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ASPECTS REGARDING GENERATION OF NON-INVOLUTE GEAR PROFILES

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Abstract: *This paper presents the theoretical determination of teeth profiles of the two gears in contact (complementary curves) in case of non-involute gear profiles generation on machine tools by rolling. A generalized solution of the gear generation is presented for the three possible cases of gear cutting tools: comb slotting cutter, wheel slotting cutter and flying milling cutter.*

Key words: *gears, non-involute profiles, rolling, cutting tools.*

1. INTRODUCTION

Generation of the complex trajectories as cover of a curve family materialized on the edge of the cutting tool, known as generation by rolling, is a mathematical application of the complementary curves.

The theory of the cover curves is known and solved for a long time now. Problems appeared when the mathematical aspects were technically applied, especially in case of non-involute profiles generation by rolling. These non-involute profiles consist of line segments, circle arcs, cycloid arcs etc.

Without details, considering the problem only in case of 2D generation, based on theoretical studies, both in mathematics and mechanics (Euler, Olivier, Willis, Camus, Gohman *et al.*), it is demonstrated that the cover of a plane curve family is obtained if it is solved the equation system consisting of the parametric equation of the curve family and first derivative as function of the family parameter:

$$f(x, y, \lambda) = 0, \quad f'_{\lambda}(x, y, \lambda) = 0. \tag{1}$$

This method is named the analytic method [4]. Researchers in mechanical engineering consider this method from the kinematic generation point of view. Consequently, a kinematic method was defined [3, 6].

In many cases is easier to determine the equation of the curve family cover if the actual coordinates of the characteristic point of the curve family are determined. This method was named geometric method [4].

There are also known graphical methods for the determination of the complementary curves [4].

The two curves, curve family and its cover, are complementary curves. Their common perpendicular in the contact point passes through the instantaneous rotation center (gearing pole).

From the point of view of generation on the machine tools, it must be determined the functional profile of the cutting tool, which will generate non-involute profiles of the workpiece by rolling. The functional profile of the cutting tool in case of generation by rolling is the cover of the workpiece profile family. The determination of the functional profile has one specific aspect: always the base (B) belongs to the workpiece and the roller (R) belongs to the cutting tool.

Generation by rolling on machine tools can be done by milling with hobbing cutter, by slotting with comb cutter or wheel cutter and by milling with flying cutter.

In the next paragraphs, the rolling mechanism features (type of base and roller, generation movements, specific reference systems) will be established for the mentioned cases. The complementary curves, gearing line and rolling domain will be determined.

2. PROFILES IN CASE OF GENERATION WITH COMB CUTTING TOOLS

This is the case of milling with hobbing cutter or slotting with comb cutter. In both cases, the complementary profile will be the functional profile of the cutting tool.

a) General case. In this generation case, the rolling mechanism consists of the base (B) being the circle having R_B radius and roller (R) being the line, tangent to the rolling circle (B) (Fig. 1). The non-involute profile (C) is fixed on the base (B). The complementary profile of (C) is (m), which determines the apparent edge.

The following reference systems are established:

- xO_sy fixed on the roller – cutting tool (R),
- fix reference system XO_pY,
- $X_1O_{p1}Y_1$ fixed on the base – workpiece (B).

Generation movements for assuring rolling are base rotation ω_B and roller translation v_R.

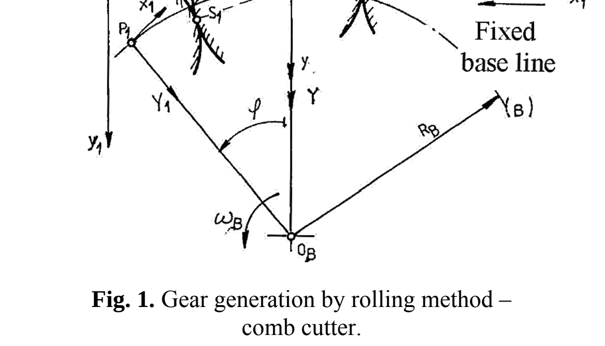

Fig. 1. Gear generation by rolling method – comb cutter.

