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THE RACK-TOOL FOR MANUFACTURING THE MULTI-LOBE ROTORS OF AXIAL HELICAL PUMPS

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Abstract: *The helical pumps with axial worm (progressive cavity pumps) are frequently used in oil industry, for circulating liquids in mixtures with solid abrasive particles. The pump worm has helical shape, with one or more lobes. Due to its complexity, this worm manufacturing is a challenging problem. In the paper, it is suggested an algorithm laying onto the "Minimum distance" method, which enables the profiling of the rack-tool generating this type of ordinate whirl of helical surfaces. The main steps of the algorithm are: the definition of the hypo-cycloid specific to pump rotor, the finding of the rotor functional profile, equidistant to the hypo-cycloid, and the finding of the rack-tool profile in the axial section of the worm. A numerical sample of implementing the developed algorithm, in the case of a rotor with four lobes, performed with the help of a dedicated MatLab soft application, is also presented, together with graphical representations of the rotor transversal section and of the corresponding rack-tool profile.*

Key words: *axial helical pumps, multi-lobes rotor, rack-tool profiling, ordinate whirl of surfaces, minimum distance method.*

1. INTRODUCTION

The progressive cavity pump [1] is a type of positive displacement pump and is also known as, Moineau pump, eccentric screw pump or axial helical pump. It transfers fluid by means of the progress, through the pump, of a sequence of small, fixed shape, discrete cavities, as its rotor is turned. This leads to the volumetric flow rate being proportional to the rotation rate (bidirectionally) and to low levels of shearing being applied to the pumped fluid [2]. These pumps have application in fluid metering and pumping of viscous or shear-sensitive materials (e.g. food and drink pumping, oil pumping, coal slurry pumping, sewage and sludge pumping, viscous chemical pumping etc.). They were invented by French engineer René Moineau.

The progressive cavity pump normally consists of a helical rotor and a twin helix, twice the wavelength helical hole in a stator. The rotor seals tightly against the stator as it rotates, forming a set of fixed-size cavities in between. The cavities move when the rotor is rotated but their shape or volume does not change. The pumped material is moved inside the cavities [3].

In what concerns the construction of these pumps, (see Fig. 1), the rotor is a worm having circular cross-section and executes an eccentric rotation inside the stator. The number of lobes of the stator, z_s, and of the rotor, z_r must obey to the following condition [4]:

$$z_s = z_r + 1. \quad (1)$$

Hereby, one can talk about single-lobe rotors (z_s = 1, Fig. 2,*a*), and multi-lobe rotors (z_s > 1, Figs. 2,*b* and *c*).

Due to its complexity, these rotors manufacturing is a challenging problem. In the present paper, it is suggested an algorithm laying onto the "Minimum distance" method.

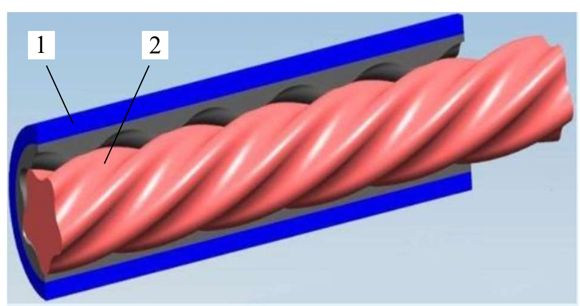

Fig. 1. The working bodies of the axial helical pump [5]: 1 – stator; 2 – rotor.

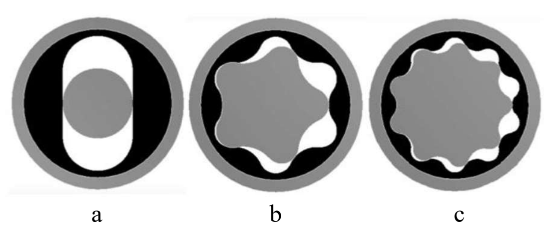

Fig. 2. Cross-sections of axial helical pumps with different kinematic ratio: *a* – 1:2; *b* – 5:6; *c* – 9:10, [5].

