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ALGORITHMS FOR REPRESENTATION BY POLES AS A WAY TO APPROXIMATE WRAPPING CURVES OF PROFILES ASSOCIATED TO ROLLING CENTRODS

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Abstract: *Representation by poles of plain curves is already known and used. In this paper, specific algorithms conceived in order to approximate, on industrial strictly acceptable conditions, wrapping profiles associated to rolling centrods are presented. This approach opens a new way to cutting tools profiling methodic.*

Key words: *representation by poles, wrapping profiles, approximation.*

1. INTRODUCTION

Many ways to describe surfaces and profiles are known, modalities of rigorously expressing – analytical or vectorial – with their variants, defined by Analytical or Differential Geometry.

We also must notice that discrete representations, closer to the physical perceptions of objects, given by measuring or calculating possibilities, squint towards a progressively wider utilization area.

In many practical cases, a profile or surface "approximate" description can satisfy the technical requirements regarding their representation in order fulfill various tasks: cutting tools profiling, cutting tools trajectories programming to generate complex surfaces, etc.

From this point of view, although the representation by poles of profiles (surfaces) is a way to approximate a geometrical profile, it was developed by Favrales (1998) and used to study wrapping surfaces (profiles) [5–7], as a variant to express the wrapping condition, on its well known forms [2], or by complementary theorems [3].

This modality of describing profiles can also be used to directly approximating the wrapping profiles, allowing simple and easy to evaluate expression forms to profile cutting tools used in generating processes.

A new methodology, based on reciprocal wrapping surfaces theory, is further presented, to express in a condensed shape, the profiles of surfaces generated by wrapping, by rolling.

2. RACK-BAR TOOL PROFILE APPROXIMATED EXPRESSION – STRAIGHT LINE SEGMENT CASE

2.1. Reference Systems

In Fig. 1, the couple of rolling centrods, the reference systems and the elements profile whose wrapping curve should be approximated are presented:

- xyz means a fix reference system, having its z axis – axoid (centrod, C_1) symmetry axis – overlaid to worked piece symmetry axis;
- XYZ – mobile system, initially overlaid to the fix one, jointed to the generated profile;
- $\xi\eta\zeta$ – mobile system, jointed to the tool centroid, C_2.

2.2. Relative Motion between Reference Systems

Relative kinematics between upper mentioned reference systems (Fig. 1) can be expressed as it follows [2]:

$$\xi = \omega_3^{\mathrm{T}}(\varphi)\cdot X - a, \tag{1}$$

where

$$a = \begin{Vmatrix} -R_{rp} \\ -R_{rp}\cdot\varphi \end{Vmatrix}. \tag{2}$$

After making substitutions and calculus, (1) becomes

$$\begin{Vmatrix} \xi \\ \eta \end{Vmatrix} = \begin{Vmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{Vmatrix} \cdot \begin{Vmatrix} P_x \\ P_y \end{Vmatrix} - \begin{Vmatrix} -R_{rp} \\ -R_{rp}\cdot\varphi \end{Vmatrix}. \tag{3}$$

In equation (3), P_X and P_Y mean the co-ordinates of current point P, placed on Σ profile, expressed by poles ([1, 7]):

$$\Sigma \begin{vmatrix} P_X = \lambda\cdot X_A + \mu\cdot X_B; \\ P_Y = \lambda\cdot Y_A + \mu\cdot Y_B; \end{vmatrix} \tag{4}$$

$$\lambda + \mu = 1, \tag{5}$$

or

$$\Sigma \begin{vmatrix} P_X = \lambda\cdot X_A + (1-\lambda) X_B; \\ P_Y = \lambda\cdot Y_A + (1-\lambda) Y_B, \end{vmatrix} \tag{6}$$

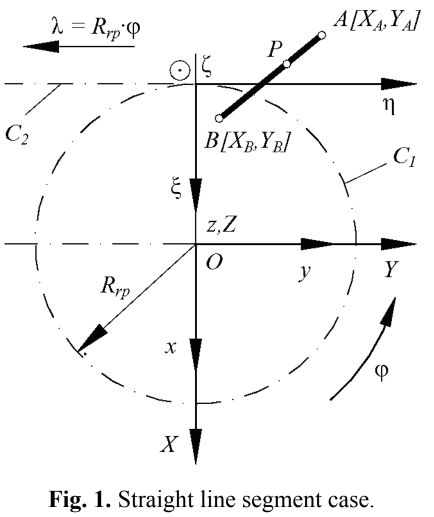

Fig. 1. Straight line segment case.

where X_A, X_B, Y_A, Y_B meaning the co-ordinates of AB segment bottom points.

