate the specific design of IR's NC axis:



a

b

c

Fig. 3. The specific design of NC axis for the reference IR model GUDEL FP 4, *a* – general view of 4 NC axis IR; *b* – C, B, A numerically controlled axis alternatives [4, 5, 6].

Accordingly the specific design of GUDEL FP 4 IR, the calculus center for axis 1, 2, 3 and 4 can be identified as located: for axis 1 (beam longitudinal translation) in the middle point of the distance between each two cam followers sustaining the longitudinal carriages C (Fig. 3a) of the both ends of beam, for axis 2 (carriage transverse translation on the beam) in the middle point of the rectangle determined by the fourth cam followers sustain the transverse carriage on the beam, for axis 3 (vertical translation of mobile element) in the middle point of the rectangle determined by the fourth cam followers sustain the mobile element of the third axis.

Then, mass of each IR's partially assembly and their specific gravitational center coordinates need to be determined. As well, inertial and gravitational loads need to be identified and located on the calculation scheme. These loads will be applied in the mass center of each IR's partial assemblies / subsystem and will be afterwards reduced in calculus center of each IR's NC axis.

3. CALCULATION STEPS

In order to carry out the design calculations of the IRs general assembly, as stated also in [1, 2, 3]:

- for determining the input data in the preliminary calculation steps, the specificity of the application in which the IR need to be integrated and the basic functional design characteristics of the IR reference model previously identified need to be considered (the most relevant of which being IR's specific constructive parameters, maximum working ranges, maximum payload and maximum speeds on the NC axes of IR, IR's specific effector and mass and dimensions of the object to be manipulated);
- to begin the design of a k translation / rotation axis within a n degrees of freedom robot (DOF), all the constructive elements should be determined by previous design and calculations for all partial assemblies corresponding to $k + 1$, $k + 2$, … n NC axes (in this case $n = 6$).

Taking into consideration all above-mentioned aspects, the main stages of calculation for the design of the general ensemble of RI are carried out in the following sequence / next steps [3]:

1. elaboration of calculus scheme of the IR to be designed;
2. center of mass localization for each major partial assembly of the IR;
3. determining the overall distribution of gravitational and inertial loads applied on the overall structure of the robot (using the most unfavorable IR load configurations;
4. identification of calculation centers for each partial IR assembly;
5. placement in the mass centers the spatially distributed loads acting across the entire structure of the robot and determination of resultant reduced forces and torques ($F_{i\ red}$, $M_{i\ red}$) on each specific calculus centers of IR's partially assemblies;
6. distribution of the previous resulting load components ($F_{i\ red}$, $M_{i\ red}$) on sets of components such as bearings

or guidance components allowing movement of each IR's mobile element and respectively on the components that included in the driving systems responsible with rotational / translational motion of each moving element on each NC axis.;

7. pre-dimensioning, preliminary selection and final verification of the assembly components used for materializing of bearings / guidance and respectively components for driving in motion of the movable elements on each NC axis;
8. preliminary selection and verification of servo-motors and position/speed encoders used on each NC axis;
9. final checking of selected servo-motors and servo-drive systems used for continuous adjustment of functional parameters of servo-motors for each NC axis;
10. performances evaluation for each IR's NC axis,
11. performances evaluation for the overall IR's general assembly.

However, due to limited allowable space, in this paper only steps 1 to 7will be detailed.

3.1. Calculation scheme elaboration for GUDEL FP 4 robot reference model

The calculus scheme is a simplified representation of the overall kinematic structure of the IR, elaborated in accordance with the general architecture and specific constructive variant of the reference model of IR to be designed. As result, it should be realized in a form of a symbolic (but as realistic as may be) representation of the IR schematics including all the major IR joints and links between them, by respect for the dimensional proportions between partial assemblies / components and highlighting all constructive and functional parameters, with respect too of marking also the eccentric disposition of the IR's components. In the representation of the calculus scheme the IR's mobile elements must be brought into the positions leading to the most unfavorable loading configuration. If it is not possible to identify a single calculation scheme corresponding to the most unfavorable load configuration of the IR, alternative calculation configurations can be defined for which the calculations next steps need to be performed in parallel up to the level of certain identification of the configuration that leads to the peak loads on IR [1, 2, 3].

In order to evaluate gravitational and inertial loads, masses of each partial assembly are considered concentrated in their related mass center located in the calculation scheme by material points identified as location by IR's constructive parameters (dimensions) and mass centers specific coordinates (previously identified) for all major partial assemblies / structural elements of the IR [1, 2, 3].

For graphical representation of overall inertial and gravitational loads acting on the IR's structure following steps can be followed: a reference system is attached to the robot in order to correctly considering overall load's distribution; then, first of all the gravitational forces acting from top to down on each mass center are represented; successively, representation of inertial forces for each active joint need to be made. For this purpose, two important issues need to be considered 3]:

- for the GUDEL FP 4 - 4 DOF robot first inertial loads are determined as joint 4 motion effect (3, 2, 1 joint being considered *locked*), then inertial loads as joint 3 motion effect (4, 2, 1 joint being *locked*) are determined and so on. Final resulting loads will be considerate simultaneously applied, so that the IR's structure will be loaded in maximum conditions (considering simultaneous movement of all IR joints);
- the action sense of each inertial load need to selected targeting to generate the most unfavorable loading schema of the studied IR (reaching the cumulative effect of as much forces acting in same sense).