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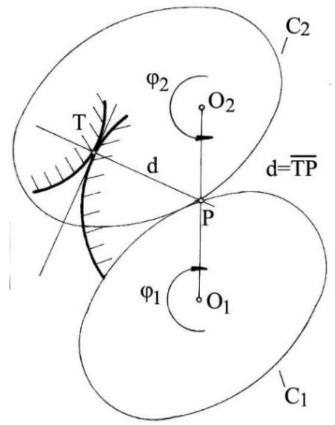

Fig. 3. Conjugated profiles & rolling centrodes [7].

According to the method, the rack-tool profiling can be performed by assimilating the rotor to an ordinate whirl of helical surfaces.

The "Minimum distance" method resulted from a new approach of well-known Willis theorem [6], applied in the case of profiles associated to a couple of rolling centrodes (C_1, C_2, see Fig. 3). According to the method [7], the envelop of a profile associated to a couple of rolling centrodes is the locus of the profile points for which, in the successive rolling positions, the distance d to gearing pole P (meaning the point of tangency between the centrodes) is minimum.

In what concerns paper structure, the next section deals with finding the equations of the transversal profile of the multi-lobe rotor. Third section deals with determining the equations of the helical surface of rotor flank. Fourth section aims to find the profile of the rack-tool for generating the rotor flanks by slotting. The fifth section presents a numerical application, while the last section is for paper conclusion.

2. THE TRANSVERSAL PROFILE OF THE MULTI-LOBE HELICAL ROTOR

The transversal profile of the multi-lobe rotor from the progressive cavity pump is, usually, a curve equidistant to a hypo-cycloid (Fig. 4).

The following reference systems are needed in order to find the rotor profile:

- xy, meaning a global system, fix, having the origin O situated on rotor symmetry axis,
- x_1y_1 - global system, fix, having the origin O_1 situated on roller symmetry axis,
- XY – local system, initially overlapped to xy, having the origin O and executing a rotation motion of φ_2 angular parameter around it, together to the base circle, and
- X_1Y_1 − local system, initially overlapped to x_1y_1, having the origin O_1 and executing a rotation motion of φ_1 angular parameter around it, together to the roller.

The hypo-cycloid profile is described by the point A, belonging to the roller of r radius, during its rolling motion to the base circle of R radius.

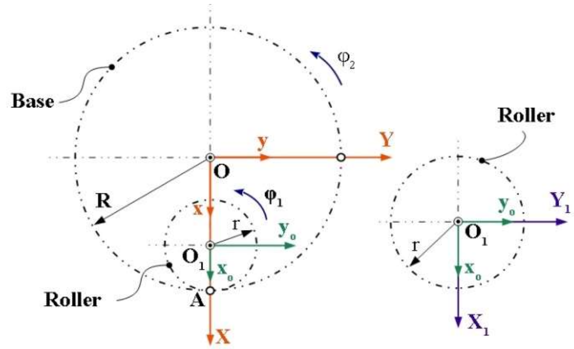

Fig. 4. The generating of hypo-cycloid curve.

The equation of roller rotation around O_1, written with matrices, is:

$$x_1 = \omega_3{}^T(\varphi_1) \cdot X_1. \quad (2)$$

The equation of base circle rotation around O, also written with matrices, is:

$$x = \omega_3{}^T(\varphi_2) \cdot X. \quad (3)$$

In the relations from above, ω_3 means the well-known matrix of coordinates transform at rotation around z axis.

