Proceedings of the 15th International Conference on Manufacturing Systems – ICMaS
Published by Editura Academiei Române, ISSN 1842-3183

"Politehnica" University of Bucharest, Machine and Manufacturing Systems Department
Bucharest, Romania, 26–27 October, 2006

CUTTING KINEMATICS ANALYSIS IN HYPOCYCLOIDAL TOOTH MILLING OF CYLINDRICAL GEARS

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Abstract: *The hypocycloidal flanks of the cylindrical gear teeth have the flank lines defined by lengthened or shortened hypocycloid and the profile involute. The curved and bulged flanks have some advantages, among them being the increased strength in bending and controlled positioning of the contact imprint. The tooth flank generation has on the basis the application of the technological process of milling using a cutting tool with multi-cutters by rolling with mobile line and continuous division. In the paper, the components of the cutting motion and their speed are analytically and numerically established. The cutting motion is the resultant of the relative motions that are simultaneously performed between the cutting edges of the cutters and tooth flanks. The real speed is determined by three groups of parameters: of motion, constructive, of position. One presents the results of some numerical and experimental applications regarding the tooth processing on a machine of FD Cugir family.*

Key words: *cylindrical gears, milling head, motion parameters, main motion, real cutting speed.*

1. INTRODUCTION

The diversity of gearing types, form sand sizes, precision, productivity, volume and efficiency of production, have determined the existence and utilization of a great number of machine types, processes, and tools for tooth generation. The increasing exigency regarding the kinematic accuracy, smooth running, durability and portant capacity of the gearings [6], make the flank processing to be one of the most complex kinematic surfaces generation modes.

The curved teeth in cylindrical gears represents preoccupations of many researcers presented in papers, doctoral thesis and/or patents, among them being [5]

The hypocycloidal flanks of the gears are defined [3] by two cycloid curves: involute – regarding the profile, and a segment of the closed hypocycloid loop, lengthened or shortened, as flank line. The two curves are generated kinematically through simple motions, strictly correlated by the generation kinematics.

2. KINEMATICS OF HYPOCYCLOIDAL FLANK GENERATION

The flank line D (closed hypocycloid hh_1 – Fig. 1) is generated kinematically in the plane $[\Gamma_D]$ as trajectory of a point belonging to the rolling curve R, which rolls inside the fixed base B. The flank lines D' are generated by transposing by rolling the plane curves D on the revolution surface Σ_c of the workpiece-gear P. The number of circles R corresponds to the ratio $i_H = R_B / R_R$ (R_B is the base radius and R_R – rolling curve radius).

These are positioned equidistantly inside the base B and revolute simultaneously with the angular speed ω_R. To each rolling curve two cutters are attached by means of a port-tool support. The cutters have principal cutting edges (T_s and T_d) and secondary ones.

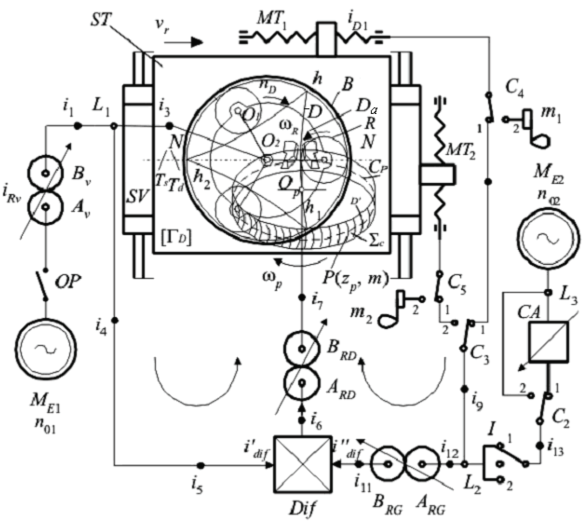

Fig. 1. Generation kinematics of hypocycloidal flank.

