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APPLICATIONS OF REPRESENTATION BY POLES AS A WAY TO APPROXIMATE WRAPPING CURVES OF PROFILES ASSOCIATED TO ROLLING CENTRODS

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Abstract: *In this paper, a development of curves representation by poles, when representing plain profiles of wrapping surfaces associated to rolling centrods, is presented. The cases of generating by using pinion cutters or rotating cutters are considered. Numerical examples of method application, in some concrete situation,s are also included.*

Key words: *representation by poles, wrapping profiles, approximation.*

1. STRAIGHT-LINE SEGMENT PROFILES GENERATING BY USING A PINION CUTTER

The case when the couple of rolling centrods is constituted by two circles, C_1 and C_2, having radius R_{rp}, respective R_{rs}, is shown in Fig. 1.

The following reference systems were considered:

– xyz is a fix reference system, having its z axis as C_1 centrod rotation axis;

– $x_0y_0z_0$ – fix system, having its z_0 axis as C_2 centrod (pinion cutter) rotation axis;

– XYZ – mobile system, attached to the generated profile – Σ;

– $\xi\eta\zeta$ – mobile system, attached to the interior pinion cutter.

Now, specific generating kinematics can be written:

– C_1 centrod rotation,

$$x = \omega_3^{\mathrm{T}}(\varphi_1)\cdot X; \tag{1}$$

– C_2 centrod rotation,

$$x_0 = \omega_3^{\mathrm{T}}(\varphi_2)\cdot \xi, \tag{2}$$

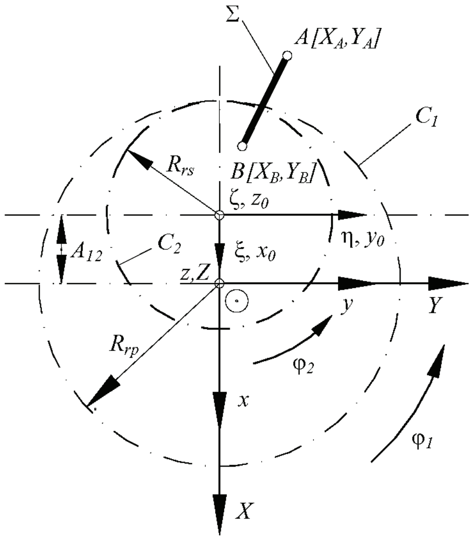

Fig. 1. Circular centrods, interior tangent.

which under rolling condition between the two centrods

$$R_{rp}\cdot\varphi_1 = R_{rs}\cdot\varphi_2. \tag{3}$$

And having in view the relative position between the two fix systems:

$$x_0 = x - A;\; A = \begin{Vmatrix} -A_{12} \\ 0 \end{Vmatrix};\; A_{12} = R_{rp} - R_{rs}, \tag{4}$$

leads to the following relative motion (between mobile reference systems) equation,

$$\xi = \omega_3(\varphi_2)\left[\omega_3^{\mathrm{T}}(\varphi_1)\cdot X - A\right]. \tag{5}$$

If now we accept straight-line segment $\overline{AB}$ to by represented by poles as:

$$\Sigma \begin{vmatrix} P_X = \lambda\cdot X_A + (1-\lambda)X_B; \\ P_Y = \lambda\cdot Y_A + (1-\lambda)Y_B, \end{vmatrix} \tag{6}$$

then Σ family of profiles, generated into $\xi\eta$ system has the form

$$\begin{Vmatrix} \xi \\ \eta \end{Vmatrix} = \begin{Vmatrix} \cos\varphi_2 & \sin\varphi_2 \\ -\sin\varphi_2 & \cos\varphi_2 \end{Vmatrix}\cdot \left[\begin{Vmatrix} \cos\varphi_1 & -\sin\varphi_1 \\ \sin\varphi_1 & \cos\varphi_1 \end{Vmatrix}\cdot\begin{Vmatrix} P_X \\ P_Y \end{Vmatrix} - \begin{Vmatrix} -A_{12} \\ 0 \end{Vmatrix}\right], \tag{7}$$

or, after development and calculus:

$$(\Sigma)_{\varphi_1} \begin{vmatrix} \xi = P_X\cos(\varphi_1-\varphi_2) - P_Y\sin(\varphi_1-\varphi_2) + A_{12}\cdot\cos\varphi_2 \\ \eta = P_X\sin(\varphi_1-\varphi_2) + P_Y\cos(\varphi_1-\varphi_2) - A_{12}\cdot\sin\varphi_2. \end{vmatrix} \tag{8}$$

Transmission ratio is defined as

$$i = \frac{\varphi_2}{\varphi_1}, \tag{9}$$

see also rolling condition (3).

