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ON THE ELASTICITY OF INVOLUTE SPUR GEARS WITH ASYMMETRIC TEETH

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Abstract: *The paper presents some research on the deformations of the involute asymmetric teeth of asymmetric spur gears. The tooth of the gear is considered as a short beam with the load applied in different points, from the bottom to the top of the tooth, of the active profile. Thus, one can study the variation of the elasticity of the pinion tooth, gear tooth and the elasticity of both teeth witch are in contact, depending on the contact point which is necessary to predict the performance. The paper offers an analysis on elasticity of the involute asymmetric teeth depending of the coefficient of asymmetry.*

Key words: *spur gears, asymmetric profiles, involute teeth, elasticity, deformations.*

1. INTRODUCTION

The paper is only a part of a study [3] of asymmetric spur gears with different coefficients of asymmetry, including the comparison with spur gears with symmetric involute teeth, with the aim of obtaining the optimal design for the mechanical transmission with spur gears with involute curves as teeth profiles.

The determination of the elasticity of the involute asymmetrical teeth makes it possible to determine the variation of the load taking into account that the contact ratio is higher than one. Considering the elasticity in the contact point for each of the two contact points on the line of action, one can solve the statically indeterminate problem of load distribution between the two pairs of teeth. If the load on the tooth can be calculated for all the contact points, is possible to determine the maximum bending stress and the contact stress, in relation with the asymmetry coefficient, for studying and optimizing the asymmetric gears parameters.

The pressure angles on the active and inactive profile can be increased or decreased to optimize any particular feature, depending on the requirements of the system that the transmission is part of.

The opposite flanks of an involute tooth, function in a different way, for the majority of gearings. The load on the active flank is significantly larger and is applied over longer periods of time than on the inactive flank.

These aspects led to the idea of using gears called asymmetric gears [5] for which, in order to build the tooth profile, two involutes are drawn from two different basic circles, involutes that are at a certain distance one from the other, in order to create the tooth body, which, this time, is no longer symmetrical as related to the radius that crosses the intersection point of the two involutes, radius which for the common toothing was the tooth symmetry axis (Fig. 1).

Thus an asymmetrical involute flank tooth results with supplementary advantages that are added to the ones of classical involute gearings.

Designing gears with asymmetrical teeth [5] is aimed at improving the performance of active flanks by reducing the performance of inactive flanks. Improving the performances can mean an increase in the loading capacity and weight, noise and vibration reduction [5].

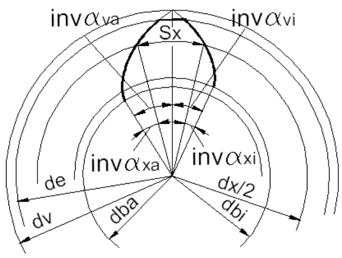

Fig. 1. Asymmetric involute tooth.

In this paper the parameters of the active profile have a as subscript and the parameters of the inactive profile have i as subscript. The degree of asymmetry is determined by the two different diameters of the basic circles d_{ba}, d_{bi}, or by the two pressure angles α_a, α_i (Fig. 1). The coefficient of asymmetry [5] can be calculated with:

$$k = \frac{d_{bi}}{d_{ba}} = \frac{\cos\alpha_i}{\cos\alpha_a}. \qquad (1)$$

Within the present paper one presents only a small part of the results obtained by the author with a Matlab modeling program of asymmetric gears, developed by the author herself. The designed program makes it possible to determine in a short time the geometrical parameters and some functional parameters such as the maximum bending and the contact stress of the asymmetric gears.

