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THEORETICAL STUDIES ABOUT THE GENERATION OF THE COMPLEX SURFACES

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Abstract: *The paper shows theoretical original aspects in the field of the generation of the complex surfaces. Starting from the general cinematic principle of their generation, there are presented the cinematic laws for the generation of the theoretical curves D and G (directrix and generatrix) of the complex surfaces. These cinematic laws are materialized in three modes that permit the distinction of three different categories of the complex surfaces: complex surfaces of first type, of second type and of third type. In the paper are also shown the cinematic particularities at the generation of each type of the complex surfaces.*

Key words: *Generation, Complex surfaces, Directrix and generatrix, Cinematic laws.*

1. INTRODUCTION

The complex surfaces are that geometric surfaces having theoretical curves D and/or G has a such geometrical form that for their generation are necessary compositions of simple movements cinematic coordinated by a R_{CCIN}.

The cinematic processes of the generation of the complex surfaces impose the presence of the *complementary generation movement*. The number of the complementary generation movements and their type (rotation or rectilinear translation)) depend by the geometrical form of the complex surface to be generated and by the adopted proceeding.

Conform to the cinematic principle of the generation of the real surfaces, for the generation of a complex surface, in the general case, the generating element $E_g(M)$ must realize a movement along the theoretical generatrix G with the speed $\vec{v}_g$ that will be named the movement $m^G(\vec{v}_g)$, and a movement along the theoretical directrix D with the speed $\vec{v}_d$ that will be named the movement $m^D(\vec{v}_d)$ (Fig. 1).

The movements $m^G(\vec{v}_g)$ and $m^D(\vec{v}_d)$ are compound movements, obtained by the composition of the main movement or of the generation feed with a complementary generation movement, cinematic cocoordinated by a R_{CCIN}.

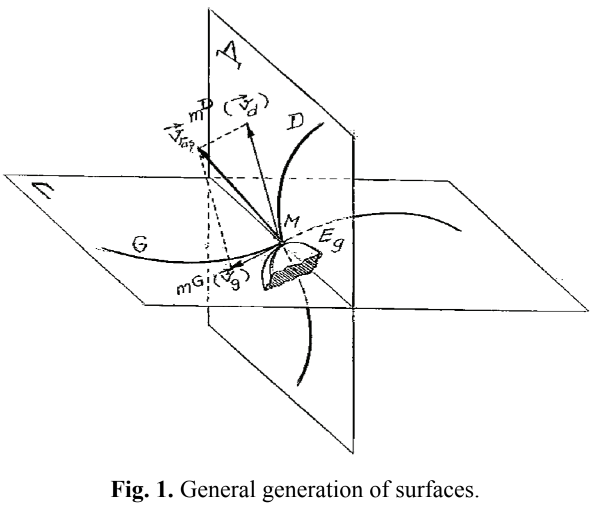

Fig. 1. General generation of surfaces.

2. CLASSIFICATION AND PARTICULARITIES OF THE COMPLEX SURFACES

Complex surfaces are classified in three categories depending on the generation mode of curves G and D:

a. complex surfaces by the first species;
b. complex surfaces by the second species;
c. complex surfaces by the third species.

2.1. Complex surfaces by the first species

There are geometrical surfaces at which:

a. G curve has a geometrical form that imposes it gene-ration by materialization on the generating edge of the cutting tool.

b. D curve has a geometrical form that imposes it generation by composing single movements, in one of the suit modes:

b$_1$. The cinematically directrix D, generated as a trajectory of the movement of a point in the plane or in the space, obtained by composition of simple movements cinematically co-coordinated by a $R^D{}_{CCIN}$.

b$_2$. The cinematically directrix D, generated as an envelope of the successive positions of a cinematic curve that displace along a space trajectory, obtained by composition of simple movements cinematically co-coordinated by a $R^D{}_{CCIN}$.

b$_3$. The materialized directrix D.

b$_4$. The scheduled directrix D.

Down it is presented the cinematic laws for the movement $m^D(\vec{v}_d)$ at the generation of the complex surfaces by the first specie with large technical utilization.

A. Cylindrical helicoidal surfaces

These surfaces has the theoretical directrix D a circular helix described on a right rotation cylinder, and the theoretical generatrix G a plane curve that constitute the profile

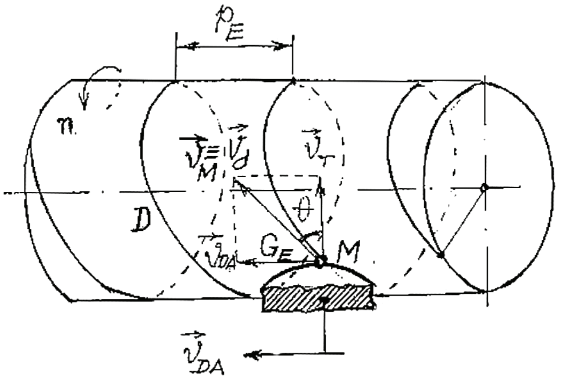

Fig. 2. The generation of a helicoidal surface.

of the helicoidal surface and that is materialized on the generating element $G_E(M)$ on the generating edge of the generating cutting tool (Fig. 2).