The relative position between the two global systems is described by the relation:

$$x_1 = x - B, \text{with } B = \begin{pmatrix} R - r \\ 0 \end{pmatrix}. \quad (4)$$

The base circle and the roller are initially tangent, in the point A. The relation between angles φ_1 and φ_2 during the rolling motion is given by the condition:

$$r \cdot \varphi_1 = R \cdot \varphi_2. \quad (5)$$

The equation of the roller motion, referred to the reference system of the base circle, can be obtained after putting together the relations (2)–(4), in the form:

$$X = \omega_3(\varphi_2)[\omega_3{}^T(\varphi_1) \cdot X_1 + B]. \quad (6)$$

After noticing that, in X_1Y_1 reference system, the point A has the coordinates (r, 0), and after some calculus, the hypo-cycloid equations result as below:

$$\begin{vmatrix} X = r \cdot \cos(\varphi_1 - \varphi_2) + (R - r)\cos\varphi_2; \\ Y = r \cdot \sin(\varphi_1 - \varphi_2) - (R - r)\sin\varphi_2. \end{vmatrix} \quad (7)$$

The shape (7) does not correspond to the technological requirements needed for pump rotor functioning. For this reason, an equidistant relative to the hypo-cycloid (7) is defined as envelop of a family of circles having r_0 radius and the centres onto the hypo-cycloid (Fig. 5). Hereby, the equations of such substitutive circle are:

$$\begin{vmatrix} X = X_h + r_0 \cos\beta; \\ Y = Y_h + r_0 \sin\beta. \end{vmatrix} \quad (8)$$

In relations (8), X_h and Y_h mean the coordinates of the generic point from hypo-cycloid (7), while β means an angle defining the position of the current point belonging to the substitutive circle (Fig. 5).

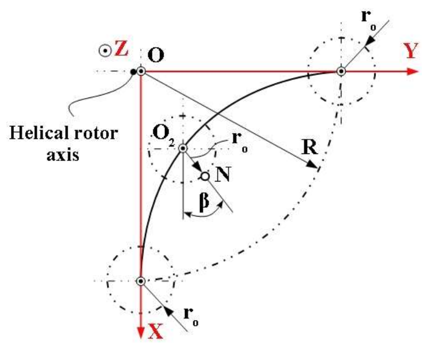

Fig. 5. The technological profile of pump rotor.

The coordinates of the current point from the equidistant to the hypo-cycloid, meaning the equations of the family of substitutive circles, result from (7) and (8):

$$(C)_\beta \begin{vmatrix} X = r\cos(\varphi_1-\varphi_2) + (R-r)\cos\varphi_2 + r_0\cos\beta; \\ Y = r\sin(\varphi_1-\varphi_2) + (R-r)\sin\varphi_2 + r_0\sin\beta. \end{vmatrix}$$

(9)

The envelop of $(C)_\beta$ family can be found according to Gohman theorem [8], by associating to equations (9) the enwrapping condition:

$$\begin{vmatrix} \dot{X}_{\varphi_1} & \dot{Y}_{\varphi_1} \\ \dot{X}_\beta & \dot{Y}_\beta \end{vmatrix} = 0, \quad (10)$$

where the four elements of the determinant are partial derivatives of X and Y from (9) against φ_1 and β. After calculus and by denoting:

$$i = \frac{\varphi_2}{\varphi_1} = \frac{r}{R}, \quad (11)$$

the four partial derivatives expressions result as:

$$\begin{aligned} &\dot{X}_{\varphi_1} = -r(1-i)\sin[(1-i)\varphi_1] - (R-r)i\sin(i\cdot\varphi_1); \\ &\dot{Y}_{\varphi_1} = r(1-i)\cos[(1-i)\varphi_1] + (R-r)i\cos(i\cdot\varphi_1); \\ &\dot{X}_\beta = -r_0\sin\beta; \quad (12) \\ &\dot{Y}_\beta = r_0\cos\beta. \end{aligned}$$

After replacing (12) in (10) and developing the determinant, the enwrapping condition gives:

$$\tan\beta = \frac{-r(1-i)\sin[(1-i)\varphi_1] - (R-r)i\sin(i\varphi_1)}{r(1-i)\cos[(1-i)\varphi_1] + (R-r)i\cos(i\varphi_1)}. \quad (13)$$

The equations (9) together to the condition (13) give the equidistant to the hypo-cycloid (7), having the general form:

$$S(\varphi_1) \begin{vmatrix} X = X(\varphi_1); \\ Y = Y(\varphi_1), \end{vmatrix} \quad (14)$$

and meaning the equations of the pump rotor flank.