2. DETERMINATION OF ASYMMETRICAL TOOTH DEFORMATIONS

We considered the asymmetrical tooth as a fixed beam, of l length, with inconstant cross section, witch is fixed on the body of the gear. The maximum cross section $S_a S_i$, and the parametrical equations of the involute and joint profiles, at the basis of the active and inactive profiles, in relations with the centroid axis $x_G y_G$, $x_G y'_G$, the angle

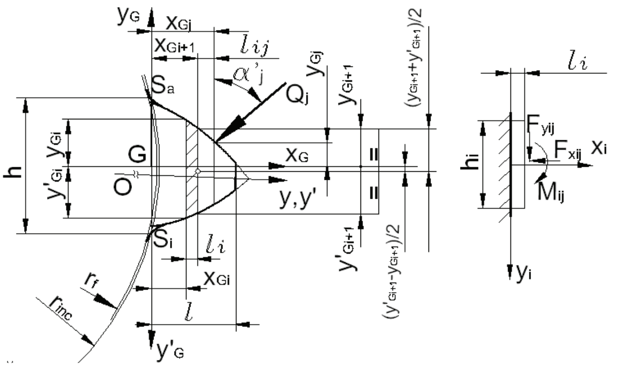

Fig. 2. The asymmetrical tooth considered as a fixed beam.

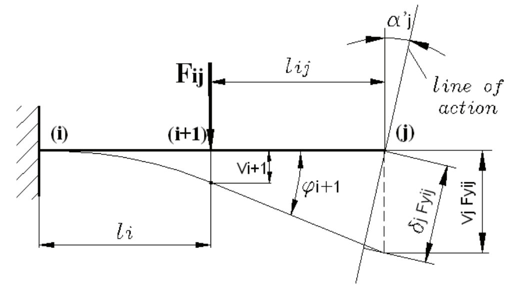

Fig. 3. The determination of the deformation corresponding to the load F_{yij}.

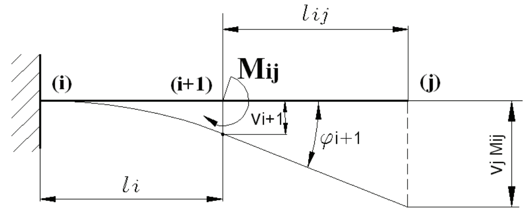

Fig. 4. The determination of the deformation corresponding to the couple M_{ij} .

α'_j between the load Q_j and the section $S_a S_i$ (Fig. 2) were calculated [3] and the results were used in this paper.

To determine the deformations of the tooth, in case of loading in different points of the active profile, we considered that the beam is made up of small length elements, with constant cross section, parallel with the maximum cross section $S_a S_i$ (the fixed support of the beam). Each of these elements, as one can see in Fig. 2, is fixed at the left end and loaded on the right end with the load F_{yij}, on the y_i axis, the load F_{xij}, on the x_i axis and the couple M_{ij} perpendicular on the plane $x_i y_i$. These loads are the result of reducing the load Q_j on the centroid of the right end section of the element l_i [2].

The dimensions of the rectangular constant cross section for the element i are gear widths b and h_i:

$$h_i = \left(y_{Gi} + y'_{Gi} + y_{Gi+1} + y'_{Gi+1}\right)/2. \tag{2}$$

The geometrical characteristics of the cross sectional area of the i element are:

$$A_i = b \cdot h_i; \quad I_i = b \cdot \left(h_i\right)^3 /12. \tag{3}$$

The three loads applied on the free ends of length l_i element are:

$$F_{yij} = Q_j \cdot \cos\alpha'_j; \tag{4}$$

$$F_{xij} = Q_j \cdot \sin\alpha'_j; \tag{5}$$

$$M_{ij} = Q_j \cdot \cos\alpha'_j \cdot \ell_{ij} - \\ -Q_j \cdot \sin\alpha'_j \cdot \left(y_{Gj} + \left(y'_{Gi+1} - y_{Gi+1}\right)/2\right). \tag{6}$$

The displacements of the contact point, on the line of action can be determined using the displacements on the axis x_G, y_G corresponding to the bending, shearing, and compression produced by these efforts.

The deformation corresponding to the bending stress is the result of the effects of the load F_{yij} and the couple M_{ij}.

