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TECHNICAL SUPPORT DESIGN OF A VIRTUAL MACHINE

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Abstract: *Parts surfaces generation is analyzed from the geometric point of view more as body surfaces and less as movements in space of the cutting tools edges. In this paper we present the technical support of surface generation on machine tools, which consist in elaborating and applying of algorithms that defines the virtual machine tool.*

Key words: *virtual machine, complex surfaces, mathematic model, cinematic methods, algorithm, support design.*

1. INTRODUCTION

The method to generate complex surfaces is based on a mathematical model with the particularity that it is used in order to determine the relationship machine tool – cutting tool – generated surface. The mathematic model solves the generation of the surfaces by space equations, without considering the perpendicularity condition of the relative speed of the surfaces with the normal vector to the contact surface in the considered point [3]. Towards the known generating methods [1, 2, 4], applying new model [5] has significant advantages: therefore: the generated surface is the real one, resulted as of the interaction between the cutting tool and the machine tool.

2. ACTUAL STATUS

Surface generation was approached using two methods.

2.1. Defining the mutual unveiling surfaces

Generating surfaces of mutual unveiling is based upon the normality condition of the relative speed vector with the commune normal to the two surfaces in the mutual contact point [2, 4]. If there is a point M [4], which scans a curve, a position vector (for point *M*) in a three-dimensional space may be represented:

- in vector form,

$$r_m = \overline{O_m M} = x_m \times i_m + y_m \times j_m + z_m \times k_m, \qquad (1)$$

- in column matrix,

$$\mathbf{r}_m = \begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix} = \begin{bmatrix} x_m & y_m & z_m \end{bmatrix}^{\mathrm{T}}, \qquad (2)$$

- in homogeneous coordinates,

$$\mathbf{r}_m = \begin{bmatrix} x_m \\ y_m \\ z_m \\ 1 \end{bmatrix} = \begin{bmatrix} x_m & y_m & z_m & 1 \end{bmatrix}^{\mathrm{T}}. \qquad (3)$$

The subscript *m* indicates that the position vector is represented in coordinate system S_m (x_m, y_m, z_m).

The same point *M* can be determined in coordinate system S_n (x_n, y_n, z_n) by the position vector,

$$\mathbf{r}_n = \begin{bmatrix} x_n & y_n & z_n & 1 \end{bmatrix}^{\mathrm{T}}, \qquad (4)$$

with the following matrix equation,

$$\mathbf{r}_n = \mathbf{M}_{mn} \times \mathbf{r}_m, \qquad (5)$$

where $\mathbf{M}_{nm}$ [4] is the transforming matrices between the coordinating systems.

To perform successive coordinate transformation, we need only to follow the product rule of matrix algebra. For instance, the matrix equation

$$\mathbf{r}_p = \mathbf{M}_{p(p-1)} \times \mathbf{M}_{(p-1)(p-2)} \times \ldots \times \mathbf{M}_{32} \times \mathbf{M}_{21} \times \mathbf{r}_1 \qquad (6)$$

represents successive coordinate transformation from S_1 to S_2, from S_2 to S_3, …, S_{p-1} to S_p.

Surfaces are described by displacing point *M* on the generating and guiding curves. In order to express the unveiling surface cinematic method [2] is used, cinematic method which used the written equation under the form:

$$\overline{n} \times \overline{v}_{21} = 0. \qquad (7)$$

In respect to this formula, as the surfaces Σ_1 and Σ_2 (Fig. 1) to be mutually unveiled, it is necessary that relative

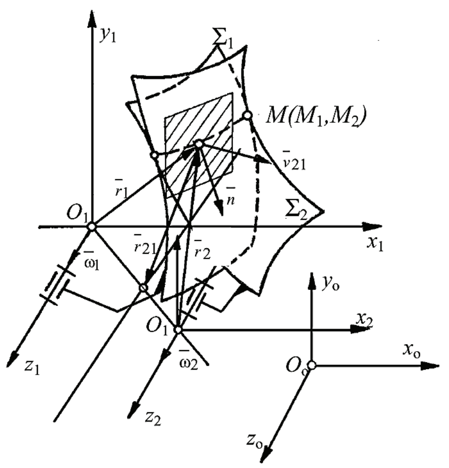

Fig. 1. Surface unveiled using the cinematic method.