The condition of rolling are:

$$v_R = R_B \cdot \omega_B \quad \text{and} \quad \text{arc}PP_1 = PO_{s1} = R_B \cdot \varphi. \tag{2}$$

In the first moment the origins of the three reference systems are identical with gear pole P. After rotation with the rolling angle φ, the mobile reference system has its origin in O_{s1}, moved by $O_sO_{s1} = R_B\varphi$ and the non-involute profile C reached the position C_1.

In the final position, the characteristic point S_1 belongs to the perpendicular to the profile C_1, which passes through the gear pole P. This point belongs also to the complementary profile (m_1), which represents the edge of the cutting tool.

Gearing line is the geometrical commonplace of the characteristic points in the fix reference system XO_pY.

Parametric equations of the gearing line are given by the coordinates of the characteristic point S in the fix reference system XO_pY.

Using this algorithm, the complementary profile of every simple non-involute profile will be determined.

b) Complementary profile of a line segment. It will be determined the complementary profile of the segment $MN = b$, positioned at a distance a, from the radius of the point P and angled by ε compared to this radius (Fig. 2).

Technologically speaking, this is the case of cutting gears used in fine mechanics with comb cutters or hobbing cutters or the case of cutting triangular spline shafts. In this case, the rolling mechanism consists of:

- base (B) – circle with the radius R_B, centered in O_B, having a rotation movement with an angular speed ω_B;
- roller (R) – line tangent to the base in gear pole P, having a translation movement with the speed v_R;
- between the two movements, the rolling requires:

$$v_R = R_B \cdot \omega_B. \tag{3}$$

For a rolling angle φ, the mobile reference systems moved in new origins O_{s1}, and O_{p1}, so $arcPP_1 = O_sO_{s1} = R_B \cdot \varphi$, and the segment MN will be positioned in M_1N_1. In this position, the characteristic point will be S_1, belonging to the perpendicular line to the segment M_1N_1 from the gear pole P.

The geometrical commonplace of the characteristic point S, in the reference system xO_sy is the complementary curve of the segment MN.

The line M_1N_1, in the mobile reference system $x_1O_{s1}y_1$, passes through the point $Q(x_Q, y_Q)$ and has its slope $m = \text{cotan}(\varphi+\varepsilon)$.

After simple calculation, the equation of the line family M_1N_1 will be:

$$\begin{aligned} y - R_B &= \\ &= \text{cotan}(\varphi+\varepsilon)[x - R_B\varphi - (a + R_B \sin\varepsilon)/\cos(\varphi+\varepsilon)] \end{aligned} \tag{4}$$

and its derivative is:

$$y\cos(\varphi+\varepsilon) + x\sin(\varphi+\varepsilon) = R_B\varphi\sin(\varphi+\varepsilon). \tag{5}$$

Between these two relations the parameter φ cannot be eliminated. This is the reason for presenting the equations of the complementary curve in parametric mode. As expected, the equations of the cover of the line positions define an elongated cycloid trajectory:

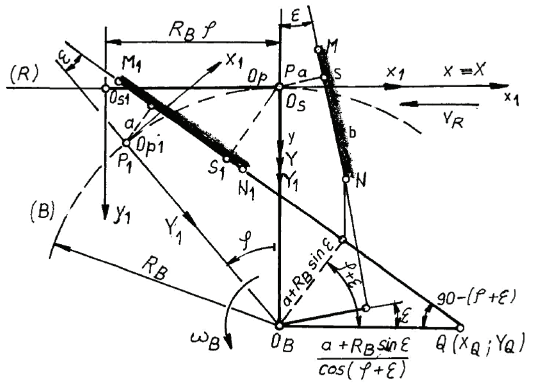

Fig. 2. Complementary profile of a line segment – comb cutter.

$$\begin{cases} x = R_B\varphi - R_B \sin(\varphi+\varepsilon)\cos(\varphi+\varepsilon) + \\ \quad + (a + R_B \sin\varepsilon)\cos(\varphi+\varepsilon) \\ y = R_B \sin^2(\varphi+\varepsilon) - (a + R_B \sin\varepsilon)\sin(\varphi+\varepsilon). \end{cases} \tag{6}$$