Note: Obviously, the angular position of the flanks of (14) type depends on the rotor number of lobes.

3. THE HELICAL SURFACE OF MULTI-LOBE ROTOR FLANKS

The helical surface of the worm-rotor flank can be found as generated by the frontal (transversal) profile of the rotor, as determined through the ensemble of equations (9) and the condition (13), which mean the equations of a plane curve, having, in principle, the representation (14).

In the reference system *XYZ*, associated to the helical surface following to be determined (Fig. 2), the plane *XY* is the plane of worm frontal profile, to whom a helical motion is given for generating the flank surface:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \omega_3^{\,T}(\phi)\cdot\begin{pmatrix} X(\varphi_1) \\ Y(\varphi_1) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ p\cdot\phi \end{pmatrix}. \quad (15)$$

In relation (15), ϕ means a variable angular parameter in the helical motion around and along Z axis, p – the helical parameter of rotor flank surface, and $\begin{pmatrix} X(\varphi_1) \\ Y(\varphi_1) \\ 0 \end{pmatrix}$ – the matrix containing the coordinates of the rotor frontal profile, (14).

After development and calculus, the parametric equations of the helical surface flank, written in *XYZ* system, result as:

$$\Sigma(\varphi_1,\phi) \begin{vmatrix} X = X(\varphi_1)\cos\phi - Y(\varphi_1)\sin\phi; \\ Y = X(\varphi_1)\sin\phi + Y(\varphi_1)\cos\phi; \\ Z = p\cdot\phi. \end{vmatrix} \quad (16)$$

4. THE PROFILE OF THE RACK-TOOL FOR GENERATING THE ROTOR FLANKS BY SLOTTING

The finding of rack-tool profile is further explained with the help of Fig. 6, which presents, in a certain position of the workpiece and tool during the generating process:

- The frontal profile of the worm to be generated, associated to circular centrode C_1 and having the equations (16),
- The rolling centrodes associated to the rotor frontal section C_1 (circle of R_r radius) and to the rack-tool C_2 (straight line tangent to C_1), and
- The generating motions between the two rolling centrodes, described through the parameters θ (for C_1 rotation) and λ (for C_2 translation).

The following reference systems are considered for finding the equations of rack-tool profile:

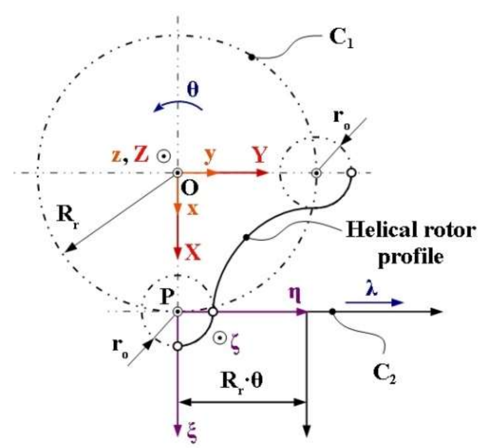

Fig. 6. The finding of rack-tool profile.

- *xyz*, meaning a global, fix system, having *z*-axis overlapped to rotor axis,
- *XYZ* – local system, initially overlapped to *xyz*, associated to the rotor frontal section and rotating together to it, and
- ξηζ – local system, associated to the rack-tool and translating to it along C_2 centrode.