speed $\overline{v}_{21}$ of point M_2 towards M_1 be placed in plan Γ tangent in point P and perpendicular on the commune normal $\overline{n}$. If absolute speeds of points M_1 and M_2 are noted with $\overline{v}_1$ and $\overline{v}_2$ it results:

$$\overline{v}_{21} = \overline{v}_2 - \overline{v}_1 = \overline{\omega}_2 \times \overline{r}_2 - \overline{r}_1 \times \overline{\omega}_1, \tag{8}$$

where $\overline{r}_1$ and $\overline{r}_2$ are the position vectors of point M_1, respective M_2, in coordinates systems S_1 and S_2.

Relation (7) is written under the form:

$$\left(v_{21}\right)_x \cdot n_x + \left(v_{21}\right)_y \cdot n_y + \left(v_{21}\right)_z \cdot n_z = 0. \tag{9}$$

where $\left(v_{21}\right)_x$, $\left(v_{21}\right)_y$, $\left(v_{21}\right)_z$ and n_x, n_y, n_z represent components of vectors $\overline{v}_{21}$ respectively $\overline{n}$ towards system S_1. For the calculation of the components of vector $\overline{n}$ the following relation is applied:

$$\overline{n} = \frac{\delta \overline{r}}{\delta u} \times \frac{\delta \overline{r}}{\delta \varphi} = n\left(u, \varphi\right), \tag{10}$$

where $\overline{r}$ represents the vector ray of a point on the generating surface defined by parameters u and φ.

Developing relationship (9), the following equation of surface results:

$$F\left(u, \varphi, \varepsilon\right) = 0, \tag{11}$$

where ε represents the generation parameter.

2.2. The machine tools generation

The theory of generating surfaces on [1] considers the movement in space of *the generating curve* (G) (Fig. 2.a). Sometimes, curve (G) is created by the movement of the *elementary curve* (G_e) (Fig. 2.b). Speeds $\vec{v}$ and $\vec{\omega}$ of point P – which belongs to the surface defined by curve (G_e) are variables as size and direction and depend on the position of the tangent position of point T. Alike, curve (D) is generated by (D_e). In the most general case, curves (G) or (D) and, respectively, curves (G_e) and (D_e) are special, and the generating condition is that tests like (G) and (G_e) and, namely, (D) and (D_e) be tangent between themselves.

Regarding the application of this method, here are some notes:

- mathematical definition of a generated surface is very commonly performed when working only with the generating curve (G) and with the guiding one (D), without considering elementary curves (G_e) and respectively, (D_e) which intercede in the most general case;

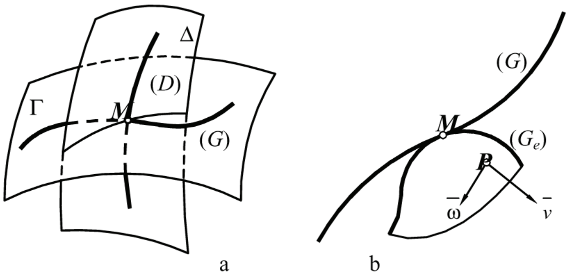

Fig. 2. Scheme of generating:
a – method on machine tools; b – curves (G).

- by using this method, it is relatively easily covered the case when generating curve (G) results as unveiling elementary generating curve (G_e);
- there are cases when the real generated surface is strongly different from the ideal (desired) surface. Such a case is the one of generating cylindrical wheels, and, to a certain extend, to those of the conic wheels with curve teeth;
- the general theory is difficultly applied in the case when the movements of point M and/or of elementary generator (G_e) are not uniform.

