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ROUGHNESS APPRAISAL OF SURFACES PROCESSED THROUGH TURNING USING WIDE CUTTING TOOLS

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Abstract: *The implementation of computerized technology within product engineering requires the use of some data bases and objective mathematical models. As very important elements, these data bases and mathematical models must also contain information regarding the roughness of the processed surfaces. This fact is important because the technological route of products depends very much on the surface roughness. In the most of the cases the surface roughness is appraised in accordance with roughness criterion R_a. The present paper presents the method that helped obtaining a mathematical model for appraising the roughness R_a of the processed surface in case of turning using wide cutting tools, more precisely in comparison with the data met within specialized literature.*

Key words: *roughness, modeling, CAD/CAM, technology.*

1. INTRODUCTION

The issue of appraising the roughness of the processed surface, in case of turning using wide cutting tools, is treated within specialized literature. As it is known, some significant results regarding this problem were obtained by Sokolovski [12, 13]. These results were ulterior overtaken and presented within Romanian specialized literature [4–7]. But, in the most of the cases the problem is not correctly treated because the models that stood at the base of appraising the roughness of the processed surface take into consideration the criterion of maximum roughness $R_{\max}$ and not the criterion R_a. The criterion R_a is mostly used in product design. Thus, the present paper approaches a new way of appraising the roughness of the surfaces processed with wide cutting tools. This new approach is supported by the geometric interpretations regarding the generation of the asperities in accordance with the papers previously specified. The interpretation of the roughness R_a is also had in mind in accordance with paper [2].

2. APPRAISAL OF THE ROUGHNESS OF THE PROCESSED SURFACE IN ACCORDANCE WITH CRITERION R_a

2.1. Generation of micro irregularities

Having in mind the geometric considerations, on the basis of Fig. 1 the generation of micro irregularities is conducted by the variation of ray R_x.

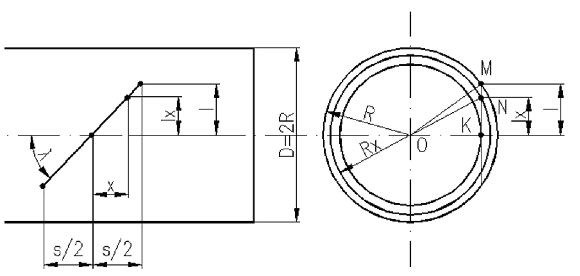

Fig. 1. Generation of micro irregularities.

This ray is calculated later within this paper.

It must be mentioned that for Figs. 1 and 2 the sources were the papers [4–7, 12, 13]. But, these figures from the mentioned papers stood at the basis of determining the maximum roughness $R_{\max}$ and not R_a.

The present paper is based on the same geometric considerations but approaches the issue in a different manner. The calculation way is different because the roughness R_a is had in mind.

From Fig. 1 the following formula results:

$$\operatorname{tg}\lambda = \frac{2\cdot l}{s} = \frac{l_x}{x}. \tag{1}$$

Knowing that:

$$R_x^2 = \overline{OK}^2 + l_x^2, \quad R^2 = \overline{OK}^2 + l^2. \tag{2}$$

ray R_x immediately results:

$$R_x = \operatorname{tg}\lambda \cdot \sqrt{\frac{R^2 - l^2}{\operatorname{tg}^2\lambda} + x^2}. \tag{3}$$

If the following notation is made:

$$A^2 = \frac{R^2 - l^2}{tg^2\lambda}, \tag{4}$$

it is obtained:

$$R_x = \operatorname{tg}\lambda \cdot \sqrt{A^2 + x^2}. \tag{5}$$

The variation of the ray R_x with value x leads to the generation of the profile of micro irregularities in accordance with Fig. 2.

Fig. 2 highlights the maximum height of micro irregularities. But the aim of this paper is to determine the micro irregularities in accordance with the roughness R_a. The criterion R_a, as it has been said, is mostly used in product design. Thus, in what comes next, this subject is approached, presenting, in the end of the paper, the formula that allows the appraisal of the roughness R_a.