The assembly formed by the simple planetary mechanism (the base and the i_H rolling curves) and the corresponding cutter couples attached to each rolling curve constitutes a cutting tool of multi-cutter (i_H =3, 4, 5 or 6) milling head (*CF*) type [5]. The cutting tool can be adapted on the tooth milling machine FD 320A-Cugir [4]. The gearing formed from the base B and the i_H rolling curves R and the revolution couples created in a driving disk D_a with the revolution axis in O_2 and in which the rolling curves axes are materialized corresponding to certain specific accuracy conditions: rigidity, running without noise and vibrations, smooth motion, backlash free and modularization.

The involute profile of the flanks is also generated kinematically [1] as envelope of the successive positions of the two cutting edges T_d and T_s [3], which materialize in the median plane of the plane wheel P and in parallel planes to this one the generating rack profile with the reference line NN tangent to the rolling circle C_P of the gear P. The reproduction of the profile is obtained by continuous division [1], in which case the revolution motions of the tool with the speeds ω_R and ω_B and of workpiece ω_P are continuous. Between the three motions and their angular speeds some relations has to be established, which are imposed by the generation kinematics, one of them being of form: $\omega_R = i_H \cdot \omega_D$.

The rolling speed v_r between the workpiece-gear (surface Σ_c) and the plane $[\Gamma_D]$ expresses the generating speed of the involute profile. The speed v_r is from the technological point of view the speed of the tangential feed motion of the saddle ST that supports the tool.

The generation kinematics of the flanks imposes the existence of the three circular motions and linear continuous motion (v_r) supplied by four generating kinematic chains represented in Fig. 1, namely: main, tangential feed, rolling for generation of the curve D, rolling for the involute profile. The trajectories of the generating motions are supplied by simple kinematic couples: shaft-bearing and saddle-guide respectively existent in the kinematic structure of the teeth processing machine.

The main kinematic chain adjusted through the change gears A_v/B_v ensures the motion of each group of cutters on hypocycloidal trajectories. Thus, the cutting edges T_d and T_s effectuate the main motion with the cutting speed tangent to the shortened hypocycloids, respectively elongated, which are flank directrices.

The feed motion with the speed v_r is obtained at the end of the tangential feed kinematic chain consisting of: $M_{E\,2} - CA - C_2$ (pos. 1) $- L_2 - C_3$ (1) $- MT_1$ and tangential saddle ST.

The rolling kinematic chain for generating the directrix hh_1 in plane $[\Gamma_D]$ is formed by: CF (Da) $- L_1 -$ $Dif.$ $- A_{RD}/B_{RD}$ and workpiece P (joint O_P, n_{Pd}). Hence, a second condition of correlation of the revolution motions is fulfilled, having the form:

$$\varepsilon_{Pd} = \varphi_D \cdot \frac{i_H}{z_p} \text{ [degrees]}, \qquad (1)$$

where ε_{Pd} represents the revolution angle of the workpiece for continuous division and φ_D – revolution angle of the driving disk of the milling head CF.

The rolling kinematic chain for generation of the involute profile is formed by the generating rack (line NN) attached to the plane $[\Gamma_D]$, and consists of: $L_2 - A_{RG}/B_{RG} -$ $- Dif - A_{RD}/B_{RD}$ and $P(n_{Pr})$. The *Botez mechanims* is formed by workpiece-gear and generating rack defined in the mean plane. On the basis of the closing condition of this kinematic chain the following relation is established:

$$\varepsilon_{\mathrm{Pr}} = \frac{\varphi_D \cdot s_T \cdot i_H}{2\pi \cdot z_P \cdot R_r} \text{ [rad]}, \qquad (2)$$

where s_T represents the tangential feed of the saddle ST, in mm/(rev. P); R_r – rolling radius of the workpiece, in mm; z_P – tooth number of the workpiece. From relations (1) and (2) the revolution angle ε_P of the workpiece results, in radians, corresponding to a certain angle φ_D, in radians, expressed by the relation:

$$\varepsilon_P = \varepsilon_{Pd} + \varepsilon_{\mathrm{Pr}} = \varphi_D \frac{i_H}{z_P}\left(1 + \frac{s_T}{2\pi \cdot R_r}\right) \text{ [rad]}, \qquad (3)$$

on which basis the workpiece speed n_P is determined.