In conclusion, we can say that using the *classical* method of generating on machine tools is limited. Starting from these observations [5], a *method* with specific hypothesis developed: considering as generating curve (G) only the cutting edges of the cutting tool (*classically* this connection is performed in two specific situations); using all the real movements of the cinematic couplets of the machine tool (*classically* not all these movements are considered); subsequently, only equations type space are used, without the implication of the normality condition of relative speed and of the normal in the contact point, specific to the mutual unveiling surfaces. The major advantage is given by the possibility to generalize the generating module of the machine tool by: simultaneous existence of curves (G_e), and (D_e), with movements in space, with or without time variation.

3. SUPORT DESIGN

Point there is considered the general theory of generating surfaces on machine tools, with the *note* that the elementary generation is defined by the cutting edge of the tool, and the movements of curves (G_e) and, respectively (D_e) are supplied by the movements of the machine tool mechanisms. Constructively, the mechanisms that ensure the movements, hereinafter referred to as simple movements, are of two types, namely: slide-guide ways and shaft bearing. For each of these movements a coordinates system is attached, with the property that one of the axis of the coordinates system be the rotation axis or the translation axis, just as the simple movement is of rotation or of translation. Mathematically [3, 5] to determine the generated surface is used the coordinate transformation. Due to the existence of the two types of mechanisms [5] in the structure of the machine tool, the roto-translation transformations are split into translation and rotation. Another *note* was that in the machine structure there can be defined three basic coordinates systems (Fig. 3), namely: a fix one $S(x, y, z)$ (belongs to the parts of the machine which do not move), one linked to the cutting edge $S_s(x_s, y_s, z_s)$ and the third one connected to piece $S_p(x_p, y_p, z_p)$. Also, there are considered the coordinates systems defined between the fix one and the one of the piece as a branch of the piece. The exemplification is performed for the hobbing machine with the hob. It is considered that a cylindrical toothed wheel is processed, with bent teeth, using a hob. It is considered that the hob has a setting movement with the angle of its propeller and one of bringing into position the tangent to the teeth

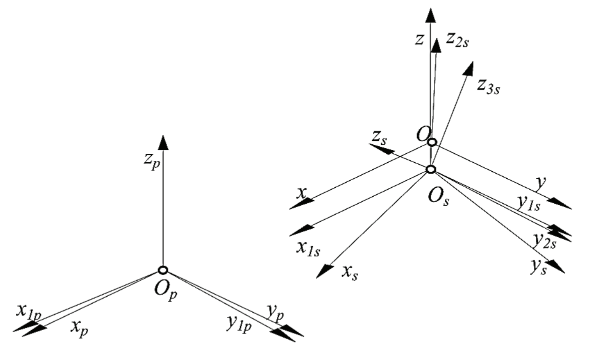

Fig. 3. Fixed coordinate systems $S(x, y, z)$, tool coordinate systems $S_s(x_s, y_s, z_s)$ and part coordinate systems $S_p(x_p, y_p, z_p)$.

direction of the piece. In the branch of the tool the following systems are defined: fix $S_s(x_s, y_s, z_s)$, with translation movement for axial advance $S_{1s}(x_{1s}, y_{1s}, z_s)$ (generation), with rotation movement of rotation with the angle of the hob $S_{2s}(x_{1s}, y_{2s}, z_{2s})$ (setting), with rotation movement with bending angle of the teeth of the piece $S_{3s}(x_{1s}, y, z_{3s})$ (setting) and for the rotation movement of the main tree $S_s(x_s, y_s, z_s)$ (generation). In the branch of the piece the following systems are defined: $S_s(x_s, y_s, z_s)$ fix, $S_{1p}(x_{1p}, y_{1p}, z_p)$ with translation movement around axis Ox with a value depending on the diameters of the piece and of the tool (setting) and $S_p(x_p, y_p, z_p)$ with rotation movement (generation) with an angle depending on the main movement or on the movement of advance (for toothed wheels with straight teeth) or depending on and for the toothed wheels with bent teeth.

