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SURFACE ROUGHNESS WHEN TURNING ONE-SHEETED REVOLUTION HYPERBOLOID SURFACE

Bruno RĂDULESCU[1], Laurențiu SLĂTINEANU[2], Mara RĂDULESCU[1], Adelina HRIȚUC[2,*]

[1)] Lecturer, Ph.D., Department of Machine Tools and Tools, "Gheorghe Asachi" Technical University of Iași, România
[2)] Prof., Ph.D., Department of Machine Manufacturing Technology, "Gheorghe Asachi" Technical University of Iași, România
[3)] Lecturer, Ph.D., Department of Machine Tools and Tools, "Gheorghe Asachi" Technical University of Iași, România
[4)] Ph.D. Student, Department of Machine Manufacturing Technology, "Gheorghe Asachi" Technical University of Iași, România

Abstract: *There are situations in industrial practice when parts that must have a one-sheeted revolution hyperboloid surface are necessary. Turning and milling are used as roughing processing for such surfaces. The emergence and development of numerically controlled lathes and milling machines have facilitated the machining of hyperboloidal surfaces when the mathematical function that defines the hyperbola in the axial plane of the hyperboloidal surface is available. In the case of turning and when the tool corner moves along a path in the shape of a hyperbola segment, it is found that there is a variation of the cutting speed along the path of the tool corner. This variation of the cutting speed could lead to a continuous change, within certain limits, of the values of some roughness parameters, such as, for example, the arithmetic mean deviation Ra of the assessed profile. To further investigate this variation in the value of the roughness parameter Ra when changing the cutting speed, an experimental research program was designed according to the requirements of a full factorial experiment with three independent variables and two levels of variation. As the independent variables, the corner radius of the lathe tool, the longitudinal feed rate, and the cutting speed were taken into account. Experimental tests were performed on aluminum test samples. The mathematical processing of the experimental results confirmed the variation of the size of the roughness parameter Ra along the hyperboloidal surface made by turning, mainly due to the change in the cutting speed. Among the input factors considered, it was found that the strongest influence on the size of the roughness parameter Ra is exerted by the corner radius of the lathe tool, followed by the size of the longitudinal feed.*

Key words: *hyperboloidal surface, turning, surface roughness, corner radius, feed, cutting speed, empirical mathematical model.*

1. INTRODUCTION

From a mathematical point of view, *the hyperbola* is a plane curve that corresponds to the geometric locus of the points in the Euclidean plane for which the absolute value of the difference in distances from two fixed points, called foci, is constant. Hyperbola is part of the family of curves called *conics*. Such curves – conics – are obtained by intersecting a straight, circular cone with a plane. If there is no closed curve (such as an ellipse or, if the plane is perpendicular to the axis of the cone, a circle) and the plane is not parallel to the axis of the cone (in which case the intersection curve will be a parabola), will get a hyperbole.

The hyperboloid is a three-dimensional surface to which the following relation corresponds:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1, \qquad (1)$$

in the case of the one-sheeted revolution hyperboloid surface and respectively:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=-1, \qquad (2)$$

in the case of the two-sheeted revolution hyperboloid surface, *a*, *b*, and *c* are constants that define the hyperboloid.

In civil engineering, hyperboloidal surfaces are used in the case of cooling towers, roofs of buildings, etc.

In the manufacture of the machines, the hyperboloidal form may be used in the case of:

- discs for driving the parts when centerless grinding;
- rollers for turning and feeding workpieces on some straightening machines;
- gears with non-parallel or intersecting axes and which contribute to the formation of hyperboloidal gears.

Such applications have led to research into the possibilities of obtaining through various processing and fuller characterization of hyperboloidal surfaces.

Thus, Bektas considered the possibilities of analyzing the design of hyperboloidal structures [5]. He appreciated that it is sometimes required that the best-fit hyperboloid fit to a set of points in practice.

* Corresponding author: D. Mangeron Blvd., 59A, 700050, Iași, România,
Tel.: +40758508159;
E-mail addresses: *bruno.radulescu@academic.tuiasi.ro* (B. Rădulescu), *slati@tcm.tuiasi.ro* (L. Slătineanu), mara.radulescu@academic.tuiasi.ro (M. Rădulescu), *adelina.hrituc@student.tuiasi.ro* (A. Hrițuc).

