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FINITE ELEMENT ANALYSIS OF METAL STRUCTURES BASED ON ELONGATED JOHNSON CUPOLA

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Abstract: *Nowadays, metal structures occupy an important segment in the field of buildable space due to the execution speed of the construction, easy assembly, the possibility of creating a modular structure, durability, etc. We find them most often in the construction of industrial halls, metal bridges, certain footbridges or overpasses, we can even find the metal structures either as urban furniture as recreation spaces, public transport waiting spaces, greenhouse structures or even in the form of furnished domes intended for various activities. This article aims to analyse the metallic structures that follow the geometry of Johnson's elongated cupola made of different metallic materials under different external static forces to observe the mechanical behaviour of the three variants of elongated cupola characterized by Johnson Norman.*

Key words: *metal structures, elongated Johnson cupola, dome structures.*

1. INTRODUCTION

Geodesic domes are structures that aim to approximate their external surface with that of a sphere, usually they are formed by Platonic polyhedra such as the icosahedron or the dodecahedron (Fig. 1) [1, 2]. Today, however, we find dome projects that no longer exactly follow the classic structures of such geometries, but an intersection of various polyhedra intended to form complex habitable structures.

Recently more and more people tend to opt for dome-type structures either for the construction of habitable homes, relaxation spaces or outbuildings in the household.

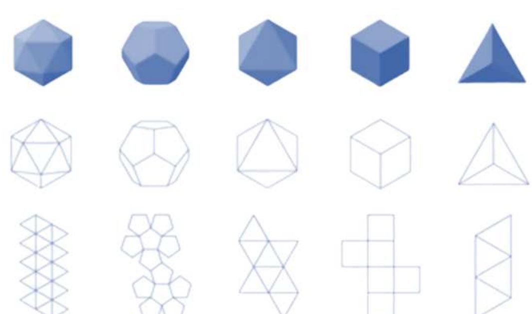

Fig. 1. Platonic polyhedra: *a* – icosahedron; *b* – dodecahedron; *c* – octahedron; *d* – cube; *e* – tetrahedron.

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They not only offer an aesthetic alternative, but also present multiple advantages such as: the possibility of reducing the amount of material used in the construction of a house compared to similar houses or structures, superior insulation and ventilation if the latest generation materials are used, these advantages given in principle by the construction form. A very important aspect is that it presents a superior perimeter resistance, a fact that implicitly determines a high seismic resistance compared to other structures but also an easy compartmentalization [3, 4].

The materials from which dome structures are usually made are wood [5] or various metal alloys [6] and are usually characterized by their framing radius or circumference radius, their surface, respectively the volume occupied and the type of structure. The shapes of the structure profiles used in the construction of domes can vary from square, circular, rectangular cross-sections, etc. depending on the dome use destination.

This article aims to analyse three domes with a metallic structure and a circular profile section structure that follows the mathematical principles of Johnson's elongated domes (elongated triangular cupola, elongated square cupola, elongated pentagonal cupola) [7].

2. METHODS

To realize the construction of the geometry following the Johnson's principles. The first step was to start from the initial modelling of the dome structure using both the 2D and 3D sketches options provided by Autodesk Inventor 2023 software application. In all three cases studied, the procedure started by framing the base of the dome in a circle with a diameter of 3000 mm, in which the hexagon, octagon and decagon bases were later placed. To generate the metallic structure a constant circular transverse profile with a diameter of 100 mm was used.

2.1. Dome based on elongated triangular cupola geometry

The elongated triangular cupola is one of the 92 strictly convex polyhedra described by Johnson Norman, also known as J_{18} [7]. This geometry can be divided into two regular polyhedra: the hexagonal prism that represents its base and the triangular cupola in the upper part [8]. This regular polyhedral contains 4 triangles, 6 squares and one hexagon, 27 edges and 15 vertices. The volume (1) and area (2) for such a solid can be calculated depending on its sides, as following:

$$V = \left(\frac{1}{6}\left(5\sqrt{2} + 9\sqrt{3}\right)\right) \cdot l^3, \tag{1}$$

$$A = \left(9 + \frac{5\sqrt{3}}{2}\right) \cdot l^2, \tag{2}$$

where l is the polyhedron side.