It is considered the case when the cutting tool has a number of cutting edges. The cutting edges are ordinated, noted (G_{ek}) for k rank and defined in the coordinates system $S_s(x_s, y_s, z_s)$ by equations:

$$\begin{aligned} x &= x_k(m) \\ y &= y_k(m) \\ z &= z_k(m). \end{aligned} \tag{12}$$

Point $M(x_m, y_m, z_m)$ of the cutting edge (G_{ek}) describes elementary director (D_{el}) of l rank and is defined in the coordinates system $S_{tr}(x_{tr}, y_{tr}, z_{tr})$ generalized by equations:

$$\begin{aligned} x &= x(t, x_m, y_m, z_m) \\ y &= y(t, x_m, y_m, z_m) \\ z &= z(t, x_m, y_m, z_m), \end{aligned} \tag{13}$$

where t represents time as parameter.

The mathematical solving is given by the use of the rotation or translation transformers applied to equations (13) just as the movements of the coordinating systems of the two branches, of the tool and of the piece, defined for the trajectory performance, (D_{el}) are of rotation or of translation.

Transformation of the coordinates system S_i into the coordinates system S_{i-1} is expressed by:

$$\mathbf{r}_{i-1} = \mathbf{M}_{i,i-1} \times \mathbf{r}_i. \tag{14}$$

If in the branch of the tool there are n coordinates systems the transformation from the branch of the too, from the system of the tool $S_s(x_s, y_s, z_s)$ to the fix one $S(x, y, z)$, is expressed by:

$$\mathbf{r} = \mathbf{M}_1 \times \mathbf{M}_2 \times \ldots \times \mathbf{M}_n \times \mathbf{r}_s, \tag{15}$$

and if in the branch of the piece there are m coordinates systems, the transformation from the branch of the tool, from fix system $S(x, y, z)$ to the one of piece $S_p(x_p, y_p, z_p)$, is expressed by:

$$\mathbf{r}_p = \mathbf{M}'_m \times \mathbf{M}'_{m-1} \times \mathbf{M}'_{m-2} \times \ldots \times \mathbf{M}'_1 \times \mathbf{r}. \tag{16}$$

Finally equations (13) which trajectory (T_{ek}) of a point $M(x_m, y_m, z_m)$ of cutting edge (G_{ek}) become after the appliance of $m+n$ rotation or translation transformation:

$$\mathbf{r}_p = \left(\mathbf{M}'_m \times \ldots \times \mathbf{M}'_1\right) \times \left(\mathbf{M}_1 \times \ldots \times \mathbf{M}_n\right) \times \mathbf{r}_p. \tag{17}$$

The generated surface is defined by topographic points determined by crossing with conveniently chosen surfaces, hereinafter named topographic surfaces, among which most used there are: the plan, the cylinder and the sphere.

4. ALGORITHMS

The elaboration of a computer program to generate a surface by a cutting edge (G_{ek}) respectively its crossing with a topographic surface is intended. The difficulty in setting an algorithm consisted in the way the problem was treated so far. To each case of generation the transformation matrixes were written, and then during the program the functions are written and with their help the generation of the surface is solved. A change no matter how insignificant of the entry data needs a readjusting of the program for the respective case. The following were considered to the generalization of the solving:

- a roto-translation transformation may be replaced by two transformations, one of translation and one of rotation, if a supplementary coordinates system is introduced;
- the description of the types of transformation matrices (translation or rotation) by files would lead to the generalization of the cases when the program could be applied;
- the modularization of the program leads to an easier treatment of the various generation cases and the possibility of treating various post-generation aspects as, for instance, those of contact for the conjugated surfaces.

A file was defined for the branch of the too and another one for the one of the piece. They have the form:

$$\begin{aligned} &n-1 \\ &x_1 \;\; y_1 \;\; z_1 \;\; (\text{transform \& ax})_1 \;\; (\text{funct})_1 \\ &x_2 \;\; y_2 \;\; z_2 \;\; (\text{transform \& ax})_2 \;\; (\text{funct})_2\,, \\ &\ldots \\ &x_n \;\; y_n \;\; z_n \;\; (\text{transform \& ax})_n \;\; (\text{funct})_n \end{aligned} \tag{18}$$

where: n is the number of coordinates systems from the respective branch (tool or piece); x_n y_n z_n coordinates of the origin of the new coordinates system in the old coordinates system; *transform&axe type* is made up of two letters: the first letter represents the transformation way (translation t, rotation r) and the second letter the axis meant to the transformation; *function* time movement function of the second coordinates system towards the first one.