Various aspects of hyperboloidal gear transmission have been addressed by Abadjiev and Abadjieva [1–4].

Some hypotheses and practical applications regarding the obtaining of hyperboloidal surfaces by wire electrical discharge were approached by C. Deneș [7–9].

A previous paper elaborated by some of the authors of this article investigated the possibility of machining one-sheeted revolution hyperboloid surfaces by milling, using an adaptable device on a universal lathe [10].

The roughness of a surface is a deviation for which the ratio between length and height is less than 50. Currently, many parameters can be used to evaluate roughness. These parameters are divided into five groups: amplitude parameters, amplitude parameters that consider the ordinate averages, step parameters, hybrid parameters, curves, and parameters associated with a curve. For the time being, in most manufacturing drawings of machine parts, the indications regarding the roughness of the different surfaces take into account only the arithmetic mean deviation *Ra* of the assessed profile and, in rarer cases, the maximum height *Rz* of the profile. Knowing the real values of the roughness parameters is important given the influence that the roughness exerts on the lubrication processes, on the maintenance over time of the character of the fits, on the tightness of the joints, on the behavior of the surfaces when wearing and rubbing, etc. [11, 12]. It is known that many groups of factors influence the values of the roughness parameter *Ra*, one of these groups being the parameters of the cutting conditions.

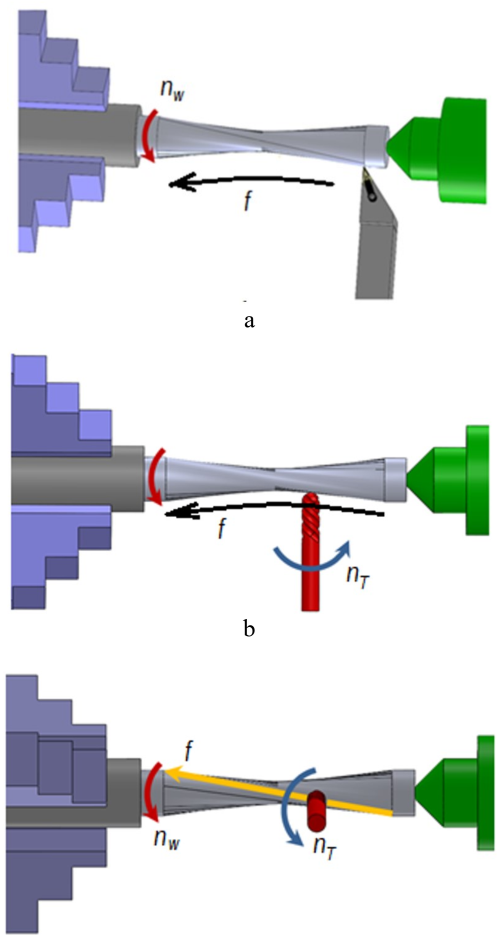

Fig. 1. Procedures for obtaining one-sheeted revolution hyperboloid surfaces on the lathe: *a* – by moving the corner of the lathe tool along a path in the form of a segment of an one-sheeted hyperbola; *b* – with the movement of an end mill having a hemispherical working area, along a path also in the form of a segment of a one-sheeted hyperbola; *c* – with the movement of an end mill having the hemispherical working area along a segment of the straight line whose rotation leads to the generation of an one sheeted revolution hyperboloid surface.

In the case of steel workpieces, experimental research has proven the existence of a maximum of the values of the roughness parameter *Ra* for a range usually located in the area of low machining speeds. The existence of this maximum is attributed to the process of formation and detachment of the built-up edge. For higher values of the cutting speed located outside the field of formation of the built-up edge, it is accepted that with the increase of the cutting speed, there is a slight decrease in the values of the roughness parameter *Ra*. The last statement remains valid for many metallic materials.

In the case of turning hyperboloidal surfaces with constant maintenance of the speed of the main shaft of the lathe, there is a continuous variation of the cutting speed due to the variation of the diameter at which the turning is performed. In this paper, it was proposed to conduct experimental research to highlight, among other things, whether the cutting speed leads to significant variation in the value of the roughness parameter *Ra* in the case of turning hyperbolic surfaces on the aluminum workpiece.