In case of convenient choice of the two reference systems (Fig. 2), the translation of the characteristic point coordinates from the mobile reference system $x_1O_{s1}y_1$ in the fix reference system XPY is done on the axis $O_s\,x$ by $PO_{s1} = -R\varphi$. Consequently, the coordinates of the instantaneous characteristic point S_1 in the fix reference system XPY are:

$$\begin{cases} x = -R_B \sin(\varphi+\varepsilon)\cos(\varphi+\varepsilon) + \\ \quad + (a + R_B \sin\varepsilon)\cos(\varphi+\varepsilon) \\ y = R_B \sin^2(\varphi+\varepsilon) - (a + R_B \sin\varepsilon)\sin(\varphi+\varepsilon). \end{cases} \tag{7}$$

For the determination of the rolling limits that define the domain of the rolling angle φ, the coordinates of the limit points M and N or the coordinate of one of them and the dimension $MN = b$ must be known.

After calculation, the range of the rolling angle φ is:

$$\varphi \in [-\varepsilon; (\arccos(1 - b/R_B)) - \varepsilon]. \tag{8}$$

This case is a generalization of the particular cases of radial segments ($a = 0$; $\varepsilon = 0$), sloped segments ($a = 0$; $\varepsilon \neq 0$) or parallel to the pole P radius ($a \neq 0$; $\varepsilon = 0$). In particular, the equations of every type of line segments can be obtained.

The algorithm is the same in case of other types of profiles (circle or cycloid arches).

3. PROFILES IN CASE OF GENERATION OF EXTERNAL SURFACES WITH WHEEL CUTTER

This is the case of slotting external gears with wheel cutter.

a) General case. In this case, the rolling mechanism consists of the base (B) being the circle having R_B radius and roller (R) being the circle having R_R radius (Fig. 3).

Base (B) and roller (R) have both rotation movements with angular speeds ω_B and ω_R being fulfilled the rolling requirement:

$$\text{arc}O_pO_{p1} = \text{arc}O_sO_{s1}, \tag{9}$$

resulting $R_B\Phi = R_R\varphi$.

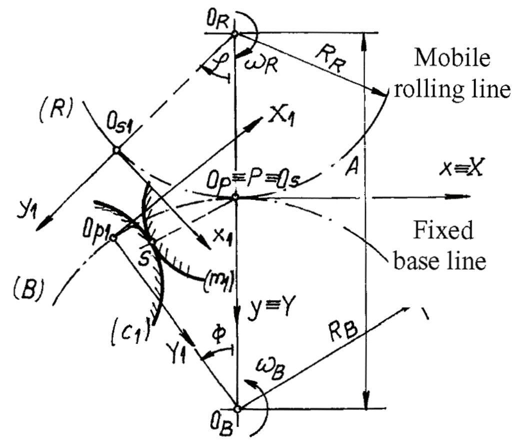

Fig. 3. General case of gear generation in case of wheel cutter.

In the first moment the origins of the three reference systems are identical with gear pole P. After rotation with the rolling angle φ, the mobile reference system fixed to the roller – cutting tool has its origin in O_{s1}. The mobile reference system fixed to the base – workpiece has its origin in O_{p1} because of base rotation by angle Φ.

Parametric equations of the gearing line are given by the coordinates of the characteristic point S_1, in the fixed reference system XO_pY.

In this generation case also, the rolling mechanism consists of the base (B) being the circle having R_B radius, centered in O_B and roller (R) being the circle having R_R radius, centered in O_R. The two circles have rotation movements with angular speed ω_B and ω_R , respectively.

b) Complementary profile of a line segment parallel to the gear pole radius will be determined for a line segment $MN = b$ positioned at a distance a from the gear pole radius, case shown in Fig. 4 (case of gear cutting of rectangular spline shafts or gears used in watches mechanisms having line profile).

Initially, the origins of the three reference systems are the same, meaning $P \equiv O_p \equiv O_s$. After rotation with the rolling angle φ, the mobile reference system fixed to the roller – cutting tool has its origin in O_{s1}, and the the mobile reference system fixed to the base – workpiece has its origin in O_{p1}, due to the base rotation by angle Φ.