In Fig. 2,*a* the approximate dimensions of the metal structure are presented, the side of 1750 mm results from the inscribed base structure in a circle of 3000 mm, and the total height of the structure can be calculated using the side dimension and the height of the triangular cupola using relation (3) [9].

$$h = \frac{\sqrt{6}}{3} \cdot l, \tag{3}$$

where l is the polyhedron side.

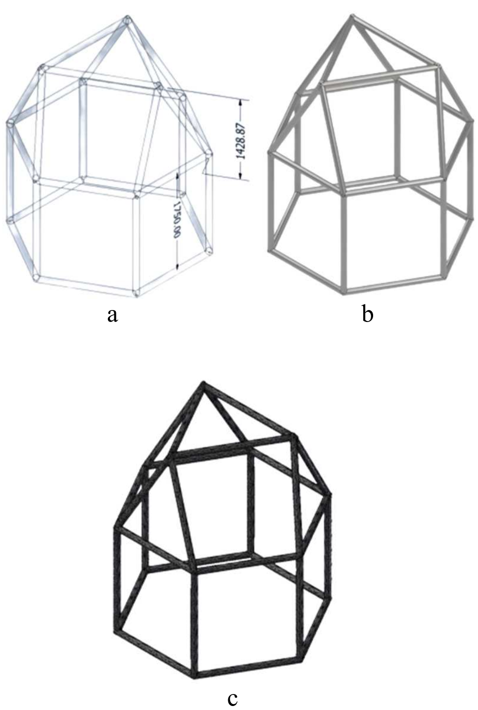


Fig. 2. Elongated triangular cupola: *a* – approximated dimensions of elongated triangular cupola structure; *b* – material applied to elongated triangular cupola structure; *c* – 3D model mesh.

2.2. Dome based on elongated square cupola geometry

The elongated square cupola is other convex Johnson polyhedron named also J_{19} [7]. This regular polyhedral can be divided into two polyhedra: one octagonal prism and one Johnson square cupola [10]. This Johnson solid contains 4 triangles, 13 squares and 1 octagon, 36 edges and 20 vertices. In this case we also treated a regular polyhedron, thus, we can calculate the area (4), volume (5) and circumradius (6) with the following formulae:

$$A = \left(15 + 2\sqrt{2} + \sqrt{3}\right) \cdot l^2, \tag{4}$$

$$V = \left(3 + \frac{8\sqrt{2}}{3}\right) \cdot l^3, \tag{5}$$

$$C = \left(\frac{1}{2}\sqrt{5 + 2\sqrt{2}}\right) \cdot l. \tag{6}$$

where l is the polyhedra side.

In Fig. 3,*a*, the approximate dimensions for dome structure are shown, the length of 1148.05 mm results from the inscribed octagon base in a circle of 3000 mm, and the total height of the structure can be calculated using the length and the height of the square cupola calculated by using relation (7) [11].

$$h = \frac{\sqrt{2}}{2} \cdot l, \tag{7}$$

where l is the polyhedron side.

2.3. Dome based on elongated pentagonal cupola geometry

The elongated pentagonal cupola is the last convex polyhedral used as a case study in this article also named J_{20}, which can be divided into a decagonal prism and a J_5 – pentagonal cupola [7]. This regular polyhedron contains 5 triangles, 15 squares, 1 pentagon and 1 decagon, 45 edges and 25 vertices [12]. The area (8) and

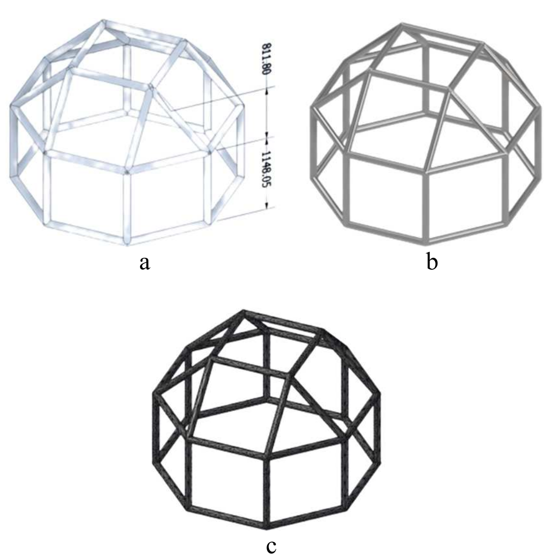


Fig. 3. Elongated squared cupola: *a* – approximated dimensions of elongated squared cupola structure; *b* – material applied to elongated squared cupola structure; *c* – 3D model mesh.