Thus, the segment Σ family, generated in ξη system during the motion expressed by equation (3), can be described through equations like:

$$(\Sigma)_\varphi \left| \begin{array}{l} \xi = P_X \cdot \cos\varphi - P_Y \cdot \sin\varphi + R_{rp} \\ \eta = P_X \cdot \sin\varphi + P_Y \cdot \cos\varphi + R_{rp} \cdot \varphi, \end{array} \right. \tag{7}$$

if the form (6) is used to express Σ profile.

2.3. Profiles Family Enveloping Curve

The enveloping curve of profiles family, expressed by the poles of straight-line segment (6), can be obtained by associating to equations (7) the enveloping condition [7] on its specific form

$$\frac{\xi'_\varphi}{\xi'_\lambda} = \frac{\eta'_\varphi}{\eta'_\lambda}, \tag{8}$$

where λ and φ are the two variables defined during the considered generating process.

Partial derivatives included in relation (8) are defined as:

$$\begin{array}{l} \xi'_\varphi = -P_X \cdot \sin\varphi - P_Y \cdot \cos\varphi, \\ \eta'_\varphi = P_X \cdot \cos\varphi - P_Y \cdot \sin\varphi + R_{rp}, \\ \xi'_\lambda = P'_{X_\lambda} \cdot \cos\varphi - P'_{Y_\lambda} \cdot \sin\varphi, \\ \eta'_\lambda = P'_{X_\lambda} \cdot \sin\varphi + P'_{Y_\lambda} \cdot \cos\varphi. \end{array} \tag{9}$$

After making (9) substitutions into (8), the enveloping condition can be expressed as

$$-P'_{X_\lambda} \cdot \cos\varphi + P'_{Y_\lambda} \cdot \sin\varphi = \frac{P_X \cdot P'_{X_\lambda} + P_Y \cdot P'_{Y_\lambda}}{R_{rp}}. \tag{10}$$

Finally, after calculus, the enveloping condition can be brought to the form

$$\begin{array}{l} -\cos(\varphi + \alpha_X) = \\ \quad = \lambda \cdot \dfrac{L}{R_{rp}} + \left[X_B \cdot \cos\alpha_X + Y_B \cdot \sin\alpha_X\right] \dfrac{1}{R_{rp}}, \end{array} \tag{11}$$

where

$$\cos\alpha_X = \frac{X_A - X_B}{L}; \quad \sin\alpha_X = \frac{Y_A - Y_B}{L}, \tag{12}$$

$$L = \sqrt{(X_A - X_B)^2 + (Y_A - Y_B)^2}. \tag{13}$$

2.4. Profile Approximated Expression Finding

In principle, the rack-bar tool profile enveloping curve, $(\Sigma)_\varphi$, is an curve arc, given into ξη system by

$$S = \left\| \begin{array}{l} \xi_1, \eta_1 \\ \xi_2, \eta_2 \\ \vdots \\ \xi_i, \eta_i \\ \vdots \\ \xi_n, \eta_n \end{array} \right\|, \tag{14}$$

matrix found by giving variation to λ and φ variables, according to (10) condition (Fig. 2).

The purpose is to define the S profile by using a second degree function as:

$$S_{app} \left| \begin{array}{l} P_X = \lambda_1^2 \cdot A_\xi + 2\mu_1 \cdot \lambda_1 \cdot B_\xi + \mu_1^2 \cdot C_\xi \\ P_Y = \lambda_1^2 \cdot A_\eta + 2\mu_1 \cdot \lambda_1 \cdot B_\eta + \mu_1^2 \cdot C_\eta, \end{array} \right. \tag{15}$$

where $\lambda_1 + \mu_1 = 1$, (16)

and A_ξ, A_η, B_ξ, B_η, C_ξ and C_η are constants that should be found.