The files for generating a cylindrical gear with 13 teeth with 2.5 module and 15 deg teeth bending using a hobbing machine, for the branch of the tool, are:

$$\begin{array}{llllc} 4 & & & & \\ 0 & 0 & 0 & tz & 3.2894737\times 0.0001\times a0 \\ 0 & 0 & 0 & rx & -2.2110940 \\ 0 & 0 & 0 & rx & -15.0002663 \\ 0 & 0 & 0 & ry & a0. \end{array}$$

Applying the algorithm lead to the elaboration of a computer program in which the first window (Fig. 4) shows the block sketches of the machine with the fix coordinates system and with the menu buttons, respectively, with the sub menu buttons generated by pressing the first working menu button.

There are introduced data on: coordinates systems; cutting tool; piece and the topographic plan. As a result there is presented (Fig. 5) the generation of a void of the wheel in the bottom connection area.

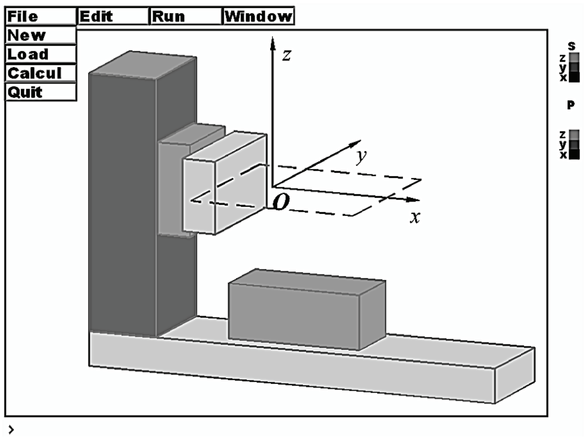

Fig. 4. Program interface.

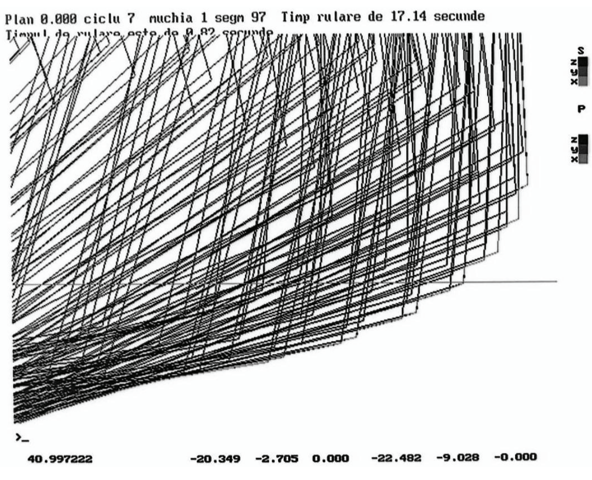

Fig. 5. Generating the void of the wheel.

5. CONCLUSIONS

The method is based on coordinates transformations between different systems, without considering the normality condition between the relative speed vector and the commune normal to the two surfaces (of the tool, respectively of the piece) in the mutual contact point.

The essential advantages of this method are:

- the generated surface is the real one, resulted by the interaction between the cutting tool and the machine tool;
- generation with the tool with multiple cutting edges;
- generation is also produced in the presence and/or cinematic errors;
- determining a form of the cutting edge in order to obtain certain real surfaces with form correction;
- reduction of time and costs of the settings of the machine tool for the making of the desired surfaces;
- determining the contact spot between the real conjugated surfaces, with the eventual change of the cutting edges;
- changing the form of the contact spot under the influence of the action of the torsion moment.

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