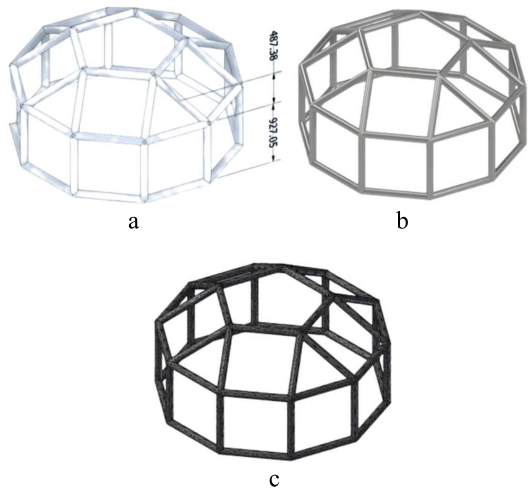


Fig. 4. Elongated pentagonal cupola: *a* – approximated dimensions of elongated pentagonal cupola structure; *b* – material applied to elongated pentagonal cupola structure; *c* – 3D model mesh.

the volume (9) can be expressed by using the following formulae:

$$A = \left(\frac{1}{4}\left(60 + \sqrt{10\left(80 + 31\sqrt{5} + \sqrt{2175 + 930\sqrt{5}}\right)}\right)\right) \cdot l^2, \quad (8)$$

$$V = \left(\frac{1}{6}\left(5 + 4\sqrt{5} + 15\sqrt{5 + 2\sqrt{5}}\right)\right) \cdot l^3, \quad (9)$$

where l is the polyhedron side.

Figure 4,*a* shows the approximate dimensions of the structure, the dimension of 927.05 mm results from the inscribed decagon base structure in a circle of 3000 mm, and the total height of the structure can be calculated using the length and the height of the pentagonal cupola using relation (10) [13].

$$h = \sqrt{\frac{5-\sqrt{5}}{10}} \cdot l, \quad (10)$$

where l is the polyhedron side.

2.4. FEA boundary conditions

A set of 10 finite element analyses was performed for each dome structure. In Fig. 5, the method of fixing geometry in their lower part, respectively the base of the dome, and the direction of the uniformly distributed force applied to the metal structures is presented. The following values of the applied forces were used: 500 N, 1000 N, 2000 N, 3000 N, 4000 N.

2.5. Materials applied

In this study, two types of material were used on each individual CAD model to observe the mechanical behaviour of these structures under different applied loads. It should be mentioned that the materials used in this study have a linear, isotropic, and homogeneous behaviour.

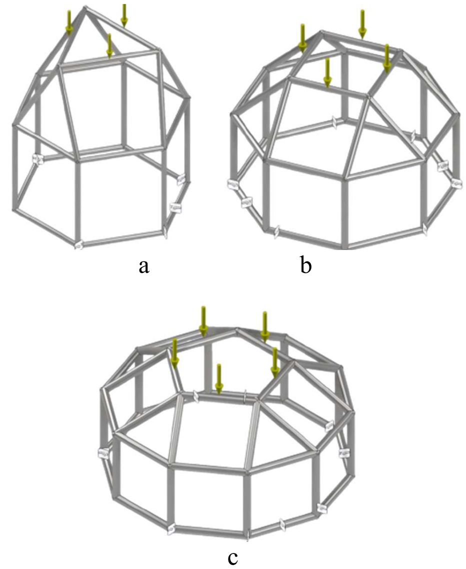


Fig. 5. FEA boundary condition: *a* – elongated triangular cupola; *b* – elongated square cupola; *c* – elongated pentagonal cupola.

Table 1

Stainless Steel – Material applied in the FEA study

Mechanical properties	Stainless Steel
Behaviour	Isotropic
Young's Modulus	205 GPa
Poisson's Ratio	0.30
Shear Modulus	80 GPa
Density	7.73 g/cm^3
Yield Strength	250 MPa
Tensile Strength	400 MPa