Thus, we finally obtain the generic form:

$$(\Sigma)_{\varphi_1} \begin{vmatrix} \xi = P_X\cos(1-i)\varphi_1 - P_Y\sin(1-i)\varphi_1 + A_{12}\cdot\cos i\varphi_1; \\ \eta = P_X\sin(1-i)\varphi_1 + P_Y\cos(1-i)\varphi_1 - A_{12}\cdot\sin i\varphi_1. \end{vmatrix} \tag{10}$$

Note: When generating with pinion-tools, exterior *gearing* case, Fig. 2, referred to the same systems as

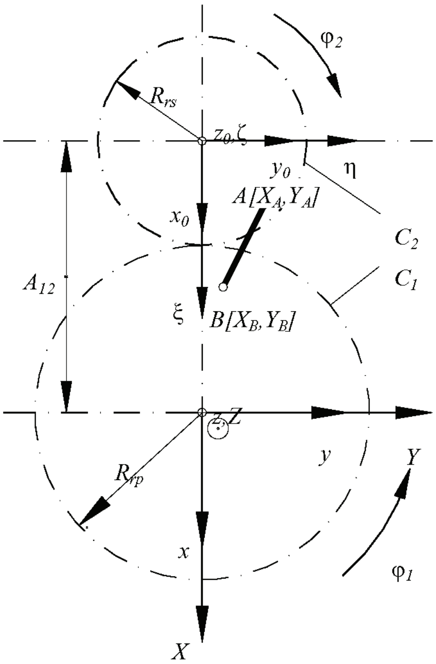

Fig. 2. Circular centrods, exterior tangent.

defined above, the family (10) of profiles equations becomes:

$$(\Sigma)_{\varphi_1}\left|\begin{array}{l}\xi = P_X\cos(1+i)\varphi_1 - P_Y\sin(1+i)\varphi_1 + A_{12}\cdot\cos i\varphi_1;\\ \eta = P_X\sin(1+i)\varphi_1 + P_Y\cos(1+i)\varphi_1 + A_{12}\cdot\sin i\varphi_1.\end{array}\right. \quad (11)$$

The envelope of (10) or (11) profiles family can be found by associating to the equations ensemble the enveloping condition, which in this case has the form

$$\lambda = \frac{R_{rp}\left[(Y_A - Y_B)\sin\varphi_1 - (X_A - X_B)\cos\varphi_1\right] - C^2}{L^2}, \quad (12)$$

where

$$C^2 = X_A\cdot X_B + Y_A\cdot Y_B - \left(X_B{}^2 + Y_B{}^2\right) \quad (13)$$

and

$$L = \sqrt{(X_A - X_B)^2 + (Y_A - Y_B)^2}. \quad (14)$$

Thus, when $\lambda\in[0,1]$, the profile conjugated to $\overline{AB}$ segment is a vector of (14) type, [1], that can be approximated by a second degree curve, expressed by poles, in accordance to the algorithm given by (1), [1], but having in view (10) or (11) profiles family (depending on the considered gearing case) and (12) specific enveloping condition also.

2. STRAIGHT-LINE SEGMENT PROFILES GENERATING BY USING A ROTATING CUTTER

The following reference systems are defined, according to Fig. 3:

- *xyz* is a fix system, having its z axis as tool rotation axis;
- $\xi\eta\zeta$ – mobile system, attached to the rotating cutter;
- *XYZ* – mobile system, attached to worked piece axial section.

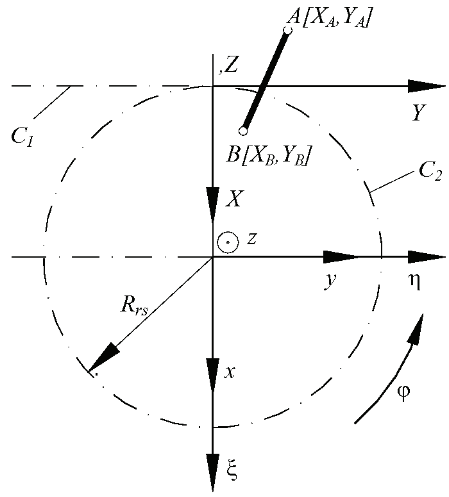

Fig. 3. Centrods when generating with rotating cutter.

Profiles family of the straight-line segment, component of worked piece axial profile, represented by poles as in relation (6), has the equations:

$$\left|\begin{array}{l}\xi = (P_X - R_{rs})\cos\varphi + (P_y - R_{rs})\sin\varphi;\\ \eta = -(P_X - R_{rs})\sin\varphi + (P_y - R_{rs})\cos\varphi.\end{array}\right. \quad (15)$$

Enveloping condition specific form is, in this case,

$$\varphi = \frac{P_X\cdot P'_{X_\lambda} + P_Y\cdot P'_{Y_\lambda}}{R_{rs}\cdot P'_{Y_\lambda}}, \quad (16)$$

where the derivatives, calculated from (6), are:

$$P'_{X_\lambda} = X_A - X_B;\; P'_{Y_\lambda} = Y_A - Y_B. \quad (17)$$

The wrapping profile can be "approximated" by specific poles, which, in the case of a second degree function, takes the expression from (37) [1], obviously by also respecting specific conditions (15) and (16).