The displacement v_{i+1} and the rotation φ_{i+1}, due to the load F_{yij} [2] in section (i + 1) can be deduced from Fig. 3 and calculated with:

$$v_{i+1,Fyij} = \frac{F_{yij} \cdot \ell_i^3}{3E_e I_i}; \quad \varphi_{i+1,Fyij} = \frac{F_{yij} \cdot \ell_i^2}{2E_e I_i}. \tag{7}$$

The displacement of point j is equal with the displacement $v_{i+1,Fyij}$ added to the displacement which is the result of $\varphi_{i+1,Fyij}$ rotation:

$$v_{jFij} = v_{i+1,Fyij} + \varphi_{i+1,Fyij} \cdot \ell_{ij}; \tag{8}$$

$$v_{jFij} == Q_j \cdot \cos\alpha'_j \cdot \left(\frac{\ell_i^3}{3E_e I_i} + \frac{\ell_i^2 \cdot \ell_j}{2E_e I_i}\right). \tag{9}$$

With the same method, one can determine the displacement produced by couple M_{ij} from Fig. 4.

The displacement v_{i+1} and rotation φ_{i+1} from couple M_{ij} in section (i + 1) are [2]:

$$v_{i+1,Mij} = \frac{M_{ij} \cdot \ell_i^2}{2E_e I_i}; \quad \varphi_{i+1,Mij} = \frac{M_{ij} \cdot \ell_i}{E_e I_i}. \tag{10}$$

The displacement of the j point due to the M_{ij} couple results:

$$v_{jMij} = \frac{M_{ij} \cdot \ell_i^2}{2E_e I_i} + \frac{M_{ij} \cdot \ell_i \cdot \ell_{ij}}{E_e I_i}; \tag{11}$$

$$v_{jMij} = Q_j \cdot \cos\alpha'_j \cdot \ell_{ij}\left(\frac{\ell_i^2}{2E_e I_i} + \frac{\ell_i \cdot \ell_{ij}}{E_e I_i}\right) - \\ -Q_j \cdot \sin\alpha'_j \cdot \left(y_{Gj} + \frac{y'_{Gi+1} - y_{Gi+1}}{2}\right) \cdot \left(\frac{\ell_i^2}{2E_e I_i} + \frac{\ell_i \cdot \ell_{ij}}{E_e I_i}\right). \tag{12}$$

In the determined relations E_e is the effective elastic modulus [1], which takes into account the ratio between the gear width b and the thickness of the tooth on the rolling circle S_r:

$$b/S_r < 5 \Rightarrow E_e = E;$$
$$b/S_r > 5, \Rightarrow E_e = E/(1-\nu^2), \qquad (13)$$

in which E is the elastic modulus in tensile of the material and ν is the Poisson ratio.

The displacements of point j corresponding to the bending of the element i of the beam results:

$$v_{ij} = v_{jFij} + v_{jMij}, \qquad (14)$$

$$v_{ij} = \frac{F_{yij}\cdot \ell_i^3}{3E_e I_i} + \frac{F_{yij}\cdot \ell_i^2 \cdot \ell_j}{2E_e I_i} + \frac{M_{ij}\cdot \ell_i^2}{2E_e I_i} + \frac{M_{ij}\cdot \ell_i \cdot \ell_{ij}}{E_e I_i}. \qquad (15)$$

The shear deformation, the displacement of section $(i+1)$ in relation with section i corresponding to the shear force F_{yij} is [3]:

$$v_{fi} = v_{fj} = 1,2\cdot F_{yij}\cdot \ell_i / G\cdot A_i, \qquad (16)$$

in which $G = E/2(1+\nu^2)$ is the elastic modulus in shear of the material.

The compression deformation of the element i is:

$$u_{ci} = u_{cj} = F_{xij}\cdot \ell_i / E_e \cdot A_i. \qquad (17)$$

The displacement of the loading point of Q_j force, on the line of the action, as a result of all deformations of element i is:

$$\delta_{ji} = \left(v_{ij} + v_{fj}\right)\cdot \cos\alpha'_j + u_{cj}\cdot \sin\alpha'_j. \qquad (18)$$

The resulting displacement of the loading point of Q_j force, on the line of action, the effect of all deformations of all elements i is:

$$\delta_{tj} = \sum_{i=1}^{n}\delta_{ji} = \sum_{i=1}^{n}\left(\left(v_{ij} + v_{fj}\right)\cdot \cos\alpha'_j + u_{cj}\cdot \sin\alpha'_j\right). \qquad (19)$$