Following all above issues Figs. 3 a, b, c illustrates the calculation scheme including the overall gravitational and inertial load's distribution for the GUDEL FP 4 robot [3, 7].

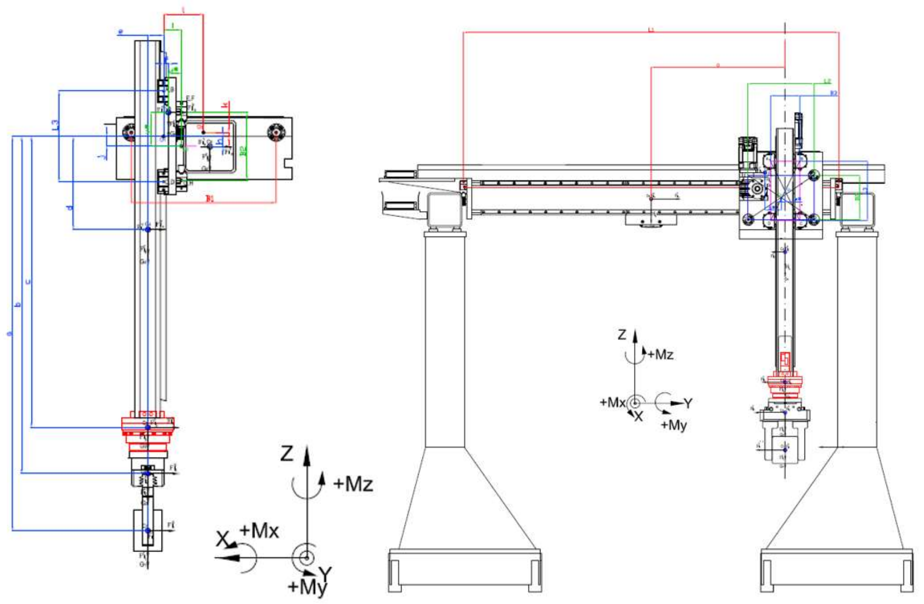


Fig. 4. Graphical representation of the calculus scheme for overall gravitational and inertial loads acting on GUDEL FP 4 IR structure in correspondence with the reference IR model [3, 7].

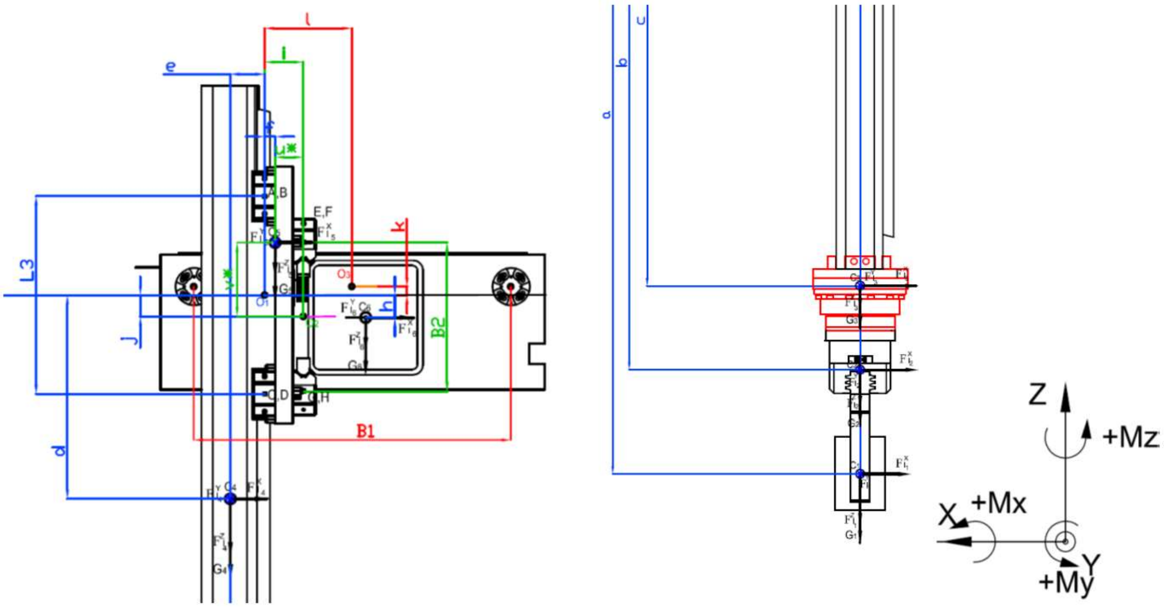


Fig. 5. Graphical representation of the calculus scheme for overall gravitational and inertial loads acting on GUDEL FP 4 IR in correspondence with the reference IR model. Detail for the spatial distribution loads from lateral view [3, 7].