3. APPLICATIONS

We further present some applications of approximating by poles wrapping curves associated to profiles frequently used in technique.

3.1. Rack-Bar Tool to Generate Exterior Slots

The case of a shaft having exterior 8 × 52 × 60 mm slots was considered (Fig. 4).

An algorithm and a special designed soft were conceived and used in order to obtain the theoretical and the approximated profiles, each one through a file of points co-ordinates.

The input data were: X_A = –30 mm; X_B = –26 mm; Y_A = Y_B = 5 mm; R_{rp} = 30 mm. The value of λ from relation (6) [1] was divided into a number of 30 values between 0 and 1.

The coordinates of points giving the two profiles, referred to $\xi\eta$ system, are presented in Table 1. As it can be observed, in the rows written with bold characters, the co-ordinates of points giving the poles are identical on both profiles.

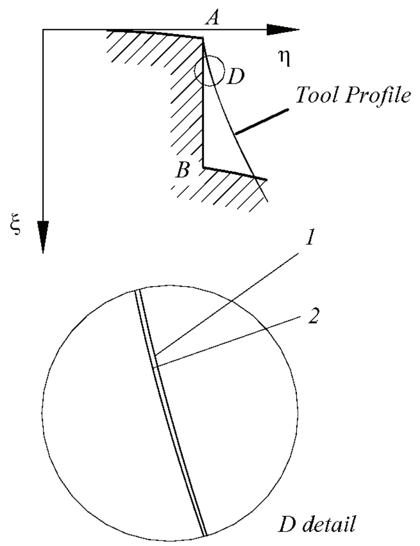

Fig. 4. Exterior slots generated by rack-bar tool case: 1 – theoretical profile; 2 – profile approximated by poles.

Table 1

Theoretical and Approximated Rack-Bar Tool Profiles

Theoretical Profile		Approximated Profile	
ξ [mm]	η [mm]	ξ [mm]	η [mm]
–0.0346	**4.9999**	**–0.0346**	**4.9999**
–0.2053	4.9945	0.0811	5.0180
–0.1344	5.0029	0.2004	5.0395
–0.0191	5.0202	0.3231	5.0644
0.1187	5.0449	0.4494	5.0928
0.2704	5.0757	0.5793	5.1246
0.4308	5.1120	0.7126	5.1598
…	…	…	…
1.8813	5.5588	1.9067	5.5647
2.0718	**5.6307**	**2.0718**	**5.6307**
2.2634	5.7057	2.2405	5.7001
…	…	…	…
4.1998	6.6056	4.1214	6.5829
4.3933	6.7088	4.3289	6.6900
4.5869	6.8145	4.5399	6.8006
4.7798	6.9221	4.7545	6.9145
4.9726	**7.0319**	**4.9726**	**7.0319**

By representing the two profiles in the same picture and by zooming and using AutoCad tools, the maximum deviation between the two curves is about 0.02 mm, which is complete acceptable for general industrial use.

3.2. Rack-Bar Tool to Generate Arc of Circle Exterior Elementary Profiles

The case of an arc of circle exterior profile is now considered (Fig. 5).

The input data were: $X_O = -50$ mm; $Y_O = 0$; $r = 10$ mm; $\hat{O} = 30°$; $R_{rp} = 60$ mm. The value of λ from relation (24) was divided into a number of 30 values between 0 and 1. The upper mentioned soft was adapted to the specific of circular elementary profile case. The coordinates of points giving the two profiles, referred to ξη system, are presented in Table 2. In this case, the precision of approximation by poles is even better, the maximum deviation being under 0.01 mm.

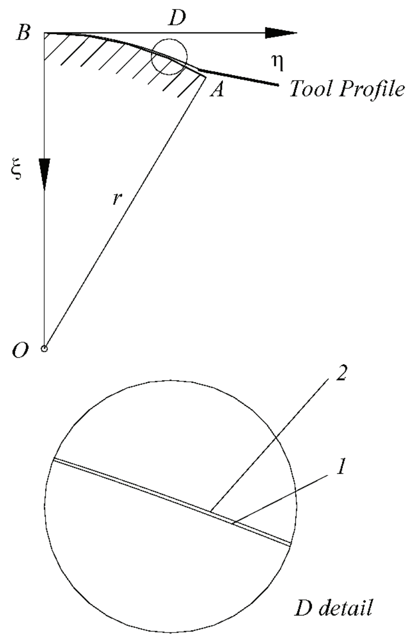

Fig. 5. Arc of circle exterior profile case: 1 – theoretical profile; 2 – profile approximated by poles.