The displacement of the contact point j, due to the contact stress on the surface of the teeth is [1]:

$$\delta_{Hj} = \frac{1}{2}\Delta_{Hj} = \frac{1,37\cdot Q_j}{2\cdot E_{12e}^{0.9}\cdot b_e \cdot Q_n^{0.1}}, \qquad (20)$$

in which:

$$E_{12e} = 2E_{1e}\cdot E_{2e}/\left(E_{1e} + E_{2e}\right) \qquad (21)$$

is the resulting elasticity modulus; Q_n is the entire load and Q_j is the load in the point j of contact.

The total displacement of j contact point, on the line of action at the asymmetrical involute tooth is:

$$\delta_j = \delta_{tj} + \delta_{Hj}. \qquad (22)$$

The elasticity of the tooth for the pinion and the gear were calculated with

$$V_{j1} = \delta_{j1}/Q_j; \qquad V_{j2} = \delta_{j2}/Q_j. \qquad (23)$$

The elasticity of the two teeth which are in contact in j point of the line of action results:

$$V_j = V_{j1} + V_{j2}. \qquad (24)$$

3. THE VARIATION OF THE ELASTICITY IN RELATION WITH THE POINT OF CONTACT

With the presented algorithm on which base we have developed the modeling program in MATLAB [4], one can determine the elasticity of the involute asymmetrical teeth for different points of contact.

The asymmetric gears can be used with high pressure angle $\alpha_a > \alpha_i$ or with low pressure angle $\alpha_a < \alpha_i$, on the active tooth side [5]. The variation of the elasticity for the same pair of gears with the coefficient of asymmetry $k = 1.22$ in the first case, and $k = 0.81$ for the second case, are presented in Fig. 5 and Fig. 6.

We can see that the maximum value of the elasticity of the pair of teeth is bigger if the pressure angle in the active profile is larger than the pressure angle in the inactive profile, for the coefficient of asymmetry bigger than the unit.

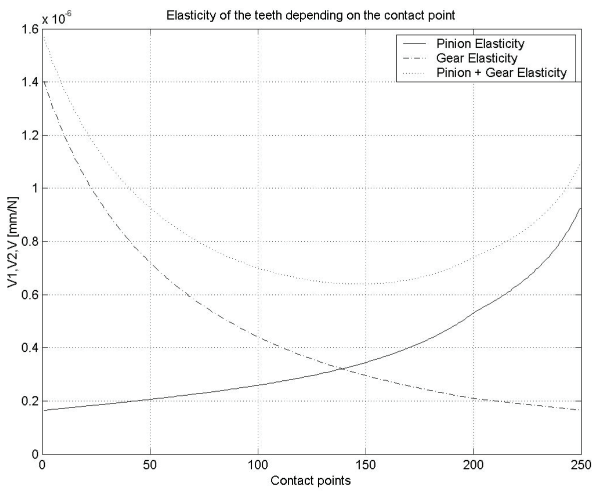

Fig. 5. Elasticity of the tooth I; k =1.22.
(z_1 = 16; z_2 = 57; a = 127,25; α_a = 40°; α_i = 20°).

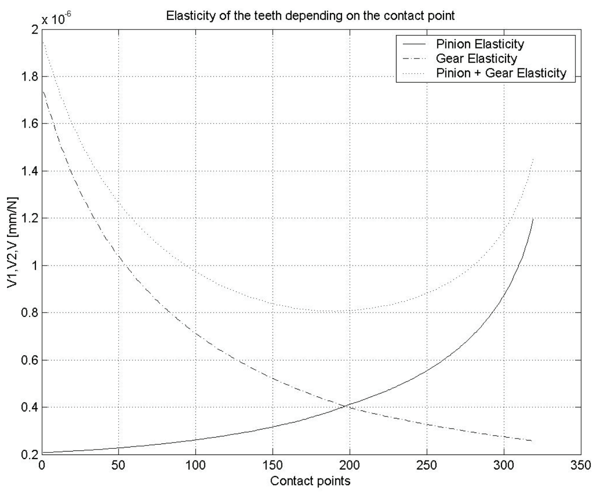

Fig. 6. Elasticity of the tooth II; k = 0.81
(z_1 = 16; z_2 = 57; a = 127,25; α_a = 20°; α_i = 40°).