The generation motion parameters are considered constant in the cutting process. The rolling motion between workpiece and CF is continuous and motion along the flank profile is discontinous.

3. ANALYSIS OF CUTTING MOTION

The cutting motion is the resultant of the relative motions that are acting simultaneously, between the workpiece P (Fig. 2) and the cutting edges T_d and T_s of the group of cutters S_1 and S_2 respectively, adapted on each rolling curve of the CF.

The motions of the cutters and gear are achieved by the four kinematic chains and satisfy the kinematic simultaneous generation condition of the profile and flank

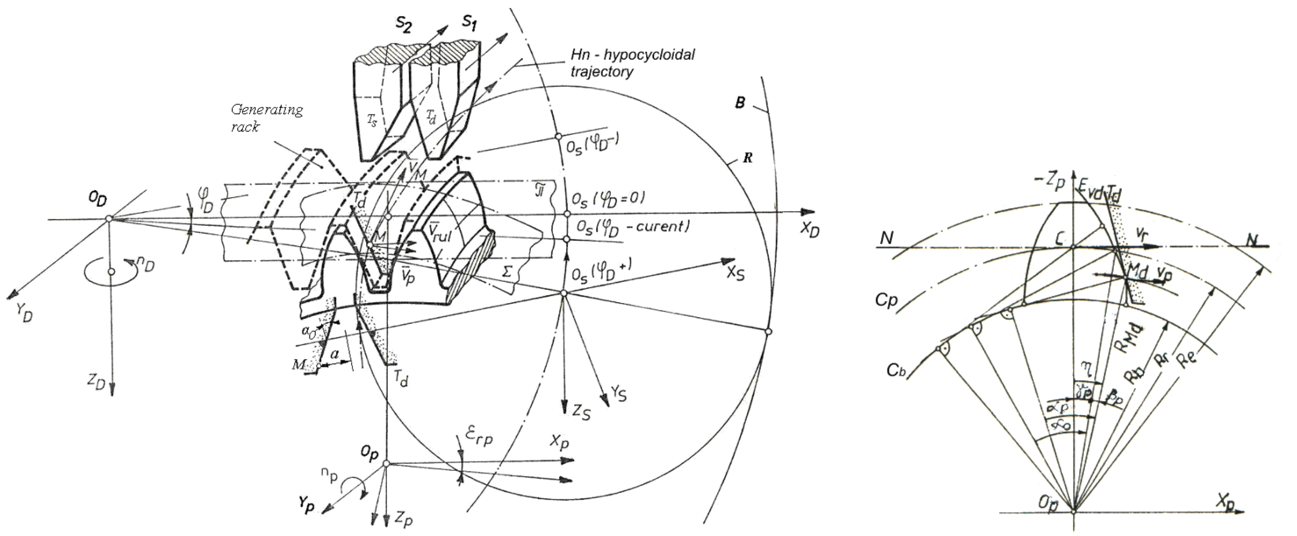

Fig. 2. Relative position tool – workpiece: a – reference frames and motions; b – definition of the current point M_d.

line. The definition of the trajectory of the cutting motion is necessary for the analytical and numerical determination of the forms of flank line and active angles of the cutting edges of the tools.

3.1. Components of the cutting motion

The revolution motion of the centres O_S of each rolling curve in regard with the centre O_D (revolution axis of *CF*) is defined through the angular parameter φ_D (Fig. 2*a*).