2. GENERATION OF HYPERBOLIC SURFACES BY CUTTING

A hyperboloidal surface can be generated, among other things, in one of three ways:

a) By rotating the curve called hyperbola around an axis passing through the two foci;
b) By moving a circle of variable diameter along the axis passing through the two foci, the circle remains tangent to a hyperbola;
c) By rotating around the axis of symmetry, passing through the two foci of a straight line segment that is not parallel and does not intersect with the axis of symmetry.

Each of the possibilities mentioned above for the theoretical generation of a hyperboloidal surface can be associated with different cutting processes. Thus, in the case of the first method, the materialization of a segment of flat hyperbola by the lathe tool corner moving along the respective segment of hyperbola can be considered (Fig. 1, *a*). If the variation of the diameter of the hyperboloidal surface generated in this way is large enough, this would mean a pronounced variation of the cutting speed and a possible variation of the size of the roughness parameter *Ra*. The situation described above is valid for those materials in which the increase of the cutting speed causes a relatively slow diminishing of the size of the roughness parameter *Ra*, due to a better flow of the material removed from the workpiece in the form of chips.

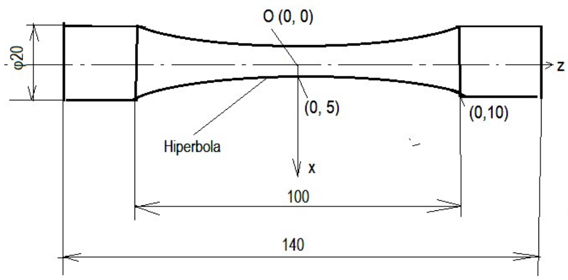

Fig. 2. The shape and dimensions of the aluminum alloy specimen after the execution of the one-sheeted revolution hyperboloid surface.

Similar statements could be made when, instead of the lathe tool, an end mill with a hemispherical active area is used (Fig. 1, *b*). In this case, the variation of the speed of movement between the hemispherical tip of the finger milling cutter and the machined surface would correspond to the longitudinal feed rate when milling a flat surface. The result should also be a slight decrease of the *Ra* parameter value when increasing the workpiece or test sample rotation speed.

Turning with a sharp-pointed tool or a milling cutter with a hemispherical active area can be performed on a numerically controlled lathe. It is necessary to take into account the corner radius of the lathe tool (if a tool with a corner radius is used) or the radius of the hemispherical surface of the end mill to determine the corrections involved in the use of such tools.

The second way of generating a hyperboloidal surface on the lathe can be considered as another way of interpreting the cutting scheme. Although the trajectory of the tool corner relative to the axis of the workpiece corresponds to a spiral on the one-sheeted hyperboloidal surface, that hyperboloidal surface can be considered as generated as a result of moving a circle of variable diameter along a plane hyperbola in the axial plane of the workpiece, the circle remaining permanently tangent to the hyperbola.

The third way of generating the hyperboloidal surface could be materialized by moving the tool's corner along a straight line segment that is not parallel and does not intersect with the axis of rotation of the workpiece. The tip of a sharp-edged lathe tool or an end mill with a hemispherical active area could be moved under controlled conditions along the segment of a straight line whose rotation could generate the intended hyperboloidal surface. It is possible to materialize such a machining scheme on some of the numerically controlled lathes.

Since the diameter at which the tool corner will act changes as the hyperboloid surface is generated, a variation in the size of the roughness parameter *Ra* is expected to occur along the hyperboloidal surface thus generated.

As expected, the size of the roughness parameter *Ra* will be affected not only by the variation of the cutting speed but will also be dependent on other factors, such as the feed rate along the workpiece rotation axis, the size of the corner radius of the lathe tool, some elements of geometry of the active area of the tool (values of side cutting edge and end cutting edge angles, back rake angle, side relief angle, etc.

Thus, it is expected that, with the increase of the size of the corner radius of the lathe tool, a decrease in the size of the roughness parameter *Ra* will be noticed. On the other hand, increasing the size of the feed will increase the value of the roughness parameter *Ra*.

We will also note that there are currently numerically controlled lathes that can ensure a constant value of the cutting speed when turning the hyperboloidal surface, which should lead to a somewhat constant value of the size of the roughness parameter *Ra* along the hyperboloidal surface if the values of the other factors influencing the roughness remain constant.

In the case of experimental research, the results of which are presented in this paper, it was preferred to use a constant speed of the lathe main shaft to observe and evaluate the change in surface roughness to the variable cutting speed determined by the variation of the diameter of the turned surface at a time.