Table 2

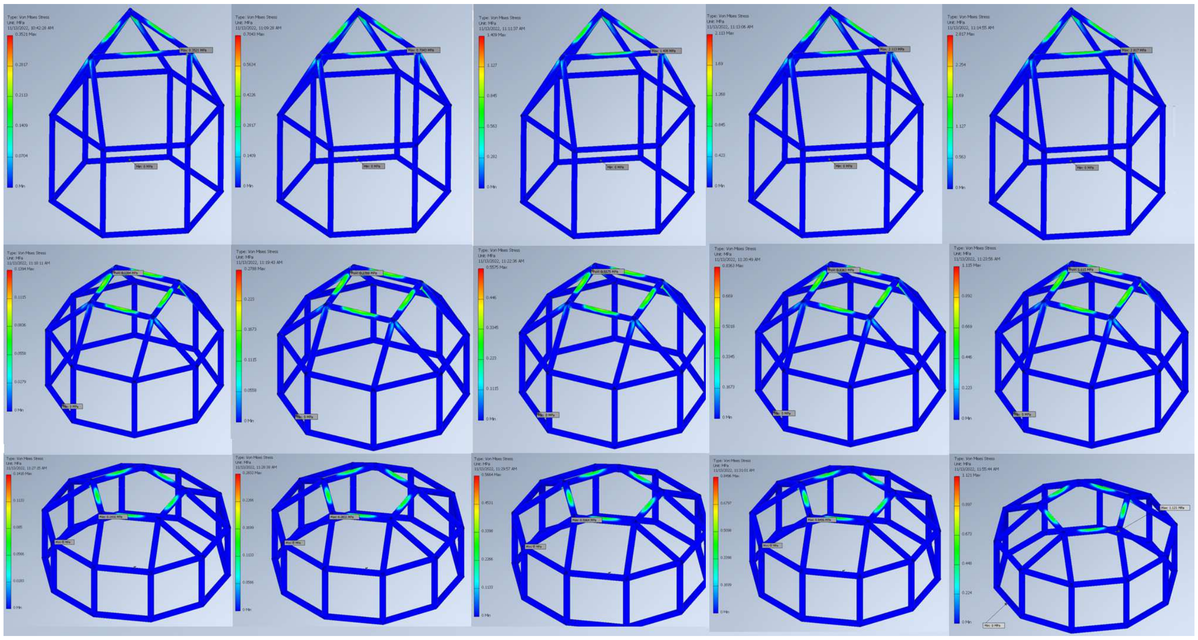
Aluminium 6061 – Material applied in the FEA study

Mechanical properties	Aluminium 6061
Behaviour	Isotropic
Young's Modulus	68.9 GPa
Poisson's Ratio	0.33
Shear Modulus	25864 MPa
Density	2.7 g/cm^3
Yield Strength	275 MPa
Tensile Strength	310 MPa

Tables 1 and 2 show the mechanical properties of aluminium 6061 and stainless steel that were considered in this study for each set of simulations. The material properties were extracted from the Inventor material library provided by Autodesk for this software application.

3. RESULTS

In Fig. 6, one can observe the maximum results on the Von Mises Stress scale for each individual geometry, in the case of stainless-steel material, depending on the



a b c d e

Fig. 6. FEA Von Mises Stress results for stainless-steel material assigned: *a* – 500 N load applied; *b* – 1000 N load applied; *c* – 2000 N load applied; *d* – 3000 N load applied; *e* – 4000 N load applied.

Table 2

FEA study results for stainless-steel material

Force [N]	**Maximum Von Mises Stress [MPa]**			**Maximum Displacement [mm]**		
	ETC	**ESC**	**EPC**	**ETC**	**ESC**	**EPC**
500	3.52×10^{-1}	1.39×10^{-1}	1.41×10^{-1}	6.24×10^{-3}	1.48×10^{-3}	9.28×10^{-4}
1000	7.04×10^{-1}	2.78×10^{-1}	2.83×10^{-1}	1.24×10^{-2}	2.97×10^{-3}	1.85×10^{-3}
2000	1.4	0.55	0.56	0.02	5.94×10^{-3}	3.71×10^{-3}
3000	2.11	0.83	0.84	3.74×10^{-2}	8.91×10^{-3}	5.57×10^{-3}
4000	2.81	1.11	1.13	4.99×10^{-2}	1.18×10^{-2}	7.42×10^{-3}

Note. ETC –elongated triangular cupola; ESC – elongated squared cupola; EPC – elongated pentagonal cupola.

force applied to them. The maximum value recorded in the case of the stainless-steel material can also be seen in Table 2, at the 4000 N applied forces: 4.99×10^{-2} mm for ETC, 1.18×10^{-2} mm for ESC, 7.42×10^{-3} mm for EPC. The elongated triangular dome results were the highest value in the case of this set of simulations. Figure 8 shows the displacement results measured in millimetres for each dome simulation, in the case of the stainless-steel material.