By the movement $m^D(\vec{v}_d)$, the element $G_E(M)$ must generate a theoretic D as a helicoidal circular trajectory with the step p and the inclination angle θ, at the r radius, given. In this purpose, the cylinder makes a rotation movement with the frequency n [rot/min], from that results the relative tangent speed $\vec{v}_T^D$, where $v_T^D = 2\pi rn$ in [mm/min] and $G_E(M)$ makes a right movement of axial displacement, normal on the rotation plane and having the speed $\vec{v}_{DA}^D$. These movements are cinematically co-coordinated by R_{CCIN}^D that imposes:

$$R_{CCIN}^D = \frac{v_{DA}^D}{v_T^D} = \text{tg}\ \theta. \quad (1)$$

After the composition of the two movements, the generator element $G_E(M)$ generates the helicoidal circular trajectory D with the speed $\vec{v}_d = \vec{v}_M$ and by consequence the cinematic law of the movement $m^D(\vec{v}_d)$ is:

$$\vec{v}_d = \vec{v}_T^D + \vec{v}_{DA}^D. \quad (2)$$

In the category of the cylindrical helicoidal surfaces are included:

a. the surfaces of the cylindrical thread, exterior and interior;

b. the surfaces of the flanks of the cylindrical creepers (worm gears) of the type ND, NG, A, E, SD and SC;

c. channel surfaces with different profiles.

Depending by the generating processes of these surfaces, the cinematic law (2) for the $m^D(\vec{v}_d)$ takes different forms.

1. Generation by turning. That is specific for the generation of the surfaces a) and b). In this case, for the both situations (θ < 45° and θ > 45°) the movement $m^D(\vec{v}_d)$ is obtained by the composition of the principal generation movement ($\vec{v}_{as}^D$) with the complementary generation movement of the directrix $D(\vec{v}_{cg}^D)$, so that the relationship (2) becomes:

$$\vec{v}_d = \vec{v}_{as}^D + \vec{v}_{cg}^D, \quad (3)$$

in which it is assured:

$$R_{CCIN}^D = \frac{v_{cg}^D}{v_{as}^D} = \text{tg}\ \theta. \quad (4)$$

2. Generation by milling. In this case, the rotation generating movement is a movement of circular feed, with a revolution circular feed n_{sc} from that results the tangential speed for the generation of the directrix D $\vec{v}_{sc}^{T\,D}$ and the movement for the axial displacement is the complementary generation movement of the directrix D ($\vec{v}_{cg}^D$). By consequence, the cinematic law (2) for the movement $m^D(\vec{v}_{cg}^D)$ becomes:

$$\vec{v}_d = \vec{v}_{sc}^{T\,D} + \vec{v}_{cg}^D, \quad (5)$$

in which is assured

$$R_{CCIN}^D = \frac{v_{cg}^D}{v_{sc}^{T\,D}} = \text{tg}\ \theta. \quad (6)$$

3. Generation by grinding. The cinematic of the generation is similar with that of the milling of the helicoidal surfaces, so that the movement $m^D(\vec{v}_d)$ is made considering the same cinematic law (5) and respecting R_{CCIN}^D gave by the relation (6).

4. The generation with screw plates and tap screw. The helicoidal generate surfaces are surfaces of the exterior respectively interior screws, the generation using generating cutting tools with the directrix D materialized. The movement $m^D(\vec{v}_d)$ is realized according with the cinematic law (3) and respecting R_{CCIN}^D gave by the relationship (4).

5. The generation by broaching. In this case, the surfaces are generated by helicoidal broaching and they are surfaces of interior channels. Like in the anterior case, the generating cutting tool has the directrix D materialized, and makes the movement $m^D(\vec{v}_d)$ according to the same cinematic law (3) and respecting the same gave by the relationship (4).

B. Conic helicoidal surfaces

There are geometric surfaces that have the theoretical directrix D a conical helix generated on a revolution right cone, and the theoretical generatrix G, like at the helicoidal right cylindrical surfaces, a plane curve that represents their profile ant that is materialized by the generator element $G_E(M)$ on the generating edge of the generating tool (Fig. 3).

By the movement $m^D(\vec{v}_d)$, the generating element $G_E(M)$ must generate the theoretic D like a conic helicoidal trajectory, having the inclination angle θ, the step p_E and the angle of the cone generatrix δ.

