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NUMERIC APPLICATIONS REGARDING DETERMINATION OF THE COMPLEMENTARY PROFILES IN GEAR HOBBING

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Abstract: *This paper is a systematization of the up-to-date researches and verification by numeric examples. The subject of this paper is gear generation by rolling without slip of the non-involute profile with cutting tools of comb type, including hobbing cutter. Theoretical aspects of this generation mode were presented in papers [3] and [7].*

Key words: *gear generation, non-involute profiles, comb type gear cutting tools, hobbing cutter.*

1. INTRODUCTION

In papers [3] and [7], based on already known theorems, the complementary profile for non-involute profile were determined for the case of gear generation by rolling without slip on machine tools. The determinations were made for usual gear non-involute profiles consisting of straight lines and circle arcs having different positions and orientations.

As well, a generalization regarding the types of cutting tools was made, from the point of view of choosing the reference systems and solving methods.

This paper has two structural parts. The first part concerns with the systematization of the results of the already mentioned papers. The second part concerns with the verification of these results, by means of numeric applications.

All the discussion concerns with comb type cutting tools for gears, the most common case in practice. Hobbing cutters are such kind of cutting tools.

2. SYSTEMATIZATION OF THE CALCULUS FORMULAE

The equations of the complementary non-involute profiles, gearing line and limits of the complementary curve were determined for the cases of gear generation with comb slotting cutter, wheel slotting cutter for both internal and external gears and also flying cutter.

Because of some technological and economical advantages, being the most usual case in practice, this paper studies gear generation with hobbing cutter, a gear cutting tool of comb type.

The determined complementary curves compose the functional profile, “the reference comb” of the hobbing cutter.

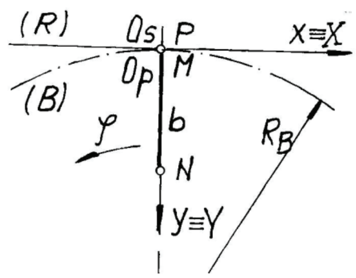
The equations were determined by means of theory of the cover curves.

Even if it may be considered having low importance, a real problem was the choice of the mobile and fix reference systems, belonging to the cutting tool and workpiece, so that the solution would have a generality level for any kind of elementary non-involute profile. Thus, only one reference system will be considered in case of non-involute profiles consisting of several elementary lines (straight lines and circle arcs, as shown in Fig. 1).

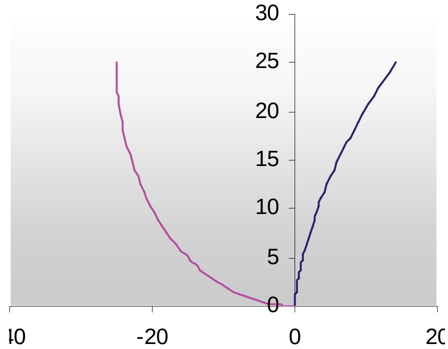
In order to be of help for the cutting tools designers, the complementary profile equations, gearing lines and limits of the rolling angle are tabled for every type of non-involute elementary profile.

In Table 1, for every elementary profile the following are given:

- explanatory figure to show fix and mobile reference systems,
- rolling angle φ,
- equations of the complementary profile in O_sxy reference system,
- equations of the gearing line in O_pXY reference system,
- variation domain for the rolling angle $\varphi \in [\varphi_{min}, \varphi_{max}]$.



a

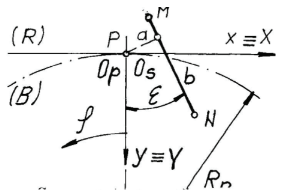
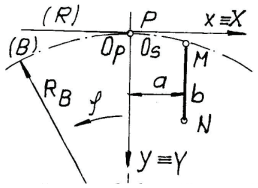
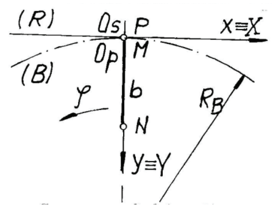
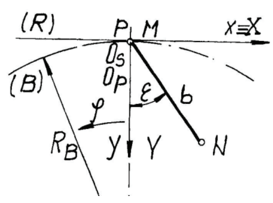
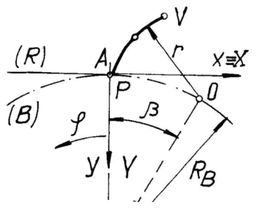
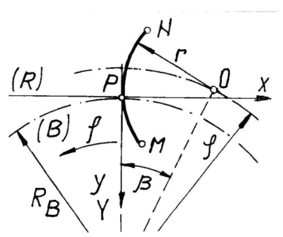


b

Fig. 1. Radial line segment: a – rolling method parameters, b – complementary profile and the gearing line.

