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AN OPTIMIZATION OF TYPE WILDHABER GEAR SETS SYNTHESIS BASED ON LOADING CAPACITY. KINEMATIC APPROACH TO CRITERIA CONSTRUCTION

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Abstract: *The contact hydrodynamic theory of lubrication takes into account hydrodynamic and heat processes in the lubrication film and the elastic deformations of the solid surfaces being in contact. In the gear-drives the load is carried by the conjugate tooth surfaces and by the lubrication film between them. Two mutual connected problems – a hydrodynamic lubrication problem and a contact problem of elasticity are presented. The paper does not give a mutual solution of the above problems. The purpose is to suggest a quality estimate of the influence of some characteristics on the gear pair loading capacity. A kinematic approach to defining characteristics for estimating the hydrodynamic loading capacity of the Wildhaber type gearing is presented.*

Key words: *Wildhaber worm gear drive, mathematical model, tooth contact synthesis, Olivier's principles.*

1. INTRODUCTION

In the present, the wormgears with cylindrical worm are the most widely applied as power transmissions [1–4]. In the class of wormgears, a special place occupies the gear pair of type Wildhaber. It consists of cylindrical gear and a globoid worm (with a toroid form) that envelops the cylindrical gear (their rotation axes are perpendicular). The cylindrical gear active tooth surfaces are planes parallel to its axis of rotation and the worm active tooth surfaces are envelopes of the gear plane teeth.

From a geometric viewpoint this gear pair can be treated as a hybrid between the cylindrical wormgears and a double enveloping gear-set since: Wildhaber gear-set similarly to cylindrical wormgears is with a single enveloping, the enveloped gear is the wormgear and it has a cylindrical form; The driving link of the gear pair is the worm, it has a toroid form that corresponds to the globoid gears geometry.

From a technological point of view, the gear-set of type Wildhaber is similar both to cylindrical wormgears and to globoid ones as its technological synthesis and manufacture are based on the second Olivier's principle. They are technologically different because of the different ways of generation of the instrumental gear tooth surfaces, namely: In the case of gear-set with cylindrical worm, the basic gear is the cylindrical worm whose active tooth surfaces are generated in a helical motion of the generating line (usually a straight-line); in the case of globoidal wormgears, the basic gear is the globoid worm whose active tooth surfaces are generated by a straight-line that performs two rotations with definite angular velocities; in the case of Wildhaber gear-set, the basic gear is the cylindrical plane-teeth gear whose plane teeth surfaces are cut by a milling cutter on the standard milling machine, the globoid worm of type Wildhaber is generated according to the second Olivier's principle on a hobbing machine.

In USA this type of gears is successfully applied as kinematic one (dividing head), and as a power transmission [5].

The study object in this article is a gear of type Wildhaber when the axes of rotation (the geometric axes of the moving links) are non-orthogonal crossed. When are synthesized the gears, kinematic criteria for quality control, which are based on the loading capacity are presented.

2. BASIC OF MODELLING FOR SYNTHESIS

The specific geometry of the skew-axes hyperbolic gear-set of type Wildhaber included the method for active tooth surfaces generation define the chosen approach to mathematical modeling of its synthesis. When synthesizing hyperbolic gears with a linear contact between their tooth surfaces, it is evident the necessity to control the quality of meshing in the whole region of mesh or in a definite part of this region [6, 7]. This approach based on "the region of mesh" is obligatory because the plane flanks have longitudinal orientation preliminary defined.

The mathematical models for synthesis based on region of mesh are not universal. The specifics of each model determines by the geometric and kinematic characteristics of the concrete region of mesh.

The kinematic scheme of the considered gear-set is shown in Fig. 1. There D_1 and D_2 are the reference surfaces of the gears that are surfaces of revolution with axes of rotations 1–1 and 2–2, respectively: D_2 is a cylinder, D_1 is an envelope of D_2. This means that D_1 and D_2 are kinematically conjugate. We use the following frames (Fig. 1): $S(O, x, y, z)$ – stationary one; $S_i(O_i, x_i, y_i, z_i)$, $(i = 1, 2)$ – firmly connected with the moving links $i = 1$ and $i = 2$ that rotate about the axes i–i, $(i = 1, 2)$ with angular velocities $\overline{\omega}_1$ and $\overline{\omega}_2$.