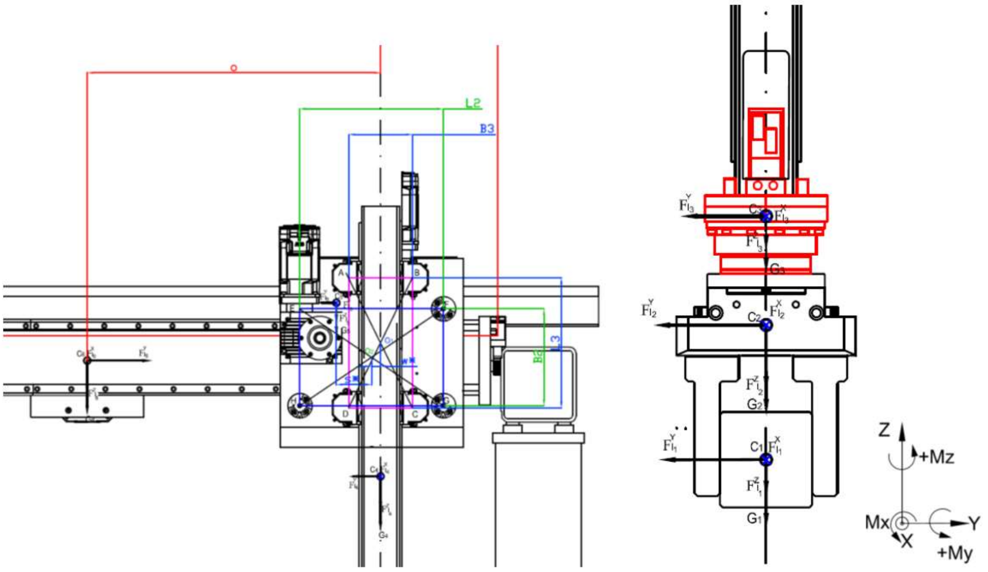


Fig. 6. Graphical representation of the calculus scheme for overall gravitational and inertial loads acting on GUDEL FP 4 IR in correspondence with the reference IR model. Detail for the spatial distribution loads from frontal view [3, 7].

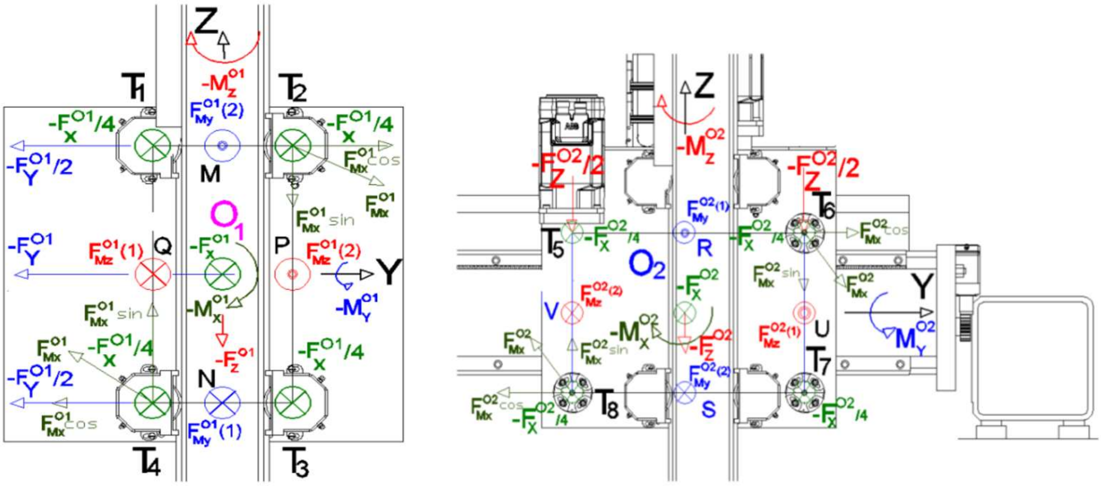


Fig. 7. Graphical representation of overall gravitational and inertial load's distribution on guiding elements of mobile elements from 3-rd IR's axis (left) and 2-nd IR's axis (right) [3, 7].

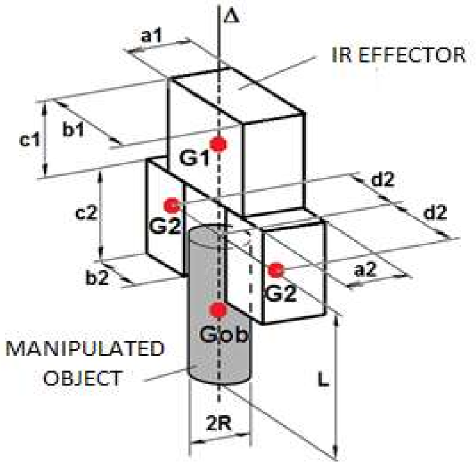


Fig. 8. Calculation schema for determining output resistive inertial momentum generated by 4th axis rotation in case of a centric mounted effector [1, 3].