The equation of a hyperbola in the *xOz* plane was taken into account to determine the coordinates of some points that could be used in the development of the numerical control program (Fig. 2):

$$\frac{x^2}{a^2} - \frac{z^2}{b^2} = 1. \quad (3)$$

From the above equation, we can arrive at:

$$x = \pm \frac{a}{b}\sqrt{b^2 + z^2}. \quad (4)$$

Intending to machine a hyperboloidal surface characterized by a maximum diameter of 20 mm, a minimum diameter of 10 mm, and a length of 100 mm (Fig. 2), we can establish the constants' values *b* in equation (4). The values *a* = 5 mm and *b* = 28.86 mm are thus reached.

Under these conditions, Eq. (4) can be written as:

$$x = 0.173\sqrt{833 + z^2}. \quad (5)$$

As mentioned above, the dimensional elements highlighted above were taken into account in developing the numerical control program.

The turning of the test samples was performed on a CNC lathe of the Mazak Quick Turn Nexus 200M type. As a workpiece, a cylindrical bar with a diameter of 30 mm was used.

Due to the rather large difference (over ten times) between the minimum diameter of the hyperboloidal surface (10 mm) and its length (103 mm), a locating and clamping of the workpiece in the universal lathe chuck and the live center located in the tailstock were used.

This solution led, on the one hand, to a decrease in the intensity of the vibrations that tended to occur and which could have affected the roughness of the machined surface. On the other hand, it allowed obtaining an area that corresponds to the dimensional requirements.

To locate and clamp the workpiece to the live center of the tailstock, it was necessary to introduce additional machining and to use two distinct components of the numerical control program for rough processing.

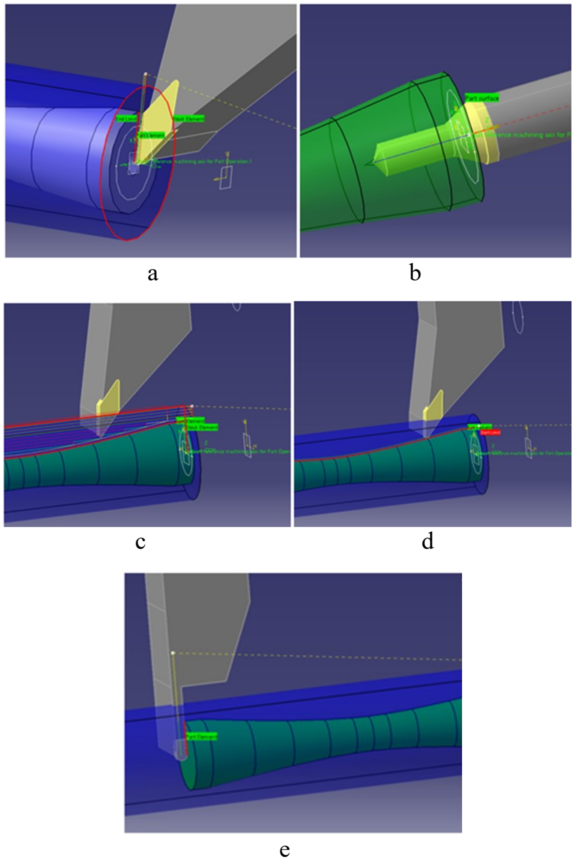

Fig. 3. The succession of the machining sequences to obtain the surface of the one-sheeted revolution hyperboloid of revolution: *a* – turning the flat front surface; *b* – machining the centering hole; *c* – roughing turning of the one-sheeted surface of the revolution hyperboloid; *d* – one-sheeted surface finishing turning of the revolution hyperboloid; *e* – cutting the one–sheeted revolution hyperboloid.

The first part of the program focused on the flat front turning, followed by executing the centering hole at the free end of the workpiece.

The second part of the program focused on the actual turning of the surface of the one sheeted rotation hyperboloid.

As the surface of the revolution, hyperboloid was characterized by a minimum diameter of 10 mm and a maximum diameter of 20 mm, and it was intended to use a constant speed of the main shaft of the lathe. The machining was carried out at a maximum rotation speed value, this being 1000 rev/min.

Note, however, that the lathe also can change the rotation speed of the lathe main shaft to the value of the diameter of the turned surface. The computer numerical control program was developed using Catia software and specific turning modules.