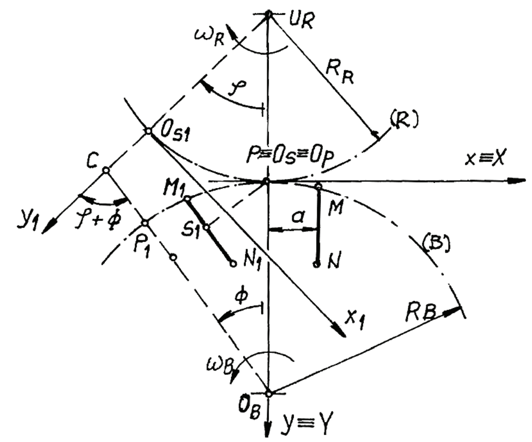

Fig. 4. Case of a line segment parallel to the gear pole radius.

Because of the rolling process, the segment MN will be positioned as M_1N_1, rotated by rolling angle of the base, Φ. The following relation is true: $\mathrm{arc}O_pO_{p1} = = \mathrm{arc}O_sO_{s1}$, and, as a consequence $R_B\Phi = R_R\varphi$. Equality of these arches leads to the relation $\varphi = R_B\Phi/R_R$ or $\Phi = R_R\varphi/R_B = e\varphi$, if it is considered $e = R_R/R_B$.

In this position, the characteristic point S_1 is on the perpendicular to the segment M_1N_1, which passes through the gear pole P. Writing the coordinates of the characteristic point S_1 in the reference system $x_1O_{s1}y_1$, after some calculation the parametric equations of the complementary profile are obtained:

$$\begin{cases} x = A\{\sin\varphi - e\sin e\varphi\cos(1+e)\varphi]/(1+e)\} \\ \quad + a\cos(1+e)\varphi \\ y = A\{\cos\varphi - e\sin e\varphi\cos(1+e)\varphi]/(1+e)\} - \\ \quad - R_R - a\sin(1+e)\varphi, \end{cases} \tag{10}$$

where $A = R_B + R_R$.

Maximum rolling angle φ_{max} is determined geometrically and, if considered the connection between the two angles, $\varphi = R_B\,\Phi/R$, the following relation is obtained:

$$\varphi_{\max} = (R_B / R_R)\arccos(\cos\varepsilon - b / R_B). \tag{11}$$

Gearing line is given by the coordinates of the characteristic point S_1, considered in the fix reference system XPY (Fig. 4). Thus, the equation of gearing line for the segment parallel to the radius, at a distance a, is:

$$\begin{cases} X = -R_B\cos e\varphi\sin e\varphi - a\cos e\varphi \\ Y = R_B\cos^2 e\varphi - a\sin e\varphi \end{cases}. \tag{12}$$

4. PROFILES IN CASE OF GENERATION WITH FLYING CUTTER

This is the case of milling with flying cutter (similar to the manufacturing of threads by means of a flying cutter or to the manufacturing of hobs by means of wheel cutter on gear milling machines with hobbing cutters).

a) General case. In this generation case, the rolling mechanism consists of the roller (R) the circle having R_R radius and base (B) being the line tangent (Fig. 5) to the rolling circle (B).

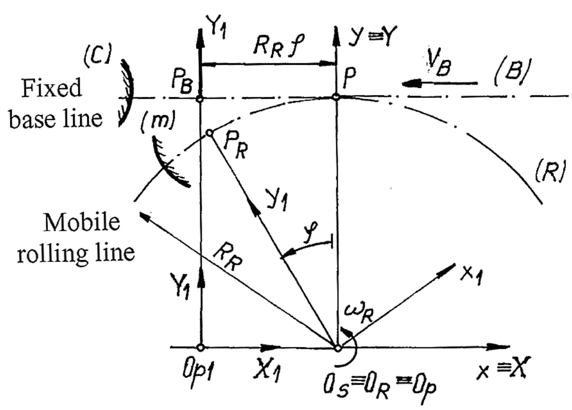

Fig. 5. General case – flying cutter.

The non-involute profile (C) is fixed on the base (B). The complementary profile of (C) is (m), which must be determined. This is the case of comb pieces cutting by means of wheel cutters.

In the first moment the origins of the three reference systems are identical. After rotation with the rolling angle φ, the mobile reference system has its origin in O_{p1}, moved by $O_pO_{p1} = R_B\varphi$.