The rolling motion between the two centrodes has to obey to the following kinematical condition:

$$\lambda = R_r \cdot \theta. \tag{17}$$

The equations of the needed generating motions [9] can be written as follows:

- The workpiece rotation, of θ angular parameter:

$$x = \omega_3{}^T(\theta) \cdot X, \tag{18}$$

- The rack-tool translation, of λ linear parameter:

$$x = \xi + C, \text{ with } C = \begin{pmatrix} R_r \\ R_r \cdot \theta \\ 0 \end{pmatrix}. \tag{19}$$

Note: Due to the particularities of the addressed generating process, the radius of base circle, previously used for hypo-cycloid definition, *R*, can be accepted as radius of C_1 centrode, R_r, hence $R_r = R$.

The equation of the workpiece motion relative to the rack-tool reference system, written with matrices, results from (18) and (19):

$$\xi = \omega_3{}^T(\theta) \cdot X - C. \tag{20}$$

The helical surface of the rotor flank generates, in the rolling motion (20) between C_1 and C_2 centrodes, a family of surfaces relative to the rack-tool reference system:

$$(\Sigma)_\theta : \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X(\varphi_1)\cos\phi - Y(\varphi_1)\sin\phi \\ X(\varphi_1)\sin\phi + Y(\varphi_1)\cos\phi \\ p\cdot\phi \end{pmatrix} - \begin{pmatrix} R_r \\ R_r \cdot \theta \\ 0 \end{pmatrix}. \tag{21}$$

In principle, the family of helical flanks $\sum(\varphi_1, \phi)$, see (16), in its motion relative to ξηζ system, determines a surfaces family having the general form:

$$\left(\xi_{(\varphi_1,\phi)}\right)_\theta : \begin{vmatrix} \xi = \xi(\varphi_1, \phi, \theta); \\ \eta = \eta(\varphi_1, \phi, \theta); \\ \zeta = \zeta(\varphi_1, \phi, \theta). \end{vmatrix} \tag{22}$$

The envelop of this family of helical surfaces will represent the flank of the generating rack-tool.

The presented problem can be significantly simplified by noticing that, in fact, it is enough to solve it in the frontal plane of the rotor worm only (see Fig. 3). In this plane, the envelop of the worm frontal profile can be found with the help of Minimum distance method [7], applied to equations (22) when $\phi = 0$.

In order to find the enwrapping condition, according to the above-mentioned method, the gearing pole (the point of tangency between C_1 and C_2 centrodes is identified at first as:

$$P \begin{vmatrix} \xi_P = 0; \\ \eta_P = -R_r \cdot \theta, \end{vmatrix} \tag{23}$$

corresponding to a generic position in the rolling process.

Then, the distance between the gearing pole and the current point of the profiles family from ξη plane, derived from (22) when $\phi = 0$:

$$\left(S_{(\varphi_1)}\right)_\theta : \begin{vmatrix} \xi = \xi(\varphi_1, \theta); \\ \eta = \eta(\varphi_1, \theta), \end{vmatrix} \tag{24}$$

is calculated as:

$$d = \sqrt{(\xi - \xi_P)^2 + (\eta - \eta_P)^2}. \tag{25}$$

Finally, according to minimum distance theorem [7], the condition of minimum is imposed to distance *d* by annulling the derivative of its expression against φ_1 leads to the relation:

$$(\xi - \xi_P) \cdot \dot{\xi}_{\varphi_1} + (\eta - \eta_P) \cdot \dot{\eta}_{\varphi_1} = 0. \tag{26}$$

The last relation represents the enwrapping condition in the specific form of the Minimum distance method. In the addressed case this enables the finding of a dependence relation of the type:

$$\theta = \theta(\varphi_1). \tag{27}$$

The ensemble formed by the equations of the profiles family (24), generated in the relative motion between the two centrodes, and the relation (26) represents the rack-tool profile in ξη plane. If the rack-tool tooth having this profile is inclined with an angle corresponding to the inclination angle of the rotor tooth helix, then the resulted tool will be able to generate the helical flank of the multi-lobe rotor of the progressive cavity pump.