Figure 3 shows the turning phases required to make the surface of the one-sheeted revolution hyperboloid.

3. EXPERIMENTAL RESEARCH

Experimental research has proposed to test the extent to which some of the input factors in the turning process taken into account influence the value of the roughness parameter *Ra*.

As a basic material for experimental research, it can be mentioned first of all the numerically controlled lathe type Mazak Quick Turn Nexus 200M (manufactured in Japan), VNMG 16 04 08-PF 4415, and VNMG 16 04 12-PM 4415 cutting carbide inserts (manufactured by Sandvik from Sweden) and having distinct corner radius values (r_ε = 0.8 mm and r_ε = 1.2 mm) with which the lathe knives were equipped, a Mitutoyo SJ 201 type roughness meter.

The principles applicable to a full factorial experiment, with three independent variables at two levels of variation [6, 11], were used to reduce the volume of experimental tests]. This led to the need to perform $N_{et} = 2^3 = 8$ experimental tests.

The values established for the input factors in the investigated process are listed in Table 1. It can be seen that the two values of the corner radius r_ε (0.8 mm and 1.2 mm), two values of the longitudinal feed (0.14 mm/rev and 0.4 mm/rev), and, respectively, two values of the rotational speed *n* of the test sample (700 rev/min and 1000 rev/min) were used.

Subsequently, it was appreciated that a more accurate and universal assessment of the influence of different input factors in the turning process on the value of the surface roughness parameter Ra is possible by taking into account the cutting speed *v* and not the rotational speed of the test sample *n*. Thus, in Table 1, separate columns were used for the rotational speed *n* of the test sample and for the cutting speed *v*.

An aluminum workpiece with a round section was used. A centering hole was made at the right end for locating in the live center at this end and locating and clamping in the universal chuck at the left end of the workpiece.

The length of the workpiece between the universal chuck and the live center corresponded to the length of a single test sample.

After machining a hyperboloidal surface on each of the test samples, they ware separated from the bar by cutting on the lathe. As previously mentioned, the values of the surface roughness parameter *Ra* were determined using the Mitutoyo SJ 201 roughness meter. These values have been entered in the last column of Table 1.

The experimental results were mathematically processed using specialized software based on the least-squares method [6]. The software allows the selection of the most appropriate empirical mathematical model from 5 available models (first and second-degree polynomial empirical models, power type function, exponential function, and a hyperbolic function, respectively).

As a criterion for assessing the adequacy of the empirical mathematical model determined to the experimental results, the value of *the so-called Gauss's criterion* was used. This criterion considers the value of the sum of the squares of the differences between the values of the ordinates of some points determined by means of the proposed mathematical model and respectively the real values of the ordinates determined experimentally.

The lower the value of the Gauss's criterion, the more appropriate the determined empirical mathematical model is for the results determined by the experimental tests.

Table 1

Experimental conditions and results

Exp. no.	**Input factors**				**Surface roughness parameter *Ra*, µm**			
	Corner radius, r_ε, mm	**Feed, *f*, mm/rev**	**Rotation speed, *n*, rev/min**	**Cutting speed, v_c, m/min**	**Measured values, µm**			**Average value, µm**
1	0.8	0.14	700	22	1.30	1.03	1.45	1.26
2	0.8	0.14	700	44	0.97	1.00	0.91	0.96
3	0.8	0.4	1000	31.4	8.40	8.28	8.63	8.43
4	0.8	0.4	1000	62.8	6.36	5.79	5.91	6.02
5	1.2	0.14	700	22	0.80	0.99	0.71	0.83
6	1.2	0.14	700	44	0.48	0.60	0.45	0,51
7	1.2	0.4	1000	31.4	1.50	1.64	1.40	1,51
8	1.2	0.4	1000	62.8	1.25	1.22	0.73	1.06

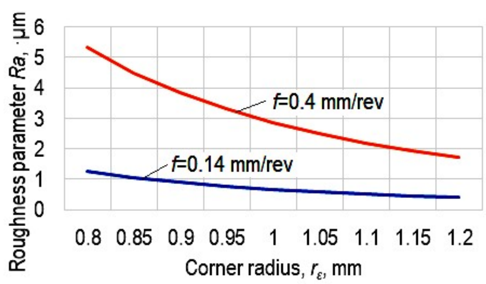

Fig. 5. The influence of the corner radius r_ε on the *Ra* surface roughness parameter value when turning a one-sheeted revolution hyperboloid surface on a test sample of aluminum (*v* = 44 m/min).