The movement $m^D(\vec{v}_d)$ is obtained from the composition of the rotation movement of the cone with the frequency n (rot/min) from which it results the tangential relative speed in the directrix generation D, $\vec{v}_T^D$, where

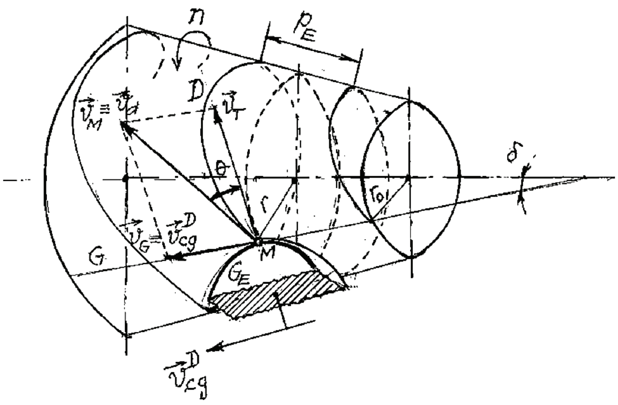

Fig. 3. The generation of a conical helix.

$v_T^D = 2\pi rn$ in [mm/min], with the complementary movement of the directrix generation D' made by $G_E(M)$, like a right movement along the cone's generatrix with the speed $\vec{v}_G = \vec{v}_{cg}^D$.

After the composition of the two movements, the generating element $G_E(M)$ generates the helicoidal conical trajectory D with the speed $\vec{v}_d = \vec{v}_M$, and by consequence the cinematic law of the movement $m^D(\vec{v}_d)$ is:

$$\vec{v}_d = \vec{v}_T^D + \vec{v}_{cg}^D. \tag{7}$$

2.2. Complex surfaces of the second species

The complex surfaces of the second specie are these geometrical surfaces at which:

1. The G curve has a geometrical form that impose it generation by composing simple movements, cinematic co-ordinates by a R_{CCIN}^G.

2. The D curve has a geometrical form that can be generated by the principal generation movement, or by a feed movement (right or circular) if the generation is produced by one of the suit modes:

a_1. The cinematic generatrix G.

a_2. The scheduled generatrix G generate as a trajectory of a movement of a point in plane, obtained by compositions of simple movements cinematically co-coordinated by a R_{CCIN}^G, that is assured by a port-schedule.

b_1. The cinematically directrix D, generated as a trajectory of the movement of a point in the plane.

b_2. The cinematically directrix D, generated as an envelope of the successive positions of a cinematic curve that displace in plane.

b_3. The cinematically directrix D, generated by transposition (printing) by rolling.

It result that at the generation of the complex surfaces by the second specie, the movements of the generating element $E_g(M)$ for the generation of the curves G and D, is made as it is presented down.

The movement $m^G(\vec{v}_g)$ is obtained by the composition of a feed movement with the speed $\vec{v}_s^G$ with a complementary generation movement of the generatrix G with the speed $\vec{v}_{cg}^G$ cinematically co-coordinated by R_{CCIN}^G, that imposes:

$$R_{CCIN}^G = \frac{v_{cg}^G}{v_s^G} = f(G). \tag{8}$$

The movement $m^D(\vec{v}_d)$ is the principal generation movement, so that $\vec{v}_d = \vec{v}_{as}^D$. At the generation by the proceedings of milling and grinding, the movement $m^D(\vec{v}_d)$ is a feed movement (right or circular), so that $\vec{v}_d = \vec{v}_s^D(\vec{v}_{sc}^{TD})$.

Down it is presented some complex surfaces by the second specie, with a large utilization in engineering.

A. Profiled rotation surfaces

These surfaces has the generatrix G a curve plane, with a certain form, and the directrix D a circle (Fig. 4).

For the generation of the generatrix G, the movement $m^G(\vec{v}_g)$ is realized by compounding the feed transversal movement of the generating element $E_g(M)$, that has the variable speed $\vec{v}_{sT}$ and that is the complementary generation movement of the generatrix G (so that, $\vec{v}_{cg}^G \equiv \vec{v}_{sT} \neq$ const.) with it longitudinal feed movement with a constant speed $\vec{v}_{sL}$ that is the movement for the advance for the generation of G (so that $\vec{v}_s^G \equiv \vec{v}_{sL} =$ const.). These movements are cinematically co-coordinated by R_{CCIN}^G that impose :

$$R_{CCIN}^G = \frac{v_{cg}^G}{v_s^G} = \frac{v_{sT}}{v_{sL}} = \text{tg}\ \alpha \neq \text{ const.}, \tag{9}$$

in which α is the slope between the tangents at the curve G and the OX axe, and have variable size and a sense depending on the geometrical form of the generatrix G.