Table 1

Gear cutting tools of comb type

No.	Figure	Equations of the complementary profile Equations of the gearing line	Rolling angle
a	 Any straight line segment	$x = 0.5R_B[2\varphi - \sin 2(\varphi+\varepsilon)] + (a + R_B\sin\varepsilon)\cos(\varphi+\varepsilon)$, $y = 0.5\,R_B[1 - \cos 2(\varphi+\varepsilon)] - (a + R_B\sin\varepsilon)\sin(\varphi+\varepsilon)$. $X = -R_B\sin(\varphi+\varepsilon)\cos(\varphi+\varepsilon) + (a + R_B\sin\varepsilon)\cos(\varphi+\varepsilon)$, $Y = R_B\sin^2(\varphi+\varepsilon) - (a + R_B\sin\varepsilon)\sin(\varphi+\varepsilon)$.	$\varphi_{min} = -\varepsilon$, $\varphi_{max} = [\arccos(1 - b/R_B)] - \varepsilon$, $\varphi \in [-\varepsilon;\ \arccos(1 - b/R_B)] - \varepsilon]$.
b	 Straight line segment with $a \neq 0$	$x = 0.5\,R_B(2\varphi - \sin 2\varphi) + a\cos\varphi$, $y = 0.5\,R_B(1 - \cos 2\varphi) - a\sin\varphi$. $X = -R_B\sin\varphi\cos\varphi + a\cos\varphi$, $Y = R_B\sin^2\varphi - a\sin\varphi$.	$\varphi_{min} = \arcsin(a/R_B)$, $\varphi_{max} = \arccos(\cos\varphi_{min} - b/R_B)$, $\varphi \in [\arcsin(a/R_B);\ \arccos(\cos\varphi_{min} - b/R_B)]$.
c	 Radial straight line segment (a=0)	$x = 0.5\,R_B(2\varphi - \sin 2\varphi)$, $y = 0.5\,R_B(1 - \cos 2\varphi)$. $X = -R_B\sin\varphi\cos\varphi$, $Y = R_B\sin^2\varphi$.	$\varphi_{min} = 0$, $\varphi_{max} = \arccos[1 - (b/R_B)]$, $\varphi \in [0;\ \arccos[1 - (b/R_B)]$.
d	 Inclined by angle ε straight line segment	$x = 0.5\,R_B[2\varphi - \sin 2(\varphi+\varepsilon)] + R_B\sin\varepsilon\cos(\varphi+\varepsilon)$, $y = 0.5\,R_B[1 - \cos 2(\varphi+\varepsilon)] - R_B\sin\varepsilon\sin(\varphi+\varepsilon)$. $X = -R_B\sin(\varphi+\varepsilon)\cos(\varphi+\varepsilon) + R_B\sin\varepsilon\cos(\varphi+\varepsilon)$, $Y = R_B\sin^2(\varphi+\varepsilon) - R_B\sin\varepsilon\sin(\varphi+\varepsilon)$.	$\varphi_{min} = 0$, $\varphi_{max} = \arccos[\cos\varepsilon - (b/R_B)] - \varepsilon$.
e	 Circle arc of r radius	$x = R_B\,\varphi - R_B\sin(\varphi-\beta) - r\cos[0.5(\varphi-\beta)]$, $y = R_B - R_B\cos(\varphi-\beta) + r\sin[0.5(\varphi-\beta)]$. $X = -R_B\sin(\varphi-\beta) - r\cos[0.5(\varphi-\beta)]$, $Y = R_B - R_B\cos(\varphi-\beta) + r\sin[0.5(\varphi-\beta)]$.	$\varphi \in [0;\ \varphi_{max}]$ $\varphi_{max} = \beta$
f	 Any circle arc	$x = -R_B\,\varphi + \rho\sin(\varphi-\beta) \pm [r\rho\sin(\varphi-\beta)]/$ $/\sqrt{\rho^2 + R_B^2 - 2\rho R_B\cos(\varphi-\beta)}$, $y = R_B - \rho\cos(\varphi-\beta) \pm [r\,\rho\cos(\varphi-\beta) - r\,R_B]/$ $/\sqrt{\rho^2 + R_B^2 - 2\rho R_B\cos(\varphi-\beta)}$. $X = -\rho\sin(\varphi-\beta) \pm [r\,\rho\sin(\varphi-\beta)]/$ $/\sqrt{\rho^2 + R_B^2 - 2\rho R_B\cos(\varphi-\beta)}$, $Y = R_B - \rho\cos(\varphi-\beta) \pm [r\,\rho\cos(\varphi-\beta) - r\,R_B]/$ $/\sqrt{\rho^2 + R_B^2 - 2\rho R_B\cos(\varphi-\beta)}$.	