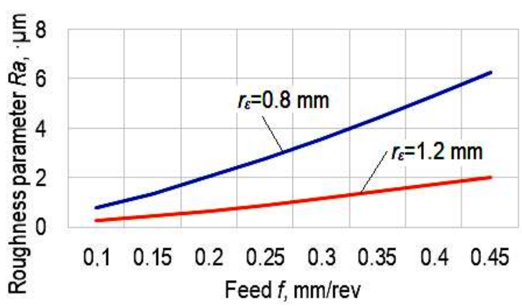

Fig. 4. The influence of the longitudinal feed *f* on the value of the *Ra* surface roughness parameter when turning a one-sheeted revolution hyperboloid surface on a test sample of aluminum (*v* = 44 m/min).

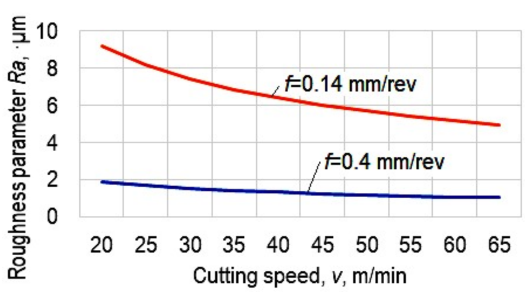

Fig. 6. The influence of the cutting speed *v* on the *Ra* surface roughness parameter when turning a one-sheeted revolution hyperboloid surface on a test sample of aluminum (r_ε = 8 mm).

By using the specialized software, of the five empirical mathematical models, the empirical mathematical model of the exponential function type was considered to be the most appropriate in relation to the experimental results, for which Gauss's criterion has the value $S_G = 1.276313$:

$$Ra = 9.962 \cdot 0.0598^{r_\varepsilon} 265^{f} 0.987^{v}. \quad (6)$$

However, since in manufacturing engineering, empirical mathematical models of power function type are used relatively frequently (there are thus mathematical models of power function type to highlight the influence of cutting conditions on the cutting tool life, on the magnitude of forces generated by cutting processes, on the values of some parameters of the characterization of the roughness of the machined,surfaces, etc.), a mathematical model of this type was also determined (Gauss's criterion having, in this case, the value $S_G = 1/320943$):

$$Ra = 73.234 r_\varepsilon^{-2.778} f^{1.383} v^{-0.522}. \quad (7)$$

Using the mathematical model constituted by Eq. (7), the graphical representations in Figs. 4, 5, and 6 were elaborated.

The analysis of the empirical mathematical model constituted by Eq. (7) and of the graphical representations in Figs. 4, 5, and 6 led to the formulation of the observations mentioned below.

It can be seen that the strongest influence on the value of the surface roughness parameter *Ra* is exerted by the corner radius r_ε of the carbide insert used since, in the mathematical relation (7), this input factor corresponds to the highest absolute value (2.778) if the absolute values of the exponents attached to each of the input factors considered are taken into account. The negative value of the exponent attached to the input factor corner radius r_ε confirms that as the magnitude of the corner radius r_ε increases, there will be a decrease in the value of the roughness parameter *Ra*.

The second factor that influences the value of the surface roughness parameter and which can be determined by analyzing the values of the exponents attached to the input factors in Eq. (7) is the feed *f*, the increase of which leads to an increase in the value of the surface roughness parameter *Ra*.

Suppose the absolute values of the exponents attached to the input factors under consideration are analyzed. In that case, it is found that the lowest such value corresponds to the cutting speed *v*. A lower influence will therefore be exerted by the cutting speed *v*, to which, in Eq. (7), the lowest value of the attached exponent is attached, to the values determined for the other input factors taken into account.