In Fig. 7, Von Mises Stress results are shown in the case of the aluminium 6061 material assigned for all dome-type structures, and in Fig. 9 displacement results are shown for each geometric structure with the same assigned material.

The maximum-recorded Von Mises Stress can be seen in Table 3, where a value of 2.81 MPa was obtained for ETC, 1.11 MPa for ESC and 1.13 MPa for EPC after applying a force of 4000 N. As could be anticipated, in the case of the elongated triangular cupola, the largest displacements were recorded for each applied force.

The results of the simulations for both materials showed that the structure, which follows Johnson's elongated triangular dome geometry, is the one that achieved the highest stress values but also the largest displacements, being followed by the dome-type structure that follows the elongated pentagonal dome geometry and finally the geometry of the dome built after the elongated square cupola, the last two domes showing very similar results.

The evolution of the FEA static simulation results is shown graphically in Figs. 10 and 12 for stainless-steel and in Figs. 11 and for aluminium 6061.

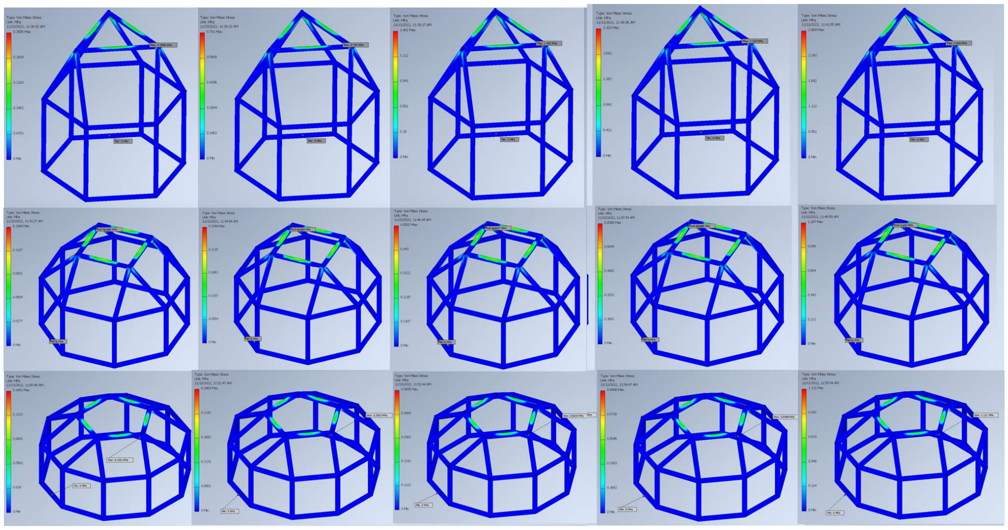


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Fig. 7. FEA Von Mises Stress results for aluminium 6061 material assigned: *a* – 500 N load applied; *b* – 1000 N load applied; *c* – 2000 N load applied; *d* – 3000 N load applied; *e* – 4000 N load applied.