Let describe the equation of the active tooth surfaces $\Sigma_2^{(j)}$ of the plane-teeth gear $i = 2$ in the frame S_2 using Fig. 1 ($\varphi_2 = 0$):

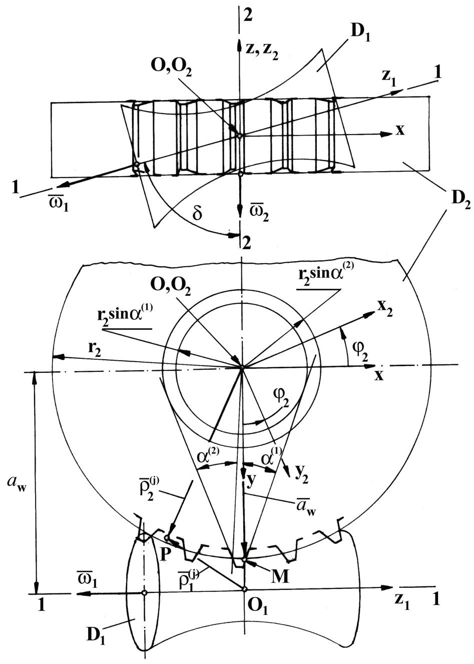

Fig. 1. Geometric and kinematic scheme of hyperbolic gear pair type Wildhaber.

$$
\begin{aligned}
x_2^{(j)} &= u^{(j)} \cdot \sin\alpha^{(j)}, \\
y_2^{(j)} &= u^{(j)} \cdot \cos\alpha^{(j)} + r_2, \\
z_2^{(j)} &= \tau^{(j)},
\end{aligned} \qquad (1)
$$

where $u^{(j)}$, $\tau^{(j)}$ are the linear parameters of the active tooth surface $\Sigma^{(j)}$; $\alpha^{(j)}$ is the profile angle of plane teeth; r_2 is the radius of the reference surface of plane-teeth gear.

In equations (1) $j = 1$ refers to the conjugate surfaces $\Sigma_1^{(1)}$ and $\Sigma_2^{(1)}$ when the gears rotate with angular velocities $\overline{\omega}_i$ $(i = 1, 2)$ (Fig. 1) and the globoid Wildhaber worm is the driving link. When the worm $i = 1$ is a driving link, rotates with an angular velocity $(-\overline{\omega}_1)$ and forces cylindrical gear $i = 2$ to rotate with an angular velocity $(-\overline{\omega}_2)$, then the tooth surfaces $\Sigma_1^{(2)}$ and $\Sigma_2^{(2)}$ are in contact.

For the parameters that take place in set (1) the following conditions are true: $u^{(j)} > 0$ for the tooth addendum; $u^{(j)} < 0$ for the tooth dedendum; $\tau^{(j)} \in [-0.5 \cdot b_2,\ 0.5 \cdot b_2]$; ($b_2$ is the cylindrical gear face width); $\alpha^{(1)} \in (0,\ 0.5 \cdot \pi)$, $\alpha^{(2)} \in (-0.5 \cdot \pi,\ 0)$.

Writing the analytical expression of the region of mesh of the synthesized gear pair requires defining the equation of meshing. In the concrete case it is of the form:

$$
\overline{V}_{12}^{(j)} \cdot \overline{n}_2^{(j)} = V_{12,x}^{(j)} \cdot n_{2,x}^{(j)} + V_{12,y}^{(j)} \cdot n_{2,y}^{(j)} + V_{12,z}^{(j)} \cdot n_{2,z}^{(j)} = 0, \qquad (2)
$$

where $\overline{V}_{12}^{(j)}$ is the velocity of the relative motion in an arbitrary contact point P of $\Sigma_1^{(j)}$ and $\Sigma_2^{(j)}$ with coordinates $V_{12,x}^{(j)}$, $V_{12,y}^{(j)}$, $V_{12,z}^{(j)}$ in the frame S; $\overline{n}_2^{(j)}$ is the normal vector to $\Sigma_2^{(j)}$ with coordinates $n_{2,x}^{(j)}$, $n_{2,y}^{(j)}$, $n_{2,z}^{(j)}$ in S.