4. CONCLUSIONS

One-sheeted hyperboloidal surfaces have found use in civil or industrial buildings and the manufacture of machines. It can be encountered in situations of the use of hyperboloidal surfaces. Such hyperboloidal surfaces of metal parts can be obtained by using turning or milling processes. Turning machining with a feed movement along a hyperbola segment of the lathe tool corner generates a continuous variation of the diameter of the hyperboloidal surface generated by turning. This variation in the diameter of the turning surface leads to a variation in the cutting speed. According to current information, the variation of the cutting speed will cause a variation of the value of the surface roughness parameter *Ra* along the hyperboloidal surface, which would mean the existence of areas of the machined surface characterized by the variation of the heights of the asperities generated by machining process. For an experimental confirmation of this hypothesis, a full factorial experiment was designed, with three independent variables at two levels of variation of the values of the independent variables. As input factors, the size of the corner radius, the size of the feed along a trajectory in the shape of a hyperbola segment, and the size of the cutting speed were taken into account. Obtaining hyperboloidal surfaces was possible on a numerically controlled lathe. The aluminum was used as a test sample material. The experimental results were processed using software based on the least-squares method. In this way, two empirical mathematical models were established, such as an exponential function and a power type function, respectively. The analysis of the values of the exponents attached to the input factors in the case of the power type function allowed us to formulate the conclusion that the strongest influence on the value of the roughness parameter *Ra* is exerted by the size of the corner radius, followed by the size of the work feed and the size of the cutting speed. Therefore, the variation of the size of the surface roughness parameter *Ra* along the hyperboloidal surface was confirmed. These observations are valid when using a constant speed of the main lathe shaft while turning the hyperboloidal surface. In the future, it is intended to continue the research by taking into account other test sample materials. The research will also be carried out to observe the extent to which the values of the surface roughness parameter *Ra* remain constant in the case of numerically controlled lathes that can change the rotation speed of the lathe main shaft even during the machining process.

REFERENCES

[1] E. Abadjieva E, V. Abadjiev, H. Kawasaki, T. Mouri, *On the synthesis of hyperboloid gear drives and technical application*, Proceedings of the ASME 2013 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference IDETC/CIE 2013, pp. 1–10, Portland, U.S.A., August, 2013.

[2] E. Abadjieva, V. Abadjiev, *Synthesis of hyperboloid gear drives: Controlling of the singularity on the active tooth surfaces*, Journal of Mechanical Engineering Research, Vol. 9, No. 3, 2017, pp. 25–33.

[3] V. Abadjiev, E. Abadjieva, *Hyperboloid gear transmissions: innovations, experimental and manufacture realization*, MATEC Web of Conferences, Vol 287, Power Transmissions 2019, Borovets, June, 2019, 04003.

[4] V. Abadjiev, E. Abadjieva, *One approach to the synthesis, design and manufacture of hyperboloid gear sets with face mating gears. Part 2: Review of practical realization*, Journal of Theoretical and Applied Mechanics, Vol. 46, No. 3, 2016, pp.3–16**.**

[5] S. Bektas, *Design of hyperboloid structures*, Journal of Architectural Research and Development, Vol. 1, No. 2, 2017, pp. 33-42.

[6] G. Crețu, *Bazele cercetării experimentale. Îndrumar de laborator* (Fundamentals of experimental research. Laboratory handbook), "Gheorghe Asachi" Technical University of Iași, 1992.

[9] C. Deneş, *Contribuţii asupra prelucrării prin eroziune electrică cu electrod filiform* (Contributions on the processing by electrical discharge machining with wire electrode), Doctoral Thesis, Universitatea "Lucian Blaga", Sibiu, România, 2002.

[8] C. Deneş, *The processing of hyperboloid surfaces using wire electrical discharge machining - practical aspects*, Nonconventional Technologies Review, Vol. 22, No. 4, 2018, pp. 3–9.

[7] C. Deneş, *The processing of hyperboloid surfaces using wire electrical discharge machining – theoretic,* Nonconventional Technologies Review, Vol. 22, No. 3, 2018, pp. 15–20.

[10] L. Slătineanu, A. Hriţuc, O. Dodun, G. Nagîţ and I. Beşliu-Băncescu*, Obtaining one-sheeted revolution hyperboloid surface by cutting*, IOP Conference Series: Materials Science and Engineering, Vol. 1235, 1235 01201, 2021.

[11] L. Slătineanu, *Bazele cercetării științifice* (Fundamentals of scientific research), Publishing House PIM, Iași, 2020.

[12] E. Străjescu, *Rugozitatea sculelor așchietoare* (Roughness of cutting tools), Publishing House Bren, Bucharest, 2004.