3.2. Evaluation of inertial reduced torque acting on axis 4 of the GUDEL FP 4 robot reference model

The 4th NC axis of presented GUDEL FP 4 robot model is a rotational axis (for effector roll motion), thus inertial loads are resumed to the sum of inertial momentum of the manipulated object, the end-effector and adapter element / flange (used for connecting the end-effector to the output element of the cycloid gearbox included in axis 4). The total value of the inertial momentum represents the resistive momentum that needs to be exceeded by the 4th axis driving system on cycloid gearbox output level). This resistive inertial momentum acting on output 4th NC axis level may be determined following annotation from Fig. 8 and equations [3].

$$M_{z4} = J_{z4}\varepsilon_4 = \left(J_{z\,ob} + J_{z\,ef}\right)\varepsilon_4$$
$$= \left(J_{z\,ob} + J_{z\,ef\,body} + J_{z\,ef\,fingers} + J_{adapt\,flange}\right)\varepsilon_4,$$
$$M_{z4} = J_{z4}\varepsilon_4 = \left(J_{z\,ob} + J_{z\,ef\,body} + 2J_{z\,cf\,ef\,fingers} + J_{adapt\,flange}\right)\varepsilon_4 \text{ [Nm]},$$
$$J_{z\,ob} = \frac{1}{2}m_{ob}R_{ob}^2 = \frac{1}{2}\rho_{ob}\pi R_{ob}^4 L_{ob},$$
$$J_{adapt\,flange} = m_{adapt\,flange}R_{adapt\,flange}^2,$$

where: $\Delta\omega_4$ is angular speed variation for 4th axis, in rad/s^2; $J_{z\,ef\,body}$ – inertial momentum of effector body, in kg m^2; $J_{z\,ef\,fingers}$ – inertial momentum of effector fingers, in kg m^2; $J_{adapt\,flange}$ – inertial momentum of the adapter flange used for mounting the end-effector, in kg m^2.

3.3. Evaluation of gravitational and inertial load acting on GUDEL FP 4 robot reference model and the reduced loads applied in C_1, C_2 and C_3 calculus centers

For gravitational forces, the direction of action is vertical, from the top to down. The numerical evaluation of the gravitational forces is done by the relation [3, 7]:

$$G_i = m_i\, g \text{ [N]}.$$

For specific partially assemblies of considered GUDEL FP 4 IR the gravitational loads can be determined by following equations:

$$G_1 = mass_{MANIPULATED\ OBJECT} \cdot g,$$
$$G_2 = mass_{END\ EFFECTOR} \cdot g,$$
$$G_3 = mass_{ORIENTATION\ SYSTEM} \cdot g,$$
$$G_4 = mass_{VERTICAL\ ELEMENT\_Z} \cdot g,$$
$$G_5 = mass_{CARRIAGE\_Y} \cdot g,$$
$$G_6 = mass_{BEAM\_X} \cdot g.$$

The numerical evaluation of an inertial force F_{ij} generated for a mass m_i by a translational movement in the joint j with the maximum speed v_j is made with the relation [3, 7]:

$$F_i^j = m_i a_{Tj},$$

where F_i^j represents the value of the inertial force, m_i is the mass and a_{Tj} is the acceleration generated by starting / stopping / speeding up / slowing down of a moving element in a translation move. Acceleration is determined by relation:

$$a_{Tj} = \frac{\Delta v_j}{t_{fr/acc}} = \frac{v_{j\,max}}{t_{fr/acc}},$$

where Δv_j is speed variation:

$$\Delta v_j = v_{j\,max} - v_{j\,min} = v_{j\,max} - 0 = v_{j\,max}.$$

Inertial force F_i^j are represented in calculation schema in parallel directions to the direction of the movement of the mobile element k (on X, Y, and Z axes). For each type of inertial forces, the specific action sense is determined in accordance with the movement of IR's mobile elements and the favorable moment of occurrence of the inertial load toked into account (start / end time of movement), for considering the maximum cumulative effect for all inertial and gravitational load (in N).

$$F_{i1X} = mass_{MANIPULATED\ OBJECT} \cdot acc_{X\,max},$$
$$F_{i2X} = mass_{END\ EFFECTOR} \cdot acc_{X\,max},$$
$$F_{i3X} = mass_{ORIENTATION\ SYSTEM} \cdot acc_{X\,max},$$
$$F_{i5X} = mass_{CARRIAGE\_Y} \cdot acc_{X\,max},$$
$$F_{i6X} = mass_{BEAM\_X} \cdot acc_{X\,max},$$

$$F_{i1Y} = mass_{MANIPULATED\ OBJECT} \cdot acc_{Y\,max},$$
$$F_{i2Y} = mass_{END\ EFFECTOR} \cdot acc_{Y\,max},$$
$$F_{i3Y} = mass_{ORIENTATION\ SYSTEM} \cdot acc_{Y\,max},$$
$$F_{i4Y} = mass_{VERTICAL\ ELEMENT\_Z} \cdot acc_{Y\,max},$$
$$F_{i5Y} = mass_{CARRIAGE\_Y} \cdot acc_{Y\,max},$$

$$F_{i1Z} = mass_{MANIPULATED\ OBJECT} \cdot acc_{Z\,max},$$
$$F_{i2Z} = mass_{END\ EFFECTOR} \cdot acc_{Z\,max},$$
$$F_{i3Z} = mass_{ORIENTATION\ SYSTEM} \cdot acc_{Z\,max},$$
$$F_{i4Z} = mass_{VERTICAL\ ELEMENT\_Z} \cdot acc_{Z\,max}.$$

When friction forces need to be considered too, they may be calculated as being:

$$F_{f1Z} = mass_{MP\ OB+E\quad EF+\quad VERT\ EL\_Z} \cdot \mu \cdot g,$$
$$F_{f2Y} = mass_{MP\ OB+END\ EF+OS\quad VERT\ EL_Z+CARR} \cdot \mu \cdot g,$$
$$F_{f3X} = mass_{MP\ OB+E\quad EF+OS+\ VERT\ EL_Z+CARR+BEAM} \cdot$$
$$\cdot \mu \cdot g\ .$$