3. NUMERIC APPLICATIONS

For some elementary profiles, theoretically solved in Table 1, the complementary profiles were determined numerically and graphically. The determinations were made on numeric data chosen by the authors.

The graphics were made in MS Excel 97, being generated point-to-point by determining the functions, which describe the complementary profile and gearing line. The increment of the rolling angle was 1º. The determinations were made inside the limits of the rolling angle $\varphi \in [\varphi_{min}, \varphi_{max}]$, these limits being calculated. Values of the other constants R_B, a, b, r were calculated separately for each case.

In the following, only some of the cases presented in Table 1 will be studied. As well, the graphics will be presented only qualitatively for the two characteristic curves: complementary profile (blue) and gearing line (red). The calculated coordinates of the points of these two curves are not given.

a) Radial line segment (segment $MN = b$ in Fig. 1.a, line c in Table 1).

The following case has been chosen: radius of the base circle $R_B = 50$ mm, length of the segment $b = 15$ mm, rolling angle $\varphi \in [0, \varphi_{max}]$. After calculations the maximum value of the rolling angle results $\varphi_{max} = 45.573º = 0.7954$ rad.

The rolling angle varied incrementally by 1º. Thus, the coordinates of the points belonging to the complementary profile and the gearing line were determined. Using these coordinates, the graphics of the two characteristic curves are presented in Fig. 1.b.

b) Line segment inclined by ε *to the pole's radius* (segment $MN = b$ in Fig. 2, a, line d in Table 1).

The following case has been chosen: radius of the base circle $R_B = 50$ mm, length of the segment $b = 15$ mm, inclination angle $\varepsilon = 15º$, rolling angle $\varphi \in [0, \varphi_{max}]$. After calculations the maximum value of the rolling angle results $\varphi_{max} = 33.2466° = 0.58026$ rad.

The graphics of the two characteristic curves are shown in Fig. 2.b.

c) Line segment parallel the pole's radius, at distance a (segment $MN = b$ in Fig. 3.a, line b in Table 1).

The following case has been chosen: radius of the base circle $R_B = 50$ mm, length of the segment $b = 15$ mm, distance $a = 10$ mm, rolling angle $\varphi \in [0, \varphi_{max}]$. After calculations the values of the rolling angle result $\varphi_{min} = 11.537º = 0.20135$ rad and $\varphi_{max} = 47.1723º = 0.82331$ rad.

Using these data, the graphics of the two characteristic curves are presented in Fig. 3.b.

d) r radius circle arc having the center on base circle (circle arc AV in Fig. 4.a , line e in Table 1).

The circle arc has δ degrees, its radius r and its center on the base circle, at the positioning angle β.

For the numeric application the following values were chosen: $R = 50$ mm, $\delta = 10°$, $\beta = 15°$. With this data the following were calculated: $r = 21.644$ mm, $\varphi = \delta = 10°$, $\varphi = 91.56°$.

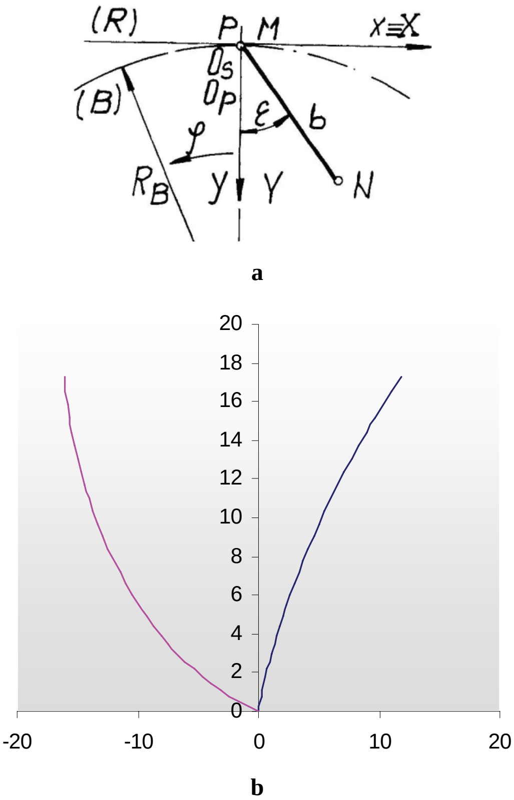


Fig. 2. Inclined line segment: a – rolling method parameters; b – complementary profile and the gearing line.