The speed of the generating element $E_g(M)$ is given by cinematic law of the movement $m^G(\vec{v}_g)$ that has the expression:

$$\vec{v}_g = \vec{v}_s^G + \vec{v}_{cg}^G = \vec{v}sL + \vec{v}_{sT}. \tag{10}$$

In a synthesis, for the generation of the generatrix G like a trajectory of the movement of the generating element $E_g(M)$ in plane (the base plane XOY), obtained from

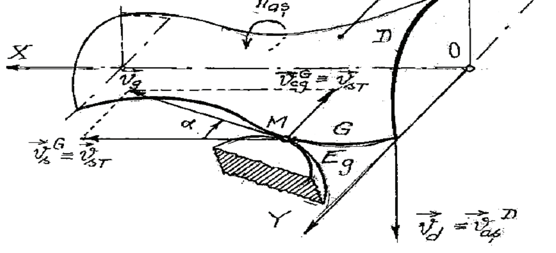

Fig. 4. The generation of a curve by composing movements.

compositions of right rectangular translation move-ments cinematically co-coordinated by R_{CCIN}^{G} it is necessary that the theoretical G is generated as a scheduled generatrix, and in this case R_{CCIN}^{G} is assured by a port program.

For the generation of the directrix D and implicitly for the generation of the profiled rotation surface (S), the movement $m^{D}(\vec{v}_{d})$ of the element $E_g(M)$ is realized by it principal rotation movement with the frequency n_{as} (so that $\vec{v}_{d} = \vec{v}_{as}^{D}$).

B. Conical right rotation surfaces

These complex surfaces are a particular case of the profiled rotation surfaces, respectively the case in which the generatrix G is a right line inclined in an axial plane against the rotation axis OX with the angle δ (Fig. 5).

2.3. Complex surfaces of the third specie

There are geometrical surfaces at which the both theoretical curves D and G are a geometrical form that imposes their generation by compositions of single movements cinematically coordinated by a R_{CCIN}^{G}, respectively R_{CCIN}^{D}. In this category with a large technical utilization, are included:

a. Involutes helicoidal cylindrical surfaces of the teeth of the cylindrical toothed wheels with inclined set of teeth.

b. Involutes helicoidal cylindrical surfaces of the teeth of the helicoidal toothed wheels.

c. Involutes curves surfaces with eloidal directrix.

d. Involutes curves surfaces with paloidal directrix.

e. Involutes curves surfaces with spiroidal directrix.

f. Certain surfaces with G and D scheduled.

Considering the geometrical form of the complex surfaces of the third specie, it results that their theoretical directrix D and theoretical generatrix G can be generate in one of the modes:

a_1. The cinematic directrix D, generated as a trajectory of a movement of a point in plane, obtained by composing sample movements cinematically co-coordinated by a R_{CCIN}^{D}.

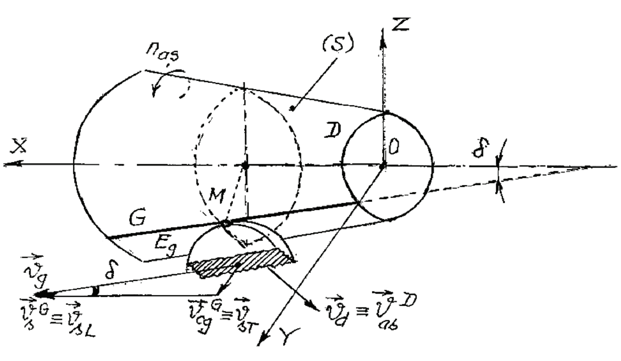

Fig. 5. The generation of a conical right rotation surface.

a_2. The cinematic directrix D, generated as a roller of the successive positions of a cinematic curve that is displaced on a spatial trajectory obtained by compositions of simple movements cinematically co-coordinated by a R_{CCIN}^{D}.

a_3. The cinematic directrix D, generated by transposition (printing) by rolling.

a_4. The scheduled directrix D.

b_1. The cinematic generatrix G, generated as an envelope of the successive positions of a curve C (conjugate of the G curve) materialized by the generating edge of the generating tool that displaced in plane.

b_2. The scheduled generatrix G.

We can observe that these kinds of generation of the two curves are a reunion of the modes for the generation of the D curve from the complex surface of the first specie with the modes of the generation of the curve G of the complex surfaces of the second specie.

We register in this way that at the generation of the complexes surface of the third specie, that are realized by the generator element $E_g(M)$, that is no more an element of the type $G_E(M)$, the movements $m^{D}(\vec{v}_{d})$ and $m^{G}(\vec{v}_{g})$ are made according to the cinematic laws of the generation of the complex surfaces of the first species, respectively of the second species.

3. CONCLUSIONS

The paper presents for the first time in the specialty literature original theoretical aspects concerning the cinematic generation of the complex surfaces. starting from the cinematic general principle of their generation, there are discovered and established the cinematic laws for the generation of the theoretical curves D and G of those surfaces. These laws make possible the classification of the complex surfaces in the three presented categories.

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