Table 2

Theoretical and Approximated Rack-Bar Tool Profiles

Theoretical Profile		Approximated Profile	
ξ [mm]	η [mm]	ξ [mm]	η [mm]
–8.5842	**5.1208**	**–8.5842**	**5.1208**
–8.6845	4.9274	–8.6797	4.9269
–8.7802	4.7355	–8.7719	4.7346
–8.8718	4.5448	–8.8608	4.5438
–8.9591	4.3557	–8.9463	4.3546
–9.0423	4.1680	–9.0285	4.1670
–9.1216	3.9817	–9.1074	3.9810
…	…	…	…
–9.6205	2.5495	–9.6182	2.5496
–9.6671	**2.3778**	**–9.6671**	**2.3778**
–9.7106	2.2074	9.7126	2.2075
…	…	…	…
–9.9751	0.5936	–9.9846	0.5918
–9.9855	0.4403	–9.9934	0.4389
–9.9931	0.2888	–9.9989	0.2875
–9.9979	0.1386	–10.0011	0.1378
–10.0000	**–0.0104**	**–10.0000**	**–0.0104**

3.3. Pinion Cutter to Generate Elementary Interior Triangular Profiles

In this paragraph, the case of an interior triangular profile (Fig. 6), generated by using a pinion cutter is exemplified. By using profile symmetry, only one of two profile flanks was considered.

The input data were: $X_A = -100$ mm; $Y_A = 0$; $X_B = -89.78$ mm; $Y_B = -6.28$ mm; $\hat{A} = 60°$; $R_{rp} = 100$ mm; $i = 2$ (transmission ratio, see (9)).

The value of λ from relation (6) was divided into a number of 30 values between 0 and 1. The upper mentioned soft was adapted to the specific of the pinion cutter case, when centrods are interior tangent.

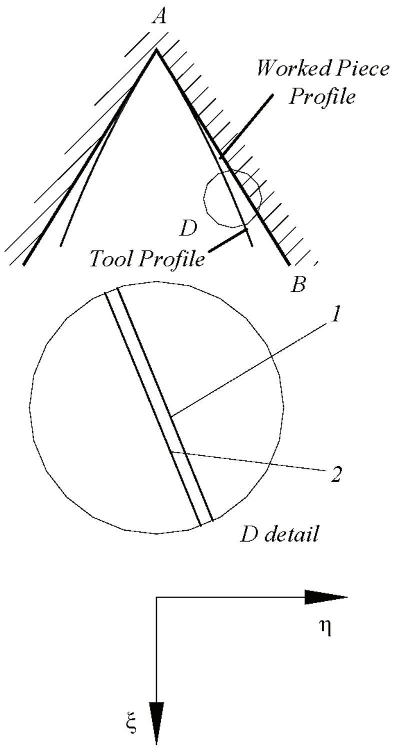

Fig. 6. Interior triangular profile case: 1 – theoretical profile; 2 – profile approximated by poles.

The coordinates of points giving the two profiles, referred to ξη system, are presented in Table 3. In this case, the precision of approximation is also acceptable, the errors being smaller than 0.02 mm.

9. CONCLUSIONS

The results of applications considered shows that the solution of approximating the enveloping curves associated to rolling centrods, by using representations by poles is completely acceptable.

The errors between the theoretical profile and the approximating one are, in all analyzed cases, smaller than 0.02 mm.

Table 3

Theoretical and Approximated Pinion Cutter Profiles

Theoretical Profile		Approximated Profile	
ξ [mm]	η [mm]	ξ [mm]	η [mm]
–50.0000	**0.0000**	**–50.0000**	**0.0000**
–49.6606	–0.2068	–49.6642	–0.1997
–49.3238	–0.4085	–49.3302	–0.3960
–48.9894	–0.6054	–48.9978	–0.5889
–486575	–0.7976	–48.6771	–0.7785
–48.3277	–0.9854	–48.3381	–0.9648
–48.0003	–1.1687	–48.0109	–1.1476
…	…	…	…
–45.4525	–2.4929	–45.4540	–2.4897
–45.1421	**–2.6423**	**–45.1421**	**–2.6423**
–44.8336	–2.7883	–44.8319	–2.7916
…	…	…	…
–41.8305	–4.0848	–41.8231	–4.0995
–41.5377	–4.1995	–41.5315	–4.2118
–41.2463	–4.3116	–41.2417	–4.3208
–40.9559	–4.4214	–40.9536	–4.4264
–40.6672	**–4.5286**	**–40.6672**	**–4.5286**

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