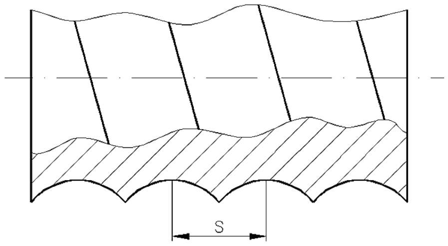

Fig. 2. Geometry of micro irregularities.

2.2. Interpretation of the roughness criterion R_a

By definition, the roughness R_a (Fig. 3) can be expressed through [2, 9–11]:

$$R_a = \frac{1}{L}\cdot\int_0^L |y(x)|\,\mathrm{d}\,x. \tag{6}$$

On the basis of formula (6) and Fig. 4, roughness R_a can be expressed as follows:

$$R_a = \frac{A_1 + A_2}{s}, \tag{7}$$

where A_1, respective A_2 represent the areas related to the reference length s for appraising the roughness R_a in accordance with the definition.

The areas A_1 and A_2 will be calculated below taking into consideration the definition of the roughness, in accordance with Fig. 3 and formula (6).

2.3. Calculation of areas A_1 and A_2

On the basis of Fig. 4 area A_1 can be determined by the means of the following formula:

$$A_1 = (R - h_2)\cdot b - 2\cdot tg\lambda\cdot\int_0^{\frac{b}{2}}\sqrt{A^2 + x^2}\,\mathrm{d}\,x. \tag{8}$$

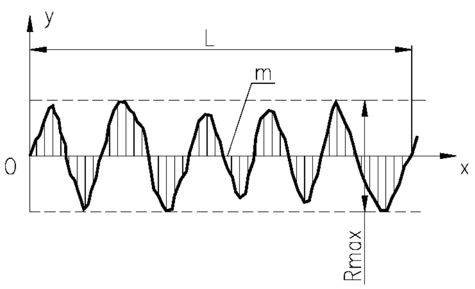

Fig. 3. R_a roughness.

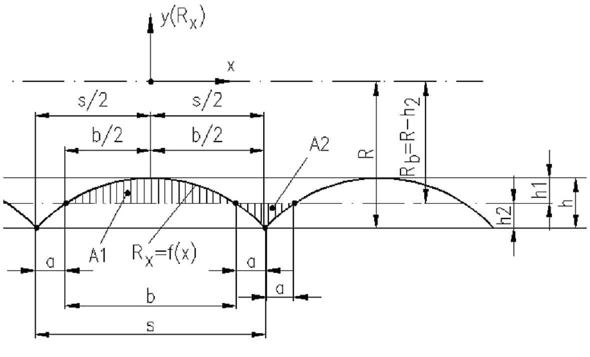

Fig. 4. Geometrical generation, detail.

Making the following notation:

$$E_1 = \int_0^{\frac{b}{2}}\sqrt{A^2 + x^2}\,\mathrm{d}\,x.$$

the solution of this integral is [3]:

$$E_1 = \int_0^{\frac{b}{2}}\sqrt{A^2 + x^2}\,\mathrm{d}x = \frac{x}{2}\sqrt{A^2 + x^2} + \frac{A^2}{2}\ln\left(x + \sqrt{A^2 + x^2}\right)\Bigg|_0^{\frac{b}{2}}. \tag{9}$$

or:

$$E_1 = \frac{b}{4}\sqrt{A^2 + \frac{b^2}{4}} + \frac{A^2}{2}\ln\left(\frac{b}{2} + \sqrt{A^2 + \frac{b^2}{4}}\right) - \frac{A^2}{2}\ln A. \tag{10}$$

Through Taylor (Mclaurin) development [1, 3] it is obtained:

$$\sqrt{A^2 + \frac{b^2}{4}} = A\cdot\left(1 + \frac{1}{2}\cdot\frac{b^2}{4\cdot A^2}\right). \tag{11}$$

and

$$\ln\left(\frac{b}{2} + \sqrt{A^2 + \frac{b^2}{4}}\right) = \ln A + \ln\left(\frac{b}{2\cdot A} + \sqrt{1 + \frac{b^2}{4\cdot A^2}}\right). \tag{12}$$

respective,

$$\ln\left(\frac{b}{2A} + \sqrt{1 + \frac{b^2}{4A}}\right) = \frac{b}{2A} - \frac{1}{6}\left(\frac{b}{2A}\right)^3 + \frac{3}{40}\left(\frac{b}{2A}\right)^5. \tag{13}$$