Obviously, the functions giving S_{app} can be rewritten, by considering (16), as:

$$S_{app} \left| \begin{array}{l} P_X = (1-\mu_1)^2 \cdot A_\xi + 2\mu_1 \cdot (1-\mu_1) \cdot B_\xi + \mu_1^2 \cdot C_\xi \\ P_Y = (1-\mu_1)^2 \cdot A_\eta + 2\mu_1 \cdot (1-\mu_1) \cdot B_\eta + \mu_1^2 \cdot C_\eta, \end{array} \right. \tag{17}$$

$\mu_1 \in [0, 1]$.

The coefficients of second degree function giving rack-bar tool profile – S_λ – can be found by imposing the condition that three points from its profile should be coincident with three points from the approximated profile, S_{app}.

Thus, the equations system which allows the constants identification can be obtained:

- when $\mu_1 = 0 \rightarrow \begin{cases} \xi_A = C_\xi; \\ \eta_A = C_\eta; \end{cases}$ (18)
- when $\mu_1 = 1 \rightarrow \begin{cases} \xi_B = A_\xi; \\ \eta_B = A_\eta; \end{cases}$ (19)
- when $\mu_1 = 0.5 \rightarrow \begin{cases} \xi_P = 0.25 \cdot A_\xi + 0.5 \cdot B_\xi + 0.25 \cdot C_\xi; \\ \eta_P = 0.25 \cdot A_\eta + 0.5 \cdot B_\eta + 0.25 \cdot C_\eta. \end{cases}$ (20)

Starting from (20) system, B_ξ and B_η can be expressed as it follows:

$$B_\xi = \frac{\xi_P - 0.25 \cdot A_\xi - 0.25 \cdot C_\xi}{0.5}, \tag{21}$$

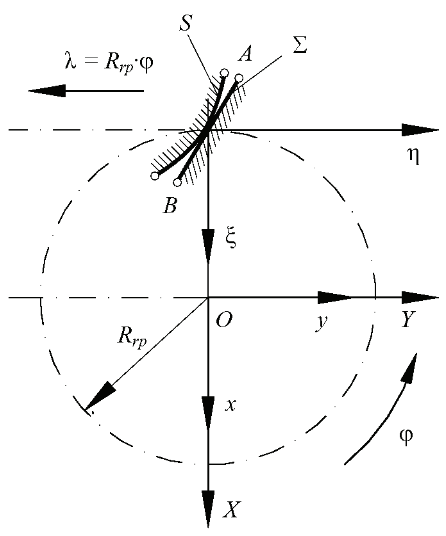

Fig. 2. Rack-bar tool profile, wrapped to Σ segment.

$$B_\eta = \frac{\eta_P - 0.25A_\eta - 0.25C_\eta}{0.5}. \qquad (22)$$

Thus, rack-bar tool profile (11), as Σ (4) straight-line segment enveloping curve, is given by a second degree curve equations – parabola (13) – when it should be necessary the identification of only three distinct points from *S* vector, (11).

It supposes (10) enveloping condition solving, depending on the coordinates accepted on Σ segment by the simple geometrical element of a composed manufactured piece profile.

3. RACK-BAR TOOL PROFILE APPROXIMATED EXPRESSION – CIRCULAR ELEMENTARY PROFILE CASE

Same way as in the case illustrated in Fig. 1, the situation when the simple geometrical profile, included into a composite manufactured piece profile, is an arc of circle, given by $\overline{AB}$ string and *r* radius magnitude.

The coordinates of string ending points

$$A[X_A,\ Y_A],\ B[X_B,\ Y_B], \qquad (23)$$

are considered known on *XYZ* system, together with the magnitude of circular profile *r* radius.