To perform calculations for reducing the overall gravitational and inertial loads and express the bending torques generated by eccentric load's application following distances and dimensional convention need to be considered (all in mm):

$$\begin{gathered}
a = d(C_1O_1)\ on\ Z\ axis, \\
b = d(C_2O_1)\ on\ Z\ axis, \\
c = d(C_3O_1)\ on\ Z\ axis, \\
d = d(C_4O_1)\ on\ Z\ axis, \\
e = d\big(C_{1,2,3,4}O_1\big)\ on\ X\ axis, \\
f = d(C_5O_1)\ on\ X\ axis, \\
h = d(C_6O_1)\ on\ Z\ axis, \\
i = d(O_1O_2)\ on\ X\ axis, \\
j = d(O_1O_2)\ on\ Z\ axis, \\
k = d(O_1O_3)\ on\ Z\ axis, \\
l = d(O_1O_3)\ on\ X\ axis, \\
r = d(O_1O_6)\ on\ X\ axis, \\
n = d(C_2C_4)\ on\ Y\ axis, \\
o = d(C_6C_4)\ on\ Y\ axis, \\
p = d(C_3C_4)\ on\ Y\ axis, \\
u = d(O_2C_5)\ on\ X\ axis, \\
v = d(O_2C_5)\ on\ Z\ axis, \\
s = d(O_2C_5)\ on\ Y\ axis, \\
w = d(O_1O_2)\ on\ Y\ axis, \\
L_1 = d(T_{10}T_{11})\ on\ Y\ axis, \\
L_2 = d(T_5T_6)\ on\ Y\ axis, \\
L_3 = d(T_2T_3)\ on\ Z\ axis, \\
B_1 = d(T_9T_{10})\ on\ X\ axis, \\
B_2 = d(T_6T_7)\ on\ Z\ axis, \\
B_3 = d(T_1T_2)\ on\ Y\ axis.
\end{gathered}$$

The total resultant reduced forces (in N) and bending torques (in Nm) acting in O_1 calculus center for Z axis may be evaluated by equations:

$$\begin{gathered}
F_{X_{O1}} = -(F_{i1X} + F_{i2X} + F_{i3X} + F_{i4X}), \\
F_{Y_{O1}} = -(F_{i1Y} + F_{i2Y} + F_{i3Y} + F_{i4Y}), \\
F_{Z_{O1}} = -(F_{i1Z} + F_{i2Z} + F_{i3Z} + F_{i4Z} + G_1 + G_2 + G_3 + \\
G_4), \\
M_{X_{O1}} = -F_{i1Y} \cdot a - F_{i2Y} \cdot b - F_{i3Y} \cdot c - F_{i4Y} \cdot d, \\
M_{Y_{O1}} = (F_{i1Z} + G_1) \cdot e + (F_{i2Z} + G_2) \cdot e + \\
(F_{i3Z} + G_3) \cdot e + (F_{i4Z} + G_4) \cdot e + F_{i1X} \cdot a + \\
+F_{i2X} \cdot b + F_{i3X} \cdot c + F_{i4X} \cdot d, \\
M_{Z_{O1}} = -(F_{i1Y} + F_{i2Y} + F_{i3Y} + F_{i4Y}) \cdot e.
\end{gathered}$$

From all six above reduced forces and torques, five of them ($F_{X_{O1}}, F_{Y_{O1}}, M_{X_{O1}}, M_{Y_{O1}}, M_{Z\_O1}$) are applied to the guiding system of vertical mobile element on axis 3, (on cam follower T_1, T_2, T_3, T_4) as shown in Fig. 7 and one of them (F_{Z_O1}) to the driving system (rack and pinion) of the same element.

As well, the total resultant reduced forces (in N) and bending torques (in Nm) acting in O_2 calculus center for Y axis may be evaluated by equations:

$$\begin{gathered}
F_{X_{O2}} = -(F_{i1X} + F_{i2X} + F_{i3X} + F_{i4X} + F_{i5X}), \\
F_{Y_{O2}} = -(F_{i1Y} + F_{i2Y} + F_{i3Y} + F_{i4Y} + F_{i5Y}), \\
F_{Z\_O2} = -(F_{i1Z} + F_{i2Z} + F_{i3Z} + F_{i4Z} + G_1 + G_2 + G_3 + \\
G_4 + G_5),
\end{gathered}$$

$$\begin{gathered}
M_{X_{O2}} = -(F_{i1Z} + F_{i2Z} + F_{i3Z} + F_{i4Z} + G_1 + G_2 + G_3 + \\
G_4) \cdot w + (G_5) \cdot s + F_{i5Y} \cdot v - \big(F_{i1Y}(a - j) + F_{i2Y}(b - \\
j) + F_{i3Y}(c - j) + F_{i4Y}(d - j)\big), \\
M_{Y_{O2}} = (F_{i1Z} + G_1) \cdot (e + i) + (F_{i2Z} + G_2) \cdot (e + i) + \\
(F_{i3Z} + G_3) \cdot (e + i) + (F_{i4Z} + G_4) \cdot (e + i) + F_{i1X} \cdot \\
(a - j) + F_{i2X} \cdot (b - j) + F_{i3X} \cdot (c - j) + F_{i4X} \cdot \\
(d - j) + (G_5) \cdot u - F_{i4X} \cdot v, \\
M_{Z_{O2}} = -(F_{i1Y} + F_{i2Y} + F_{i3Y} + F_{i4Y}) \cdot (e + i) - F_{i5Y} \cdot u \\
+ F_{i5X} \cdot s.
\end{gathered}$$