In the reference frame of the tool S_S ($O_S X_S Y_S Z_S$) the cutting edges T_s and T_d are defined by the equations:

$$\begin{cases} X_S = -(R_R + k\dfrac{\pi \cdot m}{4}) + k \cdot u \cdot \sin\alpha_0 \\ Y_S = 0 \\ Z_S = -u \cdot \cos\alpha_0, \end{cases} \tag{4}$$

in which $k = \pm 1$ corresponds to the cutting edges T_s and T_d respectively; u – parameter that defines points of the cutting edges on their length; α_0 – the angle of the reference profile.

The parametric equations of the normal hypocycloidal trajectory H_n in the reference frame S_D of *CF* have the form:

$$\begin{cases} X_D = (R_B - R_R)\cos\varphi_D - R_R\cos\dfrac{R_B - R_R}{R_R}\varphi_D \\ Y_D = (R_B - R_R)\sin\varphi_D - R_R\sin\dfrac{R_B - R_R}{R_R}\varphi_D \\ Z_D = 0, \end{cases} \tag{5}$$

where φ_D represents the parameter of the revolution motion of *CF*. The plane $O_S X_S Z_S$, which contains the cutting edges, changes continuously its position in regard with the plane $Y_D = 0$.

A current point $M_{s,d} \in T_{s,d}$ of instantaneous contact between workpiece and tool edge located at the radius $R_{Ms,d} = R_R \pm a$ will generate an elongated respectively shortened hypocycloidal trajectory.

The parameter a defines the change of point $M_{s,d}$ considered on the cutting edge T_s and T_d respectively in regard with O_S.

Main motion speed is determined with relation:

$$v_{as} = v_H = 2\pi \cdot n_D (R_B - R_R) \cdot \cdot\sqrt{1 + \frac{1}{R_R^2} f^2 - \frac{2}{R_R} f \cdot \cos\frac{R_B}{R_R}\varphi_D} \cdot 10^{-3} \tag{6}$$

where $f = \left[k \cdot u \cdot \sin\alpha_0 - (R_R + k \cdot m \cdot \pi/4)\right]$, and n_D – speed of *CF*, in rot/min. The vector of this speed is tangent to the hypocycloidal trajectory that is generated kinematically as trajectory of a point.

From relation (6) it results that the speed v_{as}, in m/min has a variable instantaneous direction (parameter φ_D) and size corresponding to the radius of current point $M_{s,d}$ (Fig. 2*b*). The generated flanks belong to the workpiece defined through the rolling circle C_P. The workpiece is framed in the frame S_P ($X_P O_P Y_P Z_P$) (Fig. 2*a*) named director kinematic system considered fixed for this study.

The components of the vector v_{as} defined in the frame S_P have the following expressions:

$$\begin{aligned} v_{asXp} &= 2\pi \cdot n_D (R_B - R_R) \cdot \left(-\sin\varphi_D - \frac{f}{R_R} \cdot \sin\frac{R_B - R_R}{R_R}\varphi_D \right), \\ v_{asYp} &= 2\pi \cdot n_D (R_B - R_R) \cdot \left(\cos\varphi_D - \frac{f}{R_R} \cdot \cos\frac{R_B - R_R}{R_R}\varphi_D \right), \\ v_{asZp} &= 0. \end{aligned} \tag{7}$$

Tangential speed v_p of the workpiece is constant and has the direction of the revolution motion and the value determined by the parameter n_p. The workpiece is considered fixed, so that the speed v_P will be equal and of contrary direction to the speed of an equivalent motion corresponding to a current point M_d (Fig. 2*b*) that belongs to the cutting edge T_d.