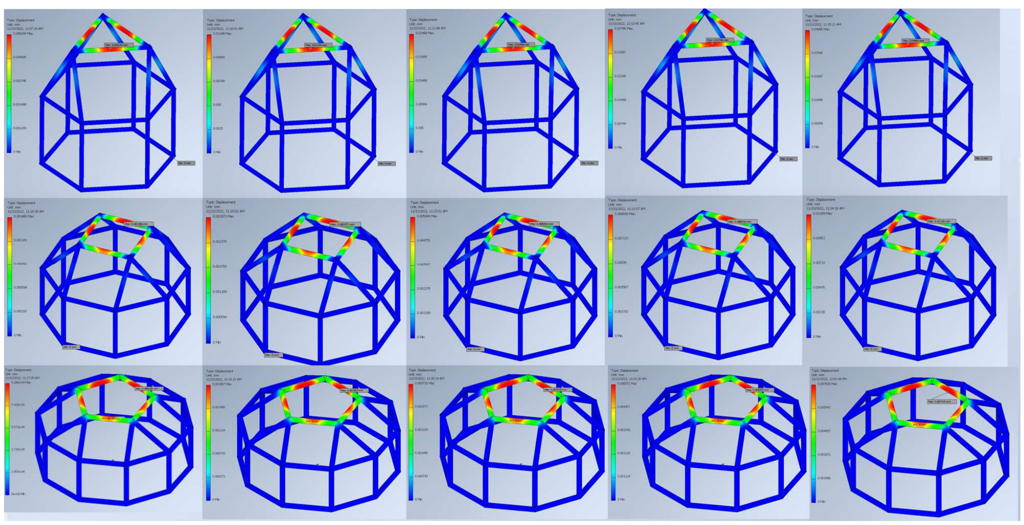

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Fig. 8. FEA displacement results for stainless-steel material assigned: *a* – 500 N load applied; *b* – 1000 N load applied; *c* – 2000 N load applied; *d* – 3000 N load applied; *e* – 4000 N load applied.

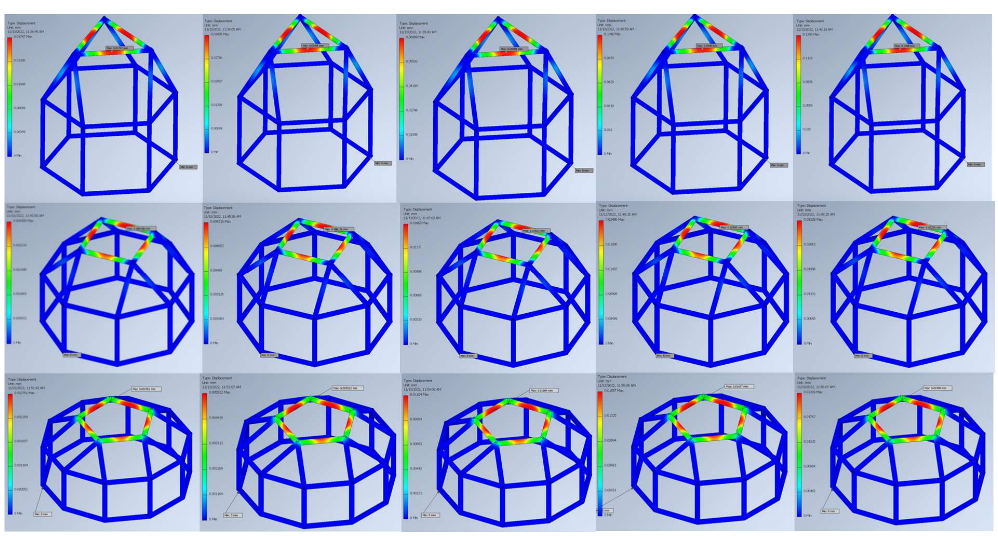

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Fig. 9. FEA displacement results for aluminium 6061 material assigned: *a* – 500 N load applied; *b* – 1000 N load applied; *c* – 2000 N load applied; *d* – 3000 N load applied; *e* – 4000 N load applied.

Table 3

FEA study results for Aluminium

Force [N]	Maximum Von Mises Stress [MPa]			Maximum Displacement [mm]		
	ETC	ESC	EPC	ETC	ESC	EPC
500	3.5×10^{-1}	1.38×10^{-1}	1.4×10^{-1}	1.74×10^{-2}	4.15×10^{-3}	2.76×10^{-3}
1000	0.7	2.76×10^{-1}	2.8×10^{-1}	3.49×10^{-2}	$8.31x10^{-3}$	5.52×10^{-3}
2000	1.4	5.53×10^{-1}	5.6×10^{-1}	6.98×10^{-2}	$1.66x10^{-2}$	1.1×10^{-2}
3000	2.1	8.3×10^{-1}	8.4×10^{-1}	1.04×10^{-1}	$2.49x10^{-2}$	1.65×10^{-2}
4000	2.8	1.1	1.12	1.39×10^{-1}	3.32×10^{-2}	2.2×10^{-2}

Note. ETC – elongated triangular cupola; ESC – elongated squared cupola; EPC – elongated pentagonal cupola.

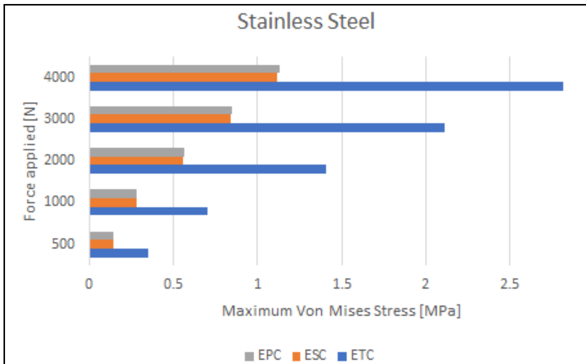

Fig. 10. Von Mises Stress results for stainless-steel material.