Due to the rolling process:

$$\mathrm{arc}PP_R = PP_B = R_B\varphi\,. \tag{13}$$

In the fix reference system, XO_pY, the parametric equations of the gearing line are given by the coordinates of the characteristic point S.

b) Complementary profile of a line segment angled to the base line. This is the case of turning leading trapezoidal threads having symmetric or non-symmetric profile by means of flying (rotating) cutter or the case of combs or hobs gear manufacturing by means of wheel cutter. In this case, the base - workpiece (B) is a line and the roller – cutting tool (R) is a circle, having R_R radius (Fig. 6). The rotation movement ω_R belongs to the roller – cutting tool and the translation movement v_R belongs to the base – workpiece. It must be determined the complementary profile of the segment $MN = b$, angled by ε compared to the base (B).

For a rolling angle φ, the roller circle (R) moved its center in O_{R1}, and O_{p1}, and the gear pole P moved in P_1. In this position, the characteristic point S_1, which is also a generating point, is given by the intersection of the segment MN with its perpendicular passing through the gear pole P. The complementary curve of the segment MN will be determined by geometrical method very quickly, in this case. After simple calculation, the parametric equations of the line family M_1N_1 will be:

$$\begin{cases} x = -R_R[\sin\varphi - \varphi\cos\varepsilon\cos(\varphi+\varepsilon)], \\ y = -R_R[\cos\varphi - \varphi\cos\varepsilon\sin(\varphi+\varepsilon)]. \end{cases} \tag{14}$$

Maximum rolling angle depends on the dimension of the segment MN, b respectively.

Generally, in case of leading trapezoidal threads manufacturing the line profile is symmetric to the line (B).

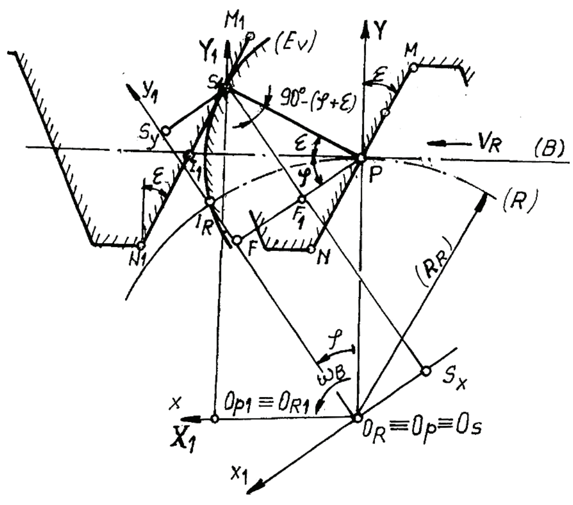

Fig. 6. Complementary profile of a line segment angled to the base line.

This is the reason the maximum rolling angle consists of two semi-angles φ_{max}, corresponding to the generation of the two semi-segments $NP = PM = b/2$ (Fig. 6). The angle φ_{max} results from the condition in which the characteristic point S_1 should cover the whole segment PM. The segment $PS_1 = R_R\varphi\sin\varepsilon$, and from the condition $PS_1 = b/2$, semi-angle φ_{max} results:

$$R_R\varphi_{max}\sin\varepsilon = b/2 \;\Rightarrow\; \varphi_{max} = (2R_R/b)\sin\varepsilon\,. \tag{15}$$

Because the two semi-segments $NP = PM = b/2$ must be generated, the maximum rolling angle is:

$$\varphi_{max} = \pm(2R_R/b)\sin\varepsilon\,. \tag{16}$$

The coordinates of the gearing line are:

$$X = -PS_1\sin\varepsilon, \quad Y = R_R + PS_1\cos\varepsilon, \tag{17}$$

and, because $PS_1 = R_R\varphi\sin\varepsilon$, the gearing line equations are:

$$\begin{cases} X = -R_R\varphi\sin^2\varepsilon, \\ Y = R_R + R_R\varphi\sin\varepsilon\cos\varepsilon. \end{cases} \tag{18}$$

The implicit form of the equation is:

$$Y = -X\tan\varepsilon + R_R\,. \tag{19}$$

This is the equation of a line, having the slope $m = -\tan\varepsilon$, being perpendicular to the segment MN.

5. CONCLUSION

This paper presents a generalization of the problem of non-involute profiles generation by means of rolling processing. Also, all the manufacturing cases were discussed: using comb cutters, wheel cutter and flying cutter.

For every case, it was presented firstly the general case, then a particular case.

There were determined the equations of the family cover, gearing line and domain of the rolling angle.

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