Note: Due to the specific choice of the rolling radius, the tool profile for generating the rotor regions having arc of circle profile is identical to these arcs, so it does not require any effort to be found.

5. NUMERICAL APPLICATION OF THE PROFILING ALGORITHM

A numerical application is further proposed, in order to sample the algorithm for profiling the rack-tool used to generate the multi-lobe helical rotor from progressive cavity pumps. The input data are (according to notations from Figs. 1 and 2):

- base circle radius $R = 32$ mm,
- roller radius $r = 8$ mm,
- substitutive circles radius $r_0 = 6$ mm, and
- parameter of rotor helix $p = 200$ mm.

As it can be easily noticed, because $R / r = 4$, the application concerns a rotor having four lobes.

The first thing to be determined is the frontal profile of the rotor. In this purpose, a dedicated MatLab application was developed.

The application works in three successive steps:

- The values of φ_1 are discretized in [0, 2π] interval in 101 points, then the values of β are calculated with (13) and the coordinates of the profile points are calculated with equations (9). These points define only the concave arc from one side of the rotor (depicted in continuous blue line in Fig. 7). Some of these values are sampled in Table 1.
- Two arcs of circle meaning a quarter of substitutive circle each (depicted in dashed red line in Fig. 7) are joined to the extremities of the arc from above; each arc is determined by the coordinates calculated in 51 points, some of them sampled in Tables 2 and 3.
- The resulted side of the rotor profile is successively rotated with 90, 180 and 270 degrees, completing the rotor frontal profile, which is represented in red in Fig. 8, together to the corresponding basic hypo-cycloid (in blue).

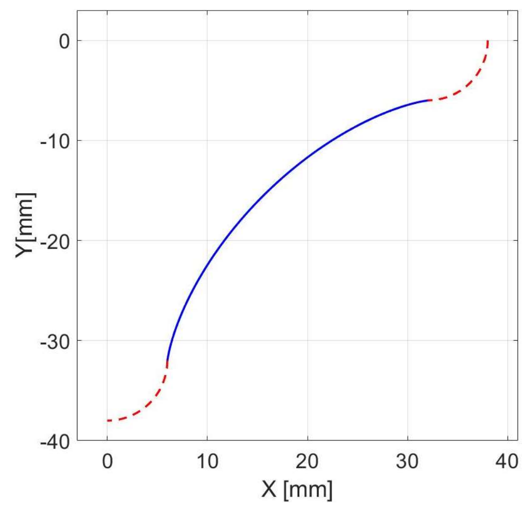

Fig. 7. The profile for one of the rotor sides.