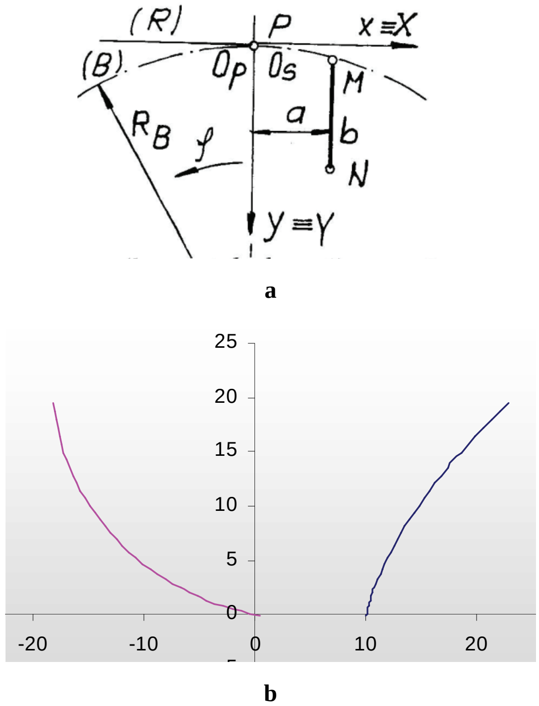
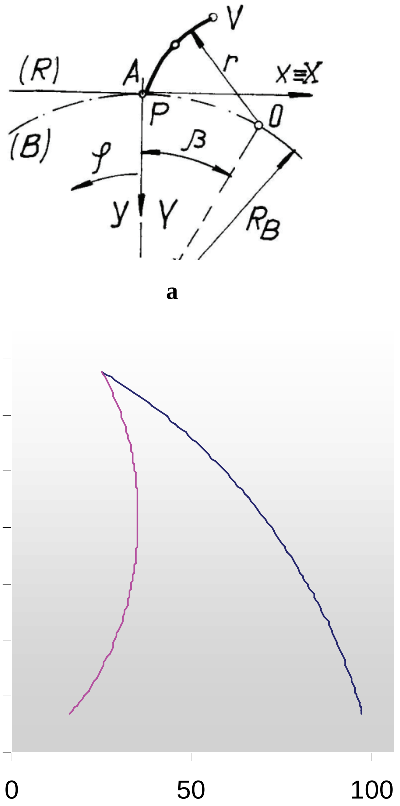


Fig. 3. Parallel line segment: a – rolling method parameters; b – complementary profile and the gearing line.

The graphics of the two characteristic curves are shown in Fig. 4.b.

4. CONCLUSION

Using the relationships in Table 1 the complementary profiles for non-involute elementary profiles as straight



a

b

Fig. 4. Circle arc: a – rolling method parameters; b – complementary profile and the gearing line.

lines and circle arcs can be determined. For a qualitative control, in the presented figures the complementary profile and the gearing line were determined in some particular cases.

REFERENCES

[1] Donin, R. (2001). *Contribuţii la generarea danturii roţilor cu profil neevolventic*, Ph.D. Thesis, "Transilvania" University, Braşov.

[2] Ghionea, A. (1983). *Contribuţii la studiul procesului de generare a danturii polihipocicloidale*, Ph.D. Thesis, Inst. Politehnic, Bucharest.

[3] Ilie, S. (2003). *Studii teoretice privind generarea prin rulare fără alunecare a profilelor neevolventice*, Report no. 2 for Ph.D. Thesis, Dept. Machines and Production Systems, "Politehnica" University, Bucharest.

[4] Litvin, F. L. (1994). *Gear Geometry and Applied Theory*, Prentice Hall, New Jersey.

[5] Miloiu, G., Dudiţă, F., Diaconescu, D. V. (1982). *Transmisii mecanice moderne*, Edit. Tehnică, Bucharest.

[6] Minciu, C., Străjescu, E., Constantin, G., *et al.* (1996). *Scule aşchietoare. Îndrumar de proiectare*, vol. 2, Edit. Tehnică, Bucharest.

[7] Minciu, C., Croitoru, S. M., Ilie, S. (2006). *Aspects regarding generation of non-involute gear profiles*, ICMaS 2006.

[8] Oancea, N. (1990, …, 2000). *Metode numerice pentru profilarea sculelor*, vol. 1–8, "Dunărea de Jos" University, Galaţi.

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