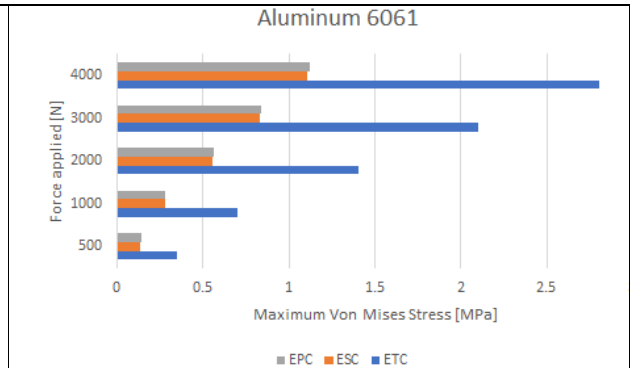

Fig. 11. Von Mises Stress results for aluminium 6061.

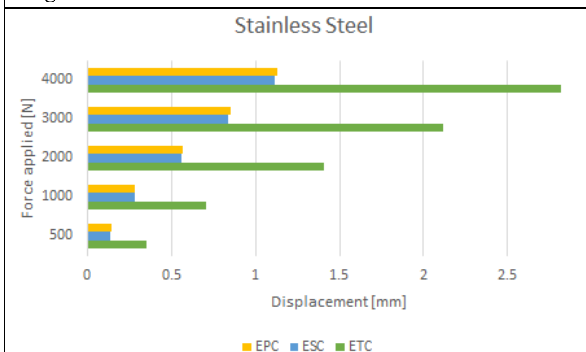

Fig. 12. Displacement results for stainless-steel material.

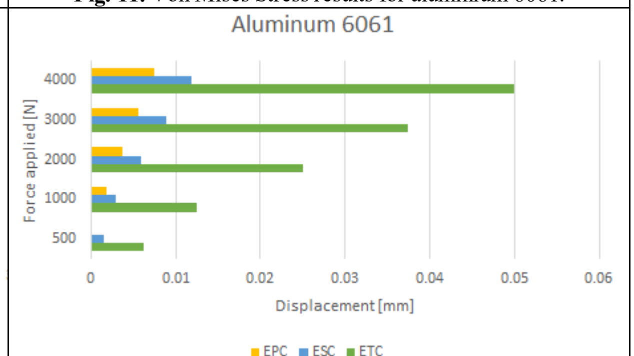

Fig. 13. Displacement results for aluminium 6061.

4. CONCLUSIONS

Dome-type structures are starting to occupy an important domain in the construction area due to the facilities with which they are built, their modularity, but also their thermal properties or distribution of external loads, resistance to possible weather conditions or seismic activities located in certain geographical areas, these being given mainly due to the materials used in their construction and the geometric shape that facilitates a good force distribution. Although in most geodesic dome type constructions Platonic polyhedra are used, the icosahedron and the dodecahedron being the most common, with the development of this sector it may be possible to introduce other types of shapes or polyhedra that can be integrated into structures of this type, structures which can serve in various ways, either as a habitable perimeter or integrating such structures as urban furniture, pavilion-type spaces, etc. Johnson's polyhedrons can represent a source of inspiration in the realization of such designs of dome-type structures, especially elongated triangle, squared and pentagonal cupola. For this purpose, the mechanical behaviour of some dome-type structures based on the geometries made by Johnson was analysed using circular profile metal made of materials often found in metal structures such as stainless steel and aluminium 6061. The static force applied on each dome-type structure showed that in both material cases used for elongated triangle cupola, the dome has vulnerabilities, gathering higher values in terms of Von Mises Stress and the displacements recorded, however, the elongated squared cupola structure has behaved best from a mechanical point of view when applying forces, including the equivalent of 4000 N. The elongated pentagonal cupola dome structure obtained values similar to those of elongated squared cupola structure, but the last one is highlighted by a set of minor values obtained at the FEA tests.

The domes based on the elongated cupola described by Johnson can represent an alternative in the construction of metal structures, but also from other materials such as wood, obtaining a positive response at static simulations carried out in this case study.

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