It should also be mentioned that the arc of circle can be convex (like it has been drawn in Fig. 3) or concave.

Thus, we can accept the circular profile to be expressed by:

$$S_{app}\left|\begin{aligned} &P_X = \lambda^2\cdot A_X + 2\lambda\cdot(1-\lambda)\cdot C_X + (1-\lambda)^2\cdot B_X \\ &P_Y = \lambda^2\cdot A_Y + 2\lambda\cdot(1-\lambda)\cdot C_Y + (1-\lambda)^2\cdot B_Y, \end{aligned}\right. \qquad (24)$$

the fact that two poles mean the string ending points, when

$$\lambda = 0 \ \rightarrow\ P_X = B_X;\ \ P_Y = B_Y; \qquad (25)$$

$$\lambda = 1 \rightarrow P_X = A_X;\ \ P_Y = A_Y \qquad (26)$$

and also that the string middle point results when

$$\lambda = 0.5\ \rightarrow\ P_X = P_{X,\lambda=0.5};\ \ P_Y = P_{Y,\lambda=0.5}\,. \qquad (27)$$

According to the notations from Fig. 3, it follows:

$$\begin{aligned} P_{X,\lambda=0.5} &= X_A - \frac{L}{2}\cdot\cos\alpha_X - \left[r - \sqrt{r^2 - \frac{L^2}{4}}\right]\cdot\sin\alpha_X, \\ P_{Y,\lambda=0.5} &= Y_A - \frac{L}{2}\cdot\sin\alpha_X - \left[r - \sqrt{r^2 - \frac{L^2}{4}}\right]\cdot\cos\alpha_X. \end{aligned} \qquad (28)$$

Then, when λ = 0.5, C_X and C_Y can be expressed from relations (24):

$$\begin{aligned} C_X &= \frac{P_{X\lambda=0.5} - 0.25\cdot A_X - 0.25\cdot B_X}{0.5}, \\ C_Y &= \frac{P_{Y\lambda=0.5} - 0.25\cdot A_Y - 0.25\cdot B_Y}{0.5}. \end{aligned} \qquad (29)$$

Thus, equations (25), (26), (28) and (29) give the poles of second degree curve to substitute *AC* arc of circle (Fig. 3). Such an “approximated” representation of the arc of circle may be accepted if the error between the substitution curve and the effective one is small enough.

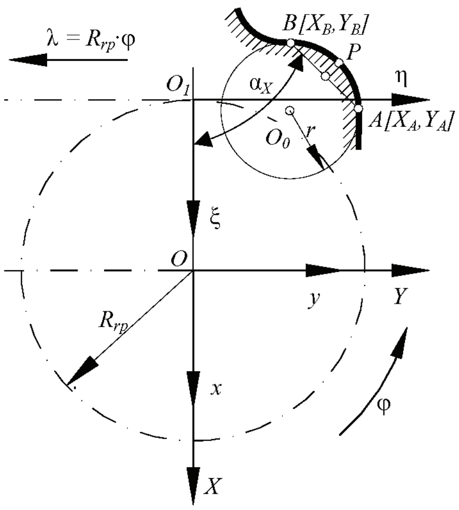

Fig. 3. Circular elementary profile case.

The profiles family can be found, into the rack-bar tool reference system, see also equations (1) and (2), in the form

$$\begin{Vmatrix}\xi\\ \eta\end{Vmatrix} = \begin{Vmatrix}\cos\varphi & -\sin\varphi\\ \sin\varphi & \cos\varphi\end{Vmatrix}\cdot\begin{Vmatrix}P_x(\lambda)\\ P_y(\lambda)\end{Vmatrix} - \begin{Vmatrix}-R_{rp}\\ -R_{rp}\cdot\varphi\end{Vmatrix}, \qquad (30)$$

with $P_X(\lambda)$ and $P_Y(\lambda)$ defined by (24).