The components of the vector v_p are determined by relations:

$$v_{pXp} = (-v_p) \cdot a_{vp}, \quad v_{pYp} = (-v_p) \cdot b_{vp}, \quad v_{pZp} = (-v_p) \cdot c_{vp} \tag{8}$$

where $v_p = 2\pi \cdot n_p \cdot R_{Md}$ and $a_{vp} = -\cos\eta$, $b_{vp} = 0$, and $c_{vp} = -\sin\eta$ represent the director cosines of the vector v_p; $\eta = (\gamma_p + \beta_p)$ – angular parameter that defines the instantaneous position of the point M_d with respect to the axis Z_P, $\gamma_p = 90°/z_p$, $\beta_p = \varepsilon_p + \alpha_0 \pm \alpha_p$ (as $R_{Md} < R_r$, respectively $R_{Md} > R_r$), z_p – tooth number of the workpiece, and $\alpha_p = \arccos\left[(R_r / R_{Md})\cos\alpha_0\right]$.

The angular parameter ε_P expresses the angular position of the workpiece, corresponding to the current point M_d. It has a dependent relation with the parameter u, namely: $u = 1/k \cdot \left[R_R \varepsilon_p + k \cdot m \cdot \pi/4\right] \cdot \sin\alpha_0$.

The point M_d radius is calculated with the relation:

$$R_{Md} = \left(X_p^2 + Z_p^2\right)^{1/2}, \tag{9}$$

where X_P and Z_P are defined in [3] and represents the coordinates of the point beleonging to the generated involute profile, corresponding to the rolling parameter ε_P. For simplifying the calculations, the mean palne $[Y_P]$ is considered, for which it corresponds $\varphi_D = 0$. After necessary computation, it results in mm:

$$R_{Md} = \sqrt{\begin{aligned} &R_r^2\left(1+\varepsilon_p^2\right) + 2 \cdot k \cdot R_r \cdot \varepsilon_p \frac{m\pi}{4} + \left(k \cdot u \frac{m \cdot \pi}{4}\right)^2 + \\ &+\left(R_r \varepsilon_p + k \cdot \frac{m \cdot \pi}{4}\right) \cdot \left(\frac{2R_r \cdot ctg\alpha_0}{K} - R_r \cdot \varepsilon_p - k \cdot \frac{m\pi}{4}\right)\sin^2\alpha_0 \end{aligned}}. \tag{10}$$

Component of the rolling motion with the speed v_r is along the line *NN* and has the direction of the revolution motion of the workpiece for rolling with speed n_{pr} (Fig. 2). The projections of the vector v_r on the axis of the frame S_P have the expressions:

$$v_{rXp} = v_r \cdot \cos\varepsilon_{\Pr} = 2\pi \cdot n_{pr} \cdot R_r \cdot \cos\varepsilon_{\Pr},$$
$$v_{rYp} = 0,\ v_{rZp} = 2\pi \cdot n_{pr} \cdot R_r \cdot \sin\varepsilon_{\Pr}, \tag{11}$$

where $n_{\Pr} = (s_T \cdot i_H / 2\pi \cdot z_P \cdot R_r) \cdot n_D$, in rot/min; s_T – tangential feed ($s_T = s_t \cdot z_P / i_H$), in mm/(rev. P); s_t – tangential feed saddle *ST*, in mm/(rev. *CF*); ε_{rp} – angular parameter of the involute profile generation.

Real cutting speed v_{ras} is determined [2] as sum of the simultaneous motion speeds, namely:

$$\vec{v}_{ras} = \vec{v}_{as} + \vec{v}_P + \vec{v}_r, \tag{12}$$

expressed by the relations (6, 8, and 11). The projections of the vector $\vec{v}_{ras}$ on the axes of the frame S_P are:

$$(v_{ras})_{Xp} = 2n_D \cdot \pi \left\{ -(R_B - R_R)\sin\varphi_D - \frac{R_B - R_R}{R_R} f \cdot \right.$$
$$\left. \cdot \sin\frac{R_B - R_R}{R_R}\varphi_D + \frac{i_H}{z_p}\left[R_{Md} \cdot \cos\eta + \frac{s_T}{2\pi} \cdot \cos\varepsilon_{\Pr} \right] \right\},$$
$$(v_{ras})_{Yp} = 2n_D \cdot \pi \left[(R_B - R_R) \cdot \cos\varphi_D - \frac{R_B - R_R}{R_R} \cdot \right. \tag{13}$$
$$\left. \cdot f \cdot \cos\frac{R_B - R_R}{R_R}\varphi_D \right],$$
$$(v_{ras})_{Zp} = 2n_D \cdot \pi \cdot \frac{i_H}{z_p}\left[R_{Md} \cdot \sin\eta + \frac{s_T}{2\pi} \cdot \sin\varepsilon_{\Pr} \right].$$