Table 1

Coordinates of profile points belonging to concave arc

Point crt. no.	φ_1 [deg]	β [deg]	X [mm]	Y [mm]
1	0.0000	−1.5707	32.0000	−6.0000
2	0.0628	−1.5550	32.0824	−5.9993
3	0.1256	−1.5393	32.1411	−5.9980
4	0.1884	−1.5236	32.1761	−5.9966
5	0.2513	−1.5079	32.1876	−5.9960
6	0.3141	−1.4922	32.1757	−5.9969
7	0.3769	−1.4765	32.1404	−6.0000
8	0.4398	−1.4608	32.0821	−6.0060
9	0.5026	−1.4451	32.0009	−6.0156
10	0.5654	−1.4294	31.8972	−6.0296
11	0.6283	−1.4137	31.7711	−6.0486
12	0.6911	−1.3980	31.6232	−6.0732
....				
43	2.6389	−0.9110	19.4639	−12.1086
44	2.7017	−0.8953	18.9622	−12.5041
45	2.7646	−0.8796	18.4628	−12.9108
46	2.8274	−0.8639	17.9664	−13.3280
47	2.8902	−0.8482	17.4738	−13.7555
48	2.9530	−0.8325	16.9858	−14.1926
49	3.0159	−0.8168	16.5031	−14.6388
50	3.0787	−0.8011	16.0264	−15.0936
51	3.1415	−0.7853	15.5563	−15.5563
52	3.2044	−0.7696	15.0936	−16.0264
53	3.2672	−0.7539	14.6388	−16.5031
54	3.3300	−0.7382	14.1926	−16.9858
55	3.3929	−0.7225	13.7555	−17.4738
56	3.4557	−0.7068	13.3280	−17.9664
57	3.5185	−0.6911	12.9108	−18.4628
58	3.5814	−0.6754	12.5041	−18.9622
....				
90	5.5920	−0.1727	6.0732	−31.6232
91	5.6548	−0.1570	6.0486	−31.7711
92	5.7176	−0.1413	6.0296	−31.8972
93	5.7805	−0.1256	6.0156	−32.0009
94	5.8433	−0.1099	6.0060	−32.0821
95	5.9061	−0.0942	6.0000	−32.1404
96	5.9690	−0.0785	5.9969	−32.1757
97	6.0318	−0.0628	5.9960	−32.1876
98	6.0946	−0.0471	5.9966	−32.1761
99	6.1575	−0.0314	5.9980	−32.1411
100	6.2203	−0.0157	5.9993	−32.0824
101	6.2831	0.0000	6.0000	−32.0000

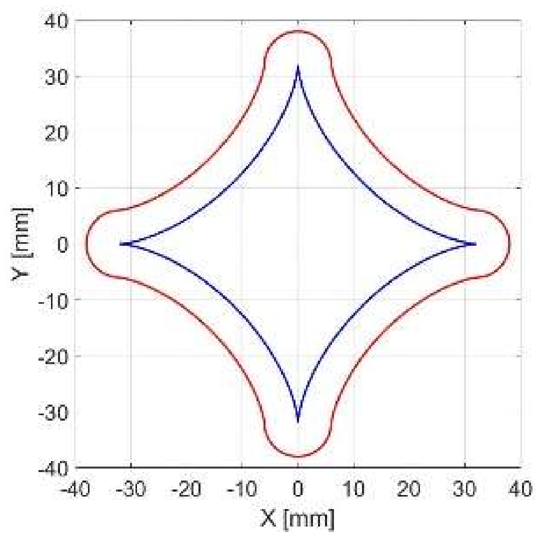

Fig. 8. The basic hypo-cycloid & full rotor profile, both in frontal section.

Table 2

Coordinates of points belonging to superior arc of circle

Point crt. no.	φ [deg]	*X* [mm]	*Y* [mm]
1	1.5707	38.0000	0.0000
2	1.5393	37.9970	−0.1884
3	1.5079	37.9881	−0.3767
4	1.4765	37.9733	−0.5646
5	1.4451	37.9526	−0.7519
....			
24	0.8482	36.5006	−3.9678
25	0.8168	36.3738	−4.1072
26	0.7853	36.2426	−4.2426
27	0.7539	36.1072	−4.3738
28	0.7225	35.9678	−4.5006
....			
47	0.1256	32.7519	−5.9526
48	0.0942	32.5646	−5.9733
49	0.0628	32.3767	−5.9881
50	0.0314	32.1884	−5.9970
51	0.0000	32.0000	−6.0000

Table 3

Coordinates of points belonging to inferior arc of circle

Point crt. no.	φ [deg]	*X* [mm]	*Y* [mm]
1	0	6	−32.0000
2	0.0314	5.9970	−32.1884
3	0.0628	5.9881	−32.3767
4	0.0942	5.9733	−32.5646
5	0.1256	5.9526	−32.7519
. . . .			
24	0.7225	4.5006	−35.9678
25	0.7539	4.3738	−36.1072
26	0.7853	4.2426	−36.2426
27	0.8168	4.1072	−36.3738
28	0.8482	3.9678	−36.5006
. . . .			
47	1.4451	0.7519	−37.9526
48	1.4765	0.5646	−37.9733
49	1.5079	0.3767	−37.9881
50	1.5393	0.1884	−37.9970
51	1.5707	0.0000	-38.0000