The enveloping condition, associated to the family of profiles that substitutes the arc of circles family, can be bring to the expression

$$-P'_{X_\lambda}\cdot\cos\varphi + P'_{Y_\lambda}\cdot\sin\varphi = \frac{P_X(\lambda)\cdot P'_{X_\lambda} + P_Y(\lambda)\cdot P'_{Y_\lambda}}{R_{rp}}, \qquad (31)$$

where:

$$P'_{X_\lambda} = 2\cdot\lambda\left(A_X - 2\cdot C_X + B_X\right) + 2\left(C_X - B_X\right), \qquad (32)$$

$$P'_{Y_\lambda} = 2\cdot\lambda\left(A_Y - 2\cdot C_Y + B_Y\right) + 2\left(C_Y - B_Y\right). \qquad (33)$$

The couple of equations (30) and (31) give the enveloping curve of arc of circle substituting profiles family, referred to ξη system, associated to the rack-bar tool. This enveloping curve means the rack-bar tool profile, into the generating plane ξη having, in principle, the form of a coordinates matrix (14).

Similarly to those presented in § 2, *S* profile can be substituted by a second degree polynomial function (or by a higher degree, in order to increase tool profile dimensional precision), see relations (18), (19), (20), (21) and (22).

Thus, in this case also, the problem can be solved by considering only three points on the generated profile, without affecting tool profile accepted precision. Otherwise, the poles of the curve to substitute the enveloping curve can be calculated by using relations of (18) … (22) type.

4. REPRESENTATION APPROACH

By considering curves representation by poles [1], an elementary profile approximated enveloping curve representation approach is suggested, as it follows.

Thus, in the case of straight line segments, the following representation is suggested (see also (6)):

P_λ	Poles	
	X	**Y**
$P_{\lambda=0}$	X_B	Y_B
$P_{\lambda=1}$	X_A	Y_A

(34)

form that, based on (7) relations and on enveloping condition, when λ has a variation between 0 and 1, leads to a enveloping curve discrete expression, as a matrix of (14) type,

$$S = \begin{Vmatrix} \xi_B, \eta_B \\ \vdots \\ \xi_P, \eta_P \\ \vdots \\ \xi_A, \eta_A \end{Vmatrix}, \quad (35)$$

where points $A[\xi_A, \eta_A] \ldots P[\xi_P, \eta_P] \ldots B[\xi_B, \eta_B]$ are points from rack-bar tool profile, reciprocal wrapped to profile to be generated – *AB* straight line segment.

Based on algorithm given through relations (15)…(25), the theoretical profile is replaced by an approximated shape – parabola given by equations (17). The representation by poles of this shape leads to a

$S_{app} =$

P_{μ_1}	Approximated Curve Poles	
	ξ	**η**
$P_{\mu_1=1}$	A_ξ	A_η
$P_{\mu_1=0.5}$	B_ξ	B_η
$P_{\mu_1=0}$	C_ξ	C_η

(36)

representation form, where A_ξ, A_η, … C_η are defined by equations (18)…(22).

Thus, a simple representation of enveloped profile – profile of rack-bar tool – is obtained for only three points – three values of both λ and μ_1 variables. The following generic scheme is suggested:

P_λ	Poles	
	X	**Y**
$P_{\lambda=0}$	X_B	Y_B
$P_{\lambda=1}$	X_A	Y_A

$\xrightarrow[\text{Env. cond. (11)},\ \lambda = 0 \ldots 1]{(\Sigma)_\varphi\ (8)}$ $\begin{Vmatrix} \xi_B, \eta_B \\ \vdots \\ \xi_P, \eta_P \\ \vdots \\ \xi_A, \eta_A \end{Vmatrix}$

$\xrightarrow{(18) \ldots (22)}$

P_{μ_1}	Approximated Curve Poles	
	ξ	**η**
$P_{\mu_1=1}$	A_ξ	A_η
$P_{\mu_1=0.5}$	B_ξ	B_η
$P_{\mu_1=0}$	C_ξ	C_η

(37)

5. CONCLUSIONS

Curves representation by poles, in the case of enveloping processes, has the advantage of a simple representation, using a small number of points (three profile points). Approximation precision may be improved by using superior degree functions. The methodic can become very useful when the tables would include pre-calculated elements.

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