From all six above reduced forces and torques, five of them ($F_{X_{O2}}, F_{Z_{O2}}, M_{X_{O2}}, M_{Y_{O2}}, M_{Z\_O2}$) are applied to the guiding system of mobile element on axis 2, (on carriage's cam follower T_5, T_6, T_7, T_8) as shown in Fig. 7 and one of them ($F_{Y_{O2}}$) to the driving system (rack and pinion) of the same element.

Finally, the total resultant reduced forces (in N) and bending torques (in Nm) acting in O_1 calculus center for X axis may be evaluated by equations:

$$\begin{gathered}
F_{X_{O3}} = -(F_{i1X} + F_{i2X} + F_{i3X} + F_{i4X} + F_{i5X} + F_{i6X}), \\
F_{Y_{O3}} = -(F_{i1Y} + F_{i2Y} + F_{i3Y} + F_{i4Y} + F_{i5Y}), \\
F_{Z\_O3} = -(F_{i1Z} + F_{i2Z} + F_{i3Z} + F_{i4Z} + G_1 + G_2 + G_3 + \\
G_4 + G_5 + G_6),
\end{gathered}$$

$$\begin{gathered}
M_{X_{O3}} = -(F_{i1Z} + F_{i2Z} + F_{i3Z} + F_{i4Z} + G_1 + G_2 + G_3 + \\
G_4) \cdot o - (G_5) \cdot (o - w) - (F_{i1Y} \cdot a + F_{i2Y} \cdot b + F_{i3Y} \cdot \\
c + F_{i4Y} \cdot d),
\end{gathered}$$

$$\begin{gathered}
M_{Y_{O2}} = [(F_{i1Z} + G_1) + (F_{i2Z} + G_2) \cdot +(F_{i3Z} + G_3) \cdot \\
+(F_{i4Z} + G_4)] \cdot (e + l) - F_{i1X} \cdot a + F_{i2X} \cdot b + F_{i3X} \cdot c + \\
F_{i4X} \cdot d + (G_5) \cdot (l - f),
\end{gathered}$$

$$\begin{gathered}
M_{Z_{O2}} = (F_{i1X} + F_{i2X} + F_{i3X} + F_{i4X} + F_{i5X}) \cdot o - \\
(F_{i1Y} + F_{i2Y} + F_{i3Y} + F_{i4Y}) \cdot (e + l) - F_{i5Y} \cdot (l - f) + \\
F_{i5X} \cdot (o - w).
\end{gathered}$$

From all six above reduced forces and torques, five of them ($F_{Y_{O3}}, F_{Z_{O3}}, M_{X_{O3}}, M_{Y_{O3}}, M_{Z\_O3}$) are applied to the guiding system of mobile element on axis 1, (on 4 cam follower, 2 of them located in the left sustaining slide of the beam and two of them located in the right sustaining slide of the beam) and one of them ($F_{X_{O3}}$) to the driving system (2 mechanisms rack and pinion located in the left side and respectively in the right side) of the same element.

3.4. Evaluation of reduced loads applied on guiding and driving system of mobile elements from axis 3 and axis 2

The five loads and torques reduced in O_1 ($F_{X_{O1}}, F_{Y_{O1}}, M_{X_{O1}}, M_{Y_{O1}}, M_{Z\_O1}$) are further reduced as radial and axial forces applied on the T_1, T_2, T_3, T_4 cam follower. For evaluating the axial and radial forces applied on this cam follower following annotations need to be considered (Fig. 7 left):

$$\begin{gathered}
T_2T_4 = \sqrt{\left({L_3}^2 + {B_3}^2\right)}\ [mm], \\
sin\,\theta_1 = \frac{T_1T_2}{T_2T_4}\ \frac{B_3}{\sqrt{\left({L_3}^2 + {B_3}^2\right)}}, \\
cos\,\theta_1 = \frac{T_2T_3}{T_2T_4} = \frac{L_3}{\sqrt{\left({L_3}^2 + {B_3}^2\right)}}.
\end{gathered}$$

Thus, axial loads applied to T_1, T_2, T_3, T_4 cam followers derived from F_X^{01} reduced force become:

$$F_{AX}^{T1} = F_{AX}^{T2} = F_{AX}^{T3} = F_{AX}^{T4} = \frac{F_X^{01}}{4},$$

and radial loads applied to T_1, T_4 cam followers derived from F_Y^{01} reduced force are:

$$F_{RAD}^{T1} = F_{RAD}^{T4} = \frac{F_Y^{01}}{2},$$

the radial loads applied to T_2, T_4 cam followers derived from M_x^{01} reduced force are: M_x^{01}

$$(F_{Mx}^{01})^{T2} = (F_{Mx}^{01})^{T4} = \frac{|M_x^{01}|}{T_2T_4},$$

the axial loads applied to T_1, T_2, T_3, T_4 cam followers derived from M_y^{01} reduced force are determined by:

$$(F_{My}^{01})^{1} = (F_{My}^{01})^{2} = \frac{|M_y^{01}|}{T_2T_3},$$

$$(F_{My}^{01})^{T1} = (F_{My}^{01})^{T2} = (F_{My}^{01})^{T3} = (F_{My}^{01})^{T4} = \frac{(F_{My}^{01})}{2},$$

the axial loads applied to T_1, T_2, T_3, T_4 cam followers derived from M_z^{01} reduced force are determined by:

$$(F_{Mz}^{01})^{1} = (F_{Mz}^{01})^{2} = \frac{|M_z^{01}|}{T_2T_3},$$

$$(F_{Mz}^{01})^{T1} = (F_{Mz}^{01})^{T2} = (F_{Mz}^{01})^{T3} = (F_{Mz}^{01})^{T4} = \frac{(F_{Mz}^{01})}{2} \text{ [N]},$$

Then, resultant load on T_1, cam follower will be:

$$F_{RAD\ res}^{T1} = \frac{F_Y^{01}}{2},$$

$$F_{AX\ res}^{T1} = F_{AX}^{T1} + (F_{My}^{01})^{T1} - (F_{Mz}^{01})^{T1} = \frac{F_X^{01}}{4} + \frac{|M_y^{01}|}{2\,T_2T_3} - \frac{|M_z^{01}|}{2\,T_2T_3},$$

resultant loads on T_2, cam follower will be:

$$F_{RAD\ res}^{T2} = \sqrt{\left(F_{Mx}^{01\ T2} \cdot \cos\theta_1\right)^2 + \left(-F_{Mx}^{01\ T2} \cdot \sin\theta_1\right)^2} = \sqrt{\left(\frac{M_x^{01}}{T_2T_4}\right)^2 \cdot ((\sin\theta_1)^2 + (\cos\theta_1)^2)} = \sqrt{\left(\frac{M_x^{01}}{T_2T_4}\right)^2},$$

$$F_{AX\ res}^{T2} = F_{AX}^{T2} + (F_{My}^{01})^{T2} + (F_{Mz}^{01})^{T2} = \frac{F_X^{01}}{4} + \frac{|M_y^{01}|}{2\,T_2T_3} + \frac{|M_z^{01}|}{2\,T_2T_3},$$

resultant loads on T_3, cam follower will be:

$$F_{RAD\ res}^{T3} = 0 \text{ [N]},$$

$$F_{AX\ res}^{T3} = F_{AX}^{T3} - (F_{My}^{01})^{T3} + (F_{Mz}^{01})^{T3} = \frac{F_X^{01}}{4} - \frac{|M_y^{01}|}{2\,T_2T_3} + \frac{|M_z^{01}|}{2\,T_2T_3},$$

resultant loads on T_4, cam follower will be:

$$F_{AX\ res}^{T4} = F_{AX}^{T4} - (F_{My}^{01})^{T4} - (F_{Mz}^{01})^{T4} == \frac{F_X^{01}}{4} - \frac{|M_y^{01}|}{2\,T_2T_3} - \frac{|M_z^{01}|}{2\,T_2T_3},$$

$$F_{RAD\ res}^{T4} = \sqrt{\left(-F_{Mx}^{01\ T4} \cdot \cos\theta_1 + \frac{F_Y^{01}}{2}\right)^2 + \left(F_{Mx}^{01\ T4} \cdot \sin\theta_1\right)^2} = \sqrt{\left(\frac{-M_x^{01}}{T_2T_4} \cdot \cos\theta_1 + \frac{F_Y^{01}}{2}\right)^2 + \left(\frac{M_x^{01}}{T_2T_4} \cdot \sin\theta_1\right)^2},$$

The five loads and torques reduced in O_2 ($F_{X_{O2}}, F_{Z_{O2}}, M_{X_{O1}}, M_{Y_{O1}}, M_{Z\_O1}$) are further reduced as radial and axial forces applied on the T_5, T_6, T_7, T_8 cam follower. For evaluating the axial and radial forces applied on this cam follower following annotations need to be considered (Fig. 7 right):

$$T_6T_8 = \sqrt{(L_2{}^2 + B_2{}^2)},$$

$$\cos\theta_2 = \frac{T_6T_7}{T_6T_8}\ \frac{B_2}{\sqrt{(L_2{}^2 + B_2{}^2)}},$$

$$\sin\theta_2 = \frac{T_6T_7}{T_6T_8} = \frac{L_2}{\sqrt{(L_2{}^2 + B_2{}^2)}}.$$

Thus, axial loads applied to T_5, T_6, T_7, T_{48} cam followers derived from F_X^{02} reduced force become:

$$F_{AX}^{T5} = F_{AX}^{T6} = F_{AX}^{T7} = F_{AX}^{T8} = \frac{F_X^{02}}{4},$$

and radial loads applied to T_5, T_6 cam followers derived from F_Z^{02} reduced force are:

$$F_{RAD}^{T5} = F_{RAD}^{T6} = \frac{F_Z^{02}}{2},$$

the radial loads applied to T_5, T_6, T_7, T_8 cam followers derived from M_x^{02} reduced force are determined by:

$$(F_{Mx}^{02})^{T6} = (F_{Mx}^{02})^{T8} = \frac{|M_x^{02}|}{T_6T_{48}},$$

the axial loads applied to T_5, T_6, T_7, T_8 cam followers derived from M_y^{02} reduced force are determined by:

$$(F_{My}^{02})^{1} = (F_{My}^{02})^{2} = \frac{|M_y^{02}|}{T_6T_7},$$

$$(F_{My}^{02})^{T5} = (F_{My}^{02})^{T6} = (F_{My}^{02})^{T7} = (F_{My}^{02})^{T8} = \frac{(F_{My}^{02})}{2},$$

the axial loads applied to T_5, T_6, T_7, T_8 cam followers derived from M_z^{02} reduced force are determined by:

$$(F_{Mz}^{02})^{1} = (F_{Mz}^{02})^{2} = \frac{|M_z^{02}|}{T_5T_6},$$

$$(F_{Mz}^{02})^{T5} = (F_{Mz}^{02})^{T6} = (F_{Mz}^{02})^{T7} = (F_{Mz}^{02})^{T8} = \frac{(F_{Mz}^{02})}{2},$$

Then, resultant load on T_5, cam follower will be:

$$F_{RAD\ res}^{T5} = \frac{F_Y^{02}}{2},$$

$$F_{AX\ res}^{T5} = F_{AX}^{T5} + (F_{My}^{02})^{T5} - (F_{Mz}^{02})^{T5} = \frac{F_X^{02}}{4} + \frac{|M_y^{02}|}{2\,T_6T_7} - \frac{|M_z^{02}|}{2\,T_5T_6},$$

resultant loads on T_6, cam follower will be:

$$F_{RAD\ res}^{T6} = \sqrt{\left(F_{Mx}^{02\ T6} \cdot \cos\theta_2\right)^2 + \left(-F_{Mx}^{02\ T6} \cdot \sin\theta_2 + \frac{F_Z^{02}}{2}\right)^2} = \sqrt{\left(\frac{M_x^{02}}{T_6T_8}\right)^2 \cdot (\cos\theta_1)^2 + \left(-\frac{M_x^{02}}{T_6T_8} \cdot \cos\theta_2 + \frac{F_Z^{02}}{2}\right)^2},$$

$$F_{AX\ res}^{T6} = F_{AX}^{T6} + (F_{My}^{02})^{T6} + (F_{Mz}^{02})^{T6} = \frac{F_X^{02}}{4} + \frac{|M_y^{02}|}{2\,T_6T_7} + \frac{|M_z^{02}|}{2\,T_5T_6},$$

resultant loads on T_7, cam follower will be:

$$F_{RAD\ res}^{T7} = 0 \text{ [N]}$$

$$F_{AX\ res}^{T7} = F_{AX}^{T7} - (F_{My}^{02})^{T7} + (F_{Mz}^{02})^{T7} = \frac{F_X^{02}}{4} - \frac{|M_y^{02}|}{2\,T_6T_7} + \frac{|M_z^{02}|}{2\,T_5T_6},$$

resultant loads on T_8, cam follower will be:

$$F_{RAD\,res}^{T8} = \sqrt{\left(-F_{Mx}^{02\ T8} \cdot cos\,\theta_2\right)^2 + \left(F_{Mx}^{02\ T8} \cdot sin\,\theta_2\right)^2} = \sqrt{\left(\frac{-M_x^{02}}{T_6T_8} \cdot cos\,\theta_2\right)^2 + \left(\frac{M_x^{02}}{T_6T_8} \cdot sin\,\theta_2\right)^2} = \sqrt{\left(\frac{M_x^{02}}{T_6T_8}\right)^2 \cdot ((sin\,\theta_1)^2 + (cos\,\theta_1)^2)} = \frac{M_x^{02}}{T_6T_8},$$

$$F_{AX\,res}^{T8} = F_{AX}^{T8} - (F_{My}^{02})^{T8} - (F_{Mz}^{02})^{T8} = \frac{F_X^{02}}{4} - \frac{|M_y^{02}|}{2\,T_6T_7} - \frac{|M_z^{02}|}{2\,T_5T_6}.$$

4. CONCLUSIONS

Present paper has presented a calculus methodology for optimal structure design of numerically controlled (NC) axes of Gantry industrial robot (IR) type.

Staring up from a selected reference model (GUDEL FP 4) of the gantry first the calculation scheme elaboration for robot reference model has been presented, including complementary details for design calculation of axes 4, 3 and 2 [3, 7].

All reduced loads above calculated for each cam followers in this paragraph may be further used for dimensioning this type of machine elements included in the guiding system of IR's axis 1, axis 2 [3].

Evaluation of inertial reduced torque acting on axis 4 of the GUDEL FP 4 robot reference model (presented in 3.2 paragraph) allows to further start up selection procedure for cycloid gearbox and the associated driving motor included in the 4th NC axis [3].

Similar procedure may be performed for loads reducing on guiding system components of mobile element from axis 1. However, for axis 1 the calculus procedure is much simpler to be performed due to the fact that each lateral (left / right) sustaining slides of the beam includes only two cam followers for guiding the mobile element from axis 1 [3].

Complementary, reduced loads $F_{Z_{O1}}$, $F_{Y_{O2}}$, $F_{X_{O3}}$, may be also used for dimensioning the rack and pinion machine elements included in the driving system of IR's axis 1, axis 2 and axis 3 [3, 7].

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