The profile of the rack-tool is determined then with another dedicated MatLab application, which determines the coordinates of a number of 101 profile points, on the base of equations (21), when $\phi = 0$, and condition (26). Because the analytical expressing of the condition would be very complicated, the couples (φ_1, β) for which this is satisfied are found with a numerical algorithm. The results are presented in Table 4 (excerpt of the points coordinates list) and Fig. 9 (where the rotor profile is depicted in red, while the tool profile – in blue).

Table 4

Coordinates of rack-tool profile points (excerpt)

Point crt. no.	φ_1 [deg]	θ [deg]	ξ [mm]	η [mm]
1	0.0000	0.0047	0.0000	−6
2	0.0628	0.0109	0.0824	−5.9993
3	0.1256	0.0251	0.2816	−5.9926
4	0.1884	0.0471	0.4229	−5.9822
5	0.2513	0.0628	0.5006	−5.9737
6	0.3141	0.0785	0.5470	−5.9672
7	0.3769	0.0942	0.5624	−5.9646
8	0.4398	0.1099	0.5474	−5.9678
9	0.5027	0.1256	0.5025	−5.9787
. . . .				
47	2.8902	0.7225	−9.7959	−21.8846
48	2.9530	0.7382	−9.8849	−22.6903
49	3.0159	0.7539	−9.9487	−23.5015
50	3.0787	0.7696	−9.9871	−24.3162
51	3.1415	0.7858	−10.0000	−25.1327
52	3.2044	0.8011	−9.9871	−25.9492
53	3.2672	0.8168	−9.9487	−26.7639
54	3.3300	0.8325	−9.8849	−27.5751
. . . .				
93	5.7805	1.4294	0.5025	−44.2664
94	5.8433	1.4451	0.5474	−44.2867
95	5.9061	1.4608	0.5624	−44.2976
96	5.9690	1.4765	0.5470	−44.3008
97	6.0318	1.4922	0.5006	−44.2982
98	6.0946	1.5079	0.4229	−44.2916
99	6.1575	1.5236	0.2816	−44.2831
100	6.2203	1.5330	0.0824	−44.2767
101	6.2831	1.5362	0.0000	−44.2723

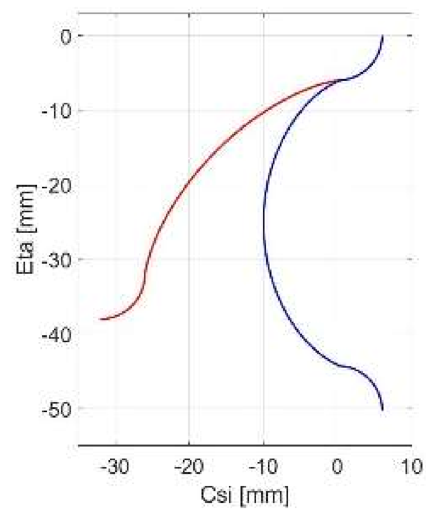

Fig. 9. The rack-tool profile corresponding to the generating of one side of the quadrilobed helical rotor.

6. CONCLUSIONS

This paper addresses the problem of profiling the rack-tool needed for generating the surface of the helical multi-lobe rotor from the progressive cavity pumps.

As above presented, the algorithm developed in this purpose supposes the covering of the following issues: *i)* the defining of the hypo-cycloid specific to pump rotor, *ii)* the finding of the technological profile of the rotor, equidistant t the hypo-cycloid, and *iii)* the profiling of the rack-tool, on the base of the Minimum distance method.

Unlike the analytical solutioning of the addressed profiling problem, which requires the manipulation of complicated expressions, having many terms, the proposed profiling algorithm is much simpler and delivers faster accurate solutions. The results of the presented numerical application confirm all thesething.

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