By replacing the expressions (11), (12) and (13) in expression (10) it is obtained:

$$E_1 = \frac{A\cdot b}{2} + \frac{b^3}{48\cdot A}. \tag{14}$$

Thus, the expression of area A_1 becomes:

$$A_1 = (R - h_2)\cdot b - 2\cdot \mathrm{tg}\lambda\cdot\left(\frac{A\cdot b}{2} + \frac{b^3}{48\cdot A}\right). \tag{15}$$

Of course, the values h_2 and b will be determined below, after the formula for A_2 is established. In a similar way, on the basis of Fig. 4 area A_2 can be expressed in the following way:

$$A_2 = 2\cdot \mathrm{tg}\lambda\cdot\int_{\frac{b}{2}}^{\frac{b}{2}+a}\sqrt{A^2 + x^2}\,\mathrm{d}x - (R - h_2)\cdot(s - b). \tag{16}$$

Knowing that (Fig. 4):

$$\frac{b}{2} + a = \frac{s}{2}. \tag{17}$$

and making the notation:

$$E_2 = \int_{\frac{b}{2}}^{\frac{s}{2}}\sqrt{A^2 + x^2}\,\mathrm{d}\,x. \tag{18}$$

a result similar with that from formula (14) is immediately obtained:

$$E_2 = \frac{A}{2} \cdot (s-b) + \frac{1}{48 \cdot A} \cdot (s^3 - b^3). \quad (19)$$

Thus, area A_2 can be expressed in the following way:

$$A_2 = 2tg\lambda \left[\frac{A}{2}(s-b) + \frac{1}{48A}(s^3 - b^3) \right] - (R - h_2)(s-b). \quad (20)$$

2.4. Establishing the position of the reference line

In accordance with the definition of roughness R_a, expressed through Fig. 3 and formula (6), the position of the reference line must result from the following condition:

$$A_1 = A_2. \quad (21)$$

Having in mind the formulas (15) and (20), in accordance with formula (21) it results:

$$h_2 = R - \text{tg}\,\lambda \cdot \left(A + \frac{s^2}{24 \cdot A} \right). \quad (22)$$

2.5. Calculation of the roughness R_a

On the basis of formula (7), knowing the expressions of areas A_1 and A_2 and expressing b as a function depending on s, as the following formula shows:

$$b = K_1 \cdot s. \quad (23)$$

It results:

$$R_a = (2K_1 - 1)(R - h_2 - A\text{tg}\lambda) - \frac{s^2\text{tg}\lambda}{12A} K_1^3 + \frac{s^2\text{tg}\lambda}{24A}. \quad (24)$$

Having also in mind the formula (22), the roughness R_a becomes:

$$R_a = \frac{s^2 \cdot \text{tg}\,\lambda}{12 \cdot A} \cdot (K_1 - K_1^3). \quad (25)$$

2.6. Determination of coefficient K_1

On the basis of Fig. 5, for the point N the following formula system can be written, observing that $x = b/2$:

$$\begin{cases} R_N = \text{tg}\,\lambda \cdot \sqrt{A^2 + \frac{b^2}{4}} \\ R_N = R - h_2 \\ h_2 = R - \text{tg}\,\lambda \cdot \left(A + \frac{s^2}{24 \cdot A} \right) \end{cases}. \quad (26)$$

The solution of the system (26) is:

$$b = s \cdot \sqrt{\frac{1}{3}} = 0.577350269 \cdot s. \quad (27)$$

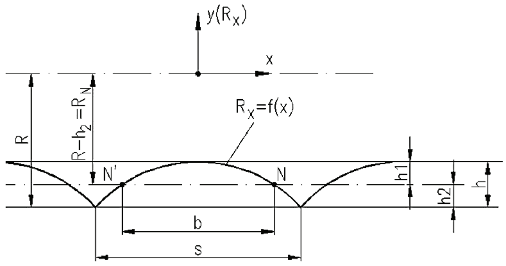

Fig. 5. Detail.