Taking into account the constructions made in Fig. 1 the coordinates of $\overline{V}_{12}^{(j)}$ are:

$$
\begin{aligned}
V_{12,x}^{(j)} &= (\cos\delta - i_{21}) \cdot y^{(j)} - a_w \cdot \cos\delta, \\
V_{12,y}^{(j)} &= \sin\delta \cdot z^{(j)} - (\cos\delta - i_{21}) \cdot x^{(j)}, \\
V_{12,z}^{(j)} &= (a_w - y^{(j)}) \cdot \sin\delta,
\end{aligned} \qquad (3)
$$

where $\delta = \angle(\overline{\omega}_1, \overline{\omega}_2)$ is the angle between the axes of rotations 1–1 and 2–2. In (3) it is assumed that $\omega_1 = 1$ rad/s whence $\omega_2 = i_{21}$ rad/s (ω_i is the magnitude of the vector $\overline{\omega}_i$, $(i = 1, 2)$). The coordinates of $\overline{n}_2^{(j)}$ and the coordinates $x^{(j)}$, $y^{(j)}$ and $z^{(j)}$ of the contact points of $\Sigma_1^{(j)}$ and $\Sigma_2^{(j)}$ in the stationary frame S are described by the relations:

$$
\begin{aligned}
& n_{2,x}^{(j)} = \cos A^{(j)}, \\
& n_{2,y}^{(j)} = -\sin A^{(j)}, \quad n_{2,z}^{(j)} = 0, \\
& A^{(j)} = (\alpha^{(j)} + \varphi_2);
\end{aligned} \qquad (4)
$$

$$
\begin{aligned}
x^{(j)} &= r_2 \cdot \sin\varphi_2 + u^{(j)} \cdot \sin A^{(j)}, \\
y^{(j)} &= r_2 \cdot \cos\varphi_2 + u^{(j)} \cdot \cos A^{(j)}, \\
z^{(j)} &= \tau^{(j)}, \\
A^{(j)} &= (\alpha^{(j)} + \varphi_2).
\end{aligned} \qquad (5)
$$

Substituting (3), (4) and (5) in (2), the analytical expression of the equation of meshing is:

$$
\begin{aligned}
& (u^{(j)} + r_2 \cdot \cos\alpha^{(j)}) \cdot (\cos\delta - i_{21}) - \tau^{(j)} \cdot \sin\delta \cdot \sin A^{(j)} - \\
& \quad - a_w \cdot \cos\delta \cdot \cos A^{(j)} = 0, \\
& \quad A^{(j)} = (\alpha^{(j)} + \varphi_2).
\end{aligned} \qquad (6)
$$

Mutual solving of equations (5) and the equation of meshing (6) gives the contact lines of the active tooth surfaces $\Sigma_1^{(j)}$ and $\Sigma_2^{(j)}$ of the globoid Wildhaber worm and of the cylindrical plane-teeth gear in the stationary space. They form the region of mesh of the synthesized gear pair.

Analyzing (5) and (6) it is seen that the common point M of the reference surfaces D_1 and D_2 of the gears (globoid worm of type Wildhaber and of the cylindrical plane-teeth gear) with coordinates $x^{(j)} = 0$, $y^{(j)} = r_2$, $z^{(j)} = 0$, $(u^{(j)} = \tau^{(j)} = \varphi_2 = 0)$ (Fig. 1), is not a common point of the active tooth surfaces $\Sigma_1^{(j)}$ and $\Sigma_2^{(j)}$ in the most common case because of the fact that their coordinates satisfy the equation of meshing (5) if the condition

$$
(a_w - r_2) \cdot \cos\delta + i_{21} \cdot r_2 = 0 \qquad (7)
$$

is fulfilled. It is possible only in one concrete case of non-orthogonal globoid gear-pair of type Wildhaber when the shaft angle $\delta = \measuredangle(\overline{\omega}_1, \overline{\omega}_2)$ is determined by $\cos\delta = i_{21} \cdot r_2 \cdot (r_2 - a_w)^{-1}$. It is clear that this value of δ belongs to the interval $(0.5 \cdot \pi,\ 1.5 \cdot \pi)$.