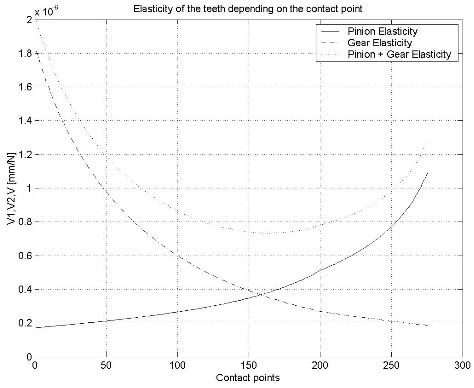

Fig. 7. Elasticity of the tooth I; k = 1,14; α_a = 35°; α_i = 20°.

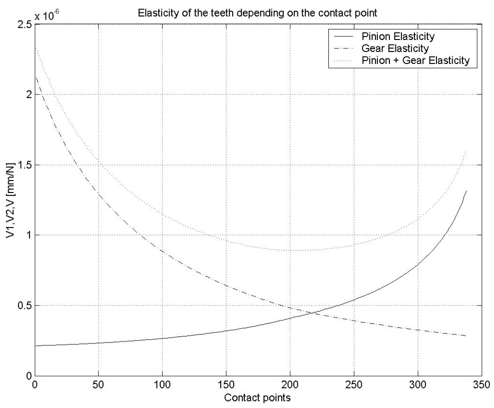

Fig. 8. Elasticity of the tooth II; k = 0.87; α_a = 20°; α_i = 35°.

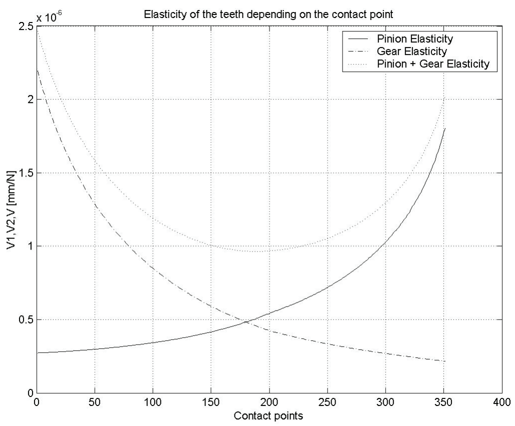

Fig. 9. Elasticity of the symmetrical teeth; k = 1 (z_1 = 16; z_2 = 57; a = 127,25; α_a = 25°; α_i = 25°).

4. THE VARIATION OF THE ELASTICITY IN RELATION WITH THE COEFFICIENT OF ASYMMETRY

For another pressure angle which means another coefficient of asymmetry, but for the same number of teeth and center distance, the variations of elasticity are presented in Fig. 7 (k = 1.14) and Fig. 8 (k = 0.87).

We can also determine the elasticity of the symmetrical gears k = 1 (Fig. 9), for the same number of teeth and center distance and equal pressure angles, and compare the values of the elasticity of the gears for different coefficients of asymmetry.

The maximum elasticity in this case is bigger in relation with the elasticity of asymmetric gears.

5. CONCLUSIONS

For the asymmetrical gears with $k > 1$ the decrease of the coefficient of asymmetry increases the elasticity .

For the asymmetrical gears with $k < 1$ the increase of the coefficient of asymmetry increases the elasticity.

The maximum elasticity in the $k = 1$ case is bigger in relation with the elasticity of asymmetric gears.

The conclusion is that increasing the value of $|k-1|$, decreases the elasticity and increases the rigidity of the teeth which are in contact.

The rigidity of the involute asymmetric teeth with coefficient of asymmetry $k \neq 1$ is larger than the rigidity of the ordinary gears $k = 1$.

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