from where it immediately results:

$$K_1 = 0.577350269. \quad (28)$$

Thus, on the basis of formula (25) and taking into consideration the substitution (4) the roughness of the processed surface in case of turning using wide cutting tools can be appraised through the following expression:

$$R_a = \frac{s^2 \cdot \text{tg}^2\lambda}{15.58845727 \cdot D} \quad (29)$$

or

$$R_a = 0.064 \cdot \frac{s^2 \cdot \text{tg}^2\,\lambda}{D}. \quad (30)$$

3. CONCLUSIONS

The formula (30) regarding the appraisal of the processed surface in case of turning with wide cutting tools represents a novelty in comparison with the data met within specialized literature, which recommends the following formula obtained by Sokolovski team and overtaken by Romanian literature for appraising the maximum roughness $R_{\max}$ and not R_a [4–7]:

$$R_{\max} = \frac{s^2 \cdot \text{tg}^2\,\lambda}{4 \cdot D} = 0.25 \cdot \frac{s^2 \cdot \text{tg}^2\,\lambda}{D}. \quad (31)$$

It results that roughness $R_{\max}$ and roughness R_a are connected through the following formula:

$$R_{\max} = 4 \cdot R_a. \quad (32)$$

On the other hand in the most of the cases the surfaces roughness is specified through the criterion R_a on the work drawing and thus, the formula (30) is better than formula (31).

In another two cases the formula (32) confirms its validity [2] and [8].

Thus, for cases of turning processing with cutting tools having no nose ray and having nose ray the formulas for appraising the roughness of processed surfaces are [2, 8, 5]:

$$R_a = 250 \cdot \frac{s}{\text{tg}\,\chi + \text{tg}\,\chi_1} = \frac{R_{\max}}{4}.$$

in case of cutting tools with no nose ray and,

$$R_a = 32 \cdot \frac{s^2}{r} \cdot K_3 = \frac{R_{\max}}{4}.$$

for cutting tools having a nose ray. In both of the formulas the roughness are given in microns.

The angles χ, χ_1 are the principle and secondary approaching angles.

Regarding the formula (30) another observation must also be made. It resulted from geometric considerations. For being an objective expression, formula (30) must be corrected with a coefficient K_2 that takes into consideration the elastic – plastic deformations of the material to be processed, meaning:

$$R_a = 0.064 \cdot \frac{s^2 \cdot \text{tg}^2\,\lambda}{D} \cdot K_2. \quad (33)$$

Future papers present results regarding this coefficient.

The formula (33) is also very important from manufacturing point of view. Such a formula must be compulsory a part of the mathematical mechanism of determining the optimal cutting parameters because the roughness R_a conditions the working parameters (especially the feed). Of course, the mathematic instrument that helps optimizing the cutting parameters contains other conditions, too, which are different from the condition regarding the roughness of the processed surface. On the basis of such reasoning it results that the mathematic model destined to optimization of working cutting parameters could be a model based on mathematic programming.

For example, the cutting parameters in case of processing through turning must result on the basis of the following model:

$$\begin{cases} \min C / \max Q = f(v, s, T) \\ r_i(v,\ s,\ T) <= a_i \quad (i = 1,\ 2,\ ...,\ m), \\ v,\ s,\ T > 0 \end{cases} \tag{34}$$

where: v, s, T are the speed, feed and tool life. C, Q are the cost and the productivity of the processing. The vector $r_i(v, s, T)$ refers to the m particular conditions for the processing. Among these conditions, in case of turning, one of the constraints must be found:

$$s <= \sqrt{(R_a \cdot D)/(64 \cdot \mathrm{tg}^2\,\lambda \cdot K_2)}\,. \tag{35}$$

if we consider the turning with wide cutting tools or [8]:

$$s <= \sqrt{(R_a \cdot r)/(32 \cdot K_3)}\,, \tag{36}$$

where r is the nose ray and K_3 has the same meaning as coefficient K_2.

Within formulas (35) and (36) the roughness of the processed surfaces must be considered in microns. Of course, the constraints regarding the roughness of the processed surface have a great importance especially for finishing processes.

Formula (33) is also very important taking into consideration the data base related to the CAD/CAM systems. In other words, the parameters which are characteristic for the formulas (35) and (36) must be stored within the common data base.

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