The real cutting speed vector is variable concernig the value and direction. The determination of its components is necessary for defining the active geometry of the cuttres edges and for analysis of the cutting process. On the basis of relations (13) the modulus of the vector $\vec{v}_{ras}$ is determined in m/min:

$$|\vec{v}_{ras}| = \sqrt{(v_{ras})^2_{Xp} + (v_{ras})^2_{Yp} + (v_{ras})^2_{Zp}} \cdot 10^{-3}. \tag{14}$$

The direction of vector $\vec{v}_{ras}$ in every point of the hypocycloidal trajectory on the length and height of generated flanks with respect to the axes of the frame S_P is deteremined by calculation of its director cosines [2].

Application. For numerical expression of the above relations it is considered the case of generation of hypocycloidal flanks by rolling, by applying the procedure presented in Fig. 3, for a cylindrical gear (workpiece) having $z_P = 52$ and $m = 2.5$ mm; milling head with 4 groups of cutters and geometric characteristics $R_B = 240$ mm and R_R = 60 mm, speed n_D = 29.2 rpm; feed s_T = 0.83 mm/(rev. *P*); u = 3.32; φ_D = 2.0 deg; k = –1 (for T_d) and α_0 = 20 deg.

On the basis of the above relations one determines: $f = -59.173$ and $f^2 = 3501.73$, $v_{aş} = 65.2$ m/min; R_{Md} = = 68.93 mm, components $(v_{ras})_{X_P} = 3.2$ m/min; $(v_{ras})_{Y_P}$ = = 65.185 m/min and $(v_{ras})_{Z_P}$ = 0.037 m/min. According to relation (13) it results $|\vec{v}_{ras}| = 65.28$ m/min. We ascertain that the difference between the values of the speeds v_{ras} and v_{as} is very small, this being the reason that in studying the cutting regime parameters one can consider only the parameter v_{as}.

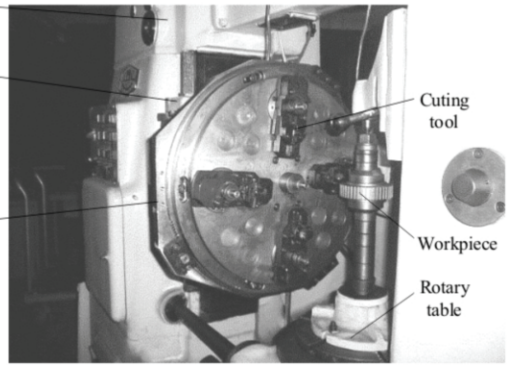

Fig. 3. Milling head with 4 groups of cutters.

4. CONCLUSIONS

The flank line and profile of the hypocycloidal curved teeth are generated kinematically by rolling with mobile line and continuous division. For processing a milling head *CF* is used, which has more couples of cutters. This tool is adapted on the tooth processing machine of FD Cugir type. The main motion has the speed v_{as} with the biggest value of the components of the real cutting speed. The size of the cutting speed is determined by the ratio i_H and the driving disk speed. Thus, for the considered numerical values, it was established the range: $v_{aş}$ = = 56.95…72.1 m/min. Knowing the projection values of the vector v_{as} in the workpiece frame and its modulus, one can determine the parameters that define the active geometry of the cutting edge of each cutter and the orientation of their faces and cutting edges in working position on the tooth processing machine.

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