Therefore, excepted the case defined by (7) it is impossible to define a pitch contact point (a pole of meshing) and pitch circles respectively when considering a hyperbolic gears of type Wildhaber with non-orthogonally skewed axes. The mathematical model under consideration defines the algorithm for basic synthesis based on region mesh of the examined gear type. The optimisation of their design needs the definition of criteria controlling their quantative characteristics of the whole region mesh. The optimization synthesis realises by a control of the kinematics (respectively geometrical) characteristics using some criteria that are presented analytically bellow.

3. KINEMATIC CRITERIA FOR OPTIMIZATION ON LOADING CAPACITY

The contact hydrodynamic theory of lubrication as a part of fluid mechanics aims at explaining the phenomenon of mutual action both between solid bodies moving in a fluid so as of solid bodies and the fluid. This theory takes into account hydrodynamic and heat processes in the lubrication film and the elastic deformations of the solid surfaces being in contact. In the gear-drives the load is carried by the conjugate tooth surfaces and by the lubrication film between them. As a result a hydrodynamic pressure between the tooth surfaces appears. The hydrodynamic elevating force balances the external forces applied to the gears. At the same time, the hydrodynamic pressure causes elastic deformation of the contacting tooth surfaces that influences on the backlash configuration between them. They define the character of the diagram of the hydrodynamic pressure. Thus two mutual connected problems – a hydrodynamic lubrication problem and a contact problem of elasticity are presented. The paper does not give a mutual solution of the above problems. The purpose is to suggest a quality estimate of the influence of some characteristics (going in their basic equations) on the gear pair loading capacity. A kinematic approach to defining characteristics for estimating the hydrodynamic loading capacity of the Wildhaber type gearing is presented.

3.1. Singular points of first order

The analysis of the character and the position of the contact lines in the mesh region is of great significance before constructing the mathematical model for synthesis based on the region of mesh. The presence of singular points of first order on the contact lines yields to a decrease of the efficiency and loading capacity of the designed gears since in these points a semi-dry and dry frictions can appear.

The so-called **ordinary nodes** are the points of contact or of intersection of the contact lines. In such points the oil film between the tooth surfaces breaks and the pressure in it decreases.

The condition for existence of ordinary nodes in the region of mesh of hyperbolic gears of type Wildhaber is:

$$\frac{\partial}{\partial \varphi_2}\left(\overline{V}_{12}^{(j)} \cdot \overline{n}_2^{(j)}\right) = 0. \tag{8}$$

Then taking into account (5) and (6) we obtain the following analytical form of the line of ordinary nodes in the stationary space:

$$\begin{aligned}
&x^{(j)} = r_2 \cdot \sin\varphi_2 + u^{(j)} \cdot \sin A^{(j)},\\
&y^{(j)} = r_2 \cdot \cos\varphi_2 + u^{(j)} \cdot \cos A^{(j)},\\
&z^{(j)} = \tau^{(j)},\\
&(u^{(j)} + r_2 \cdot \cos\alpha^{(j)}) \cdot (\cos\delta - i_{21}) - \tau^{(j)} \cdot \sin\delta \cdot \sin A^{(j)} - \\
&\quad - a_w \cdot \cos\delta \cdot \cos A^{(j)} = 0,\\
&\tau^{(j)} \cdot \sin\delta \cdot \cos A^{(j)} - a_w \cdot \cos\delta \cdot \sin A^{(j)} = 0,\\
&A^{(j)} = (\alpha^{(j)} + \varphi_2).
\end{aligned} \tag{9}$$

The equations (9) defined an ordinary nodes line in the mesh region. The objective of the optimization synthesis is the entire removal of singular points of first order from the mesh region. This task can be achieved by a suitable variation of the geometrical gear characteristics, the dimensions of the active teeth surfaces, dimensions of the crude metal for the gear wheels and so on.

3.2. A total circumferential velocity orientation in the mesh region

The theory of hydrodynamics of the modern hyperbolic gear-sets with a linear contact is not sufficiently worked out. Although, one quality estimate could be the value of the angle between the total circumferential velocity $\overline{V}_{\Sigma}^{(j)}$ and the tangent to the contact line in its arbitrary point. In the concrete case it is the angle between $\overline{V}_{\Sigma}^{(j)}$ and the straight-line $D_{12}^{(j)}$, that is the contact line relevant to a definite parameter of meshing φ_2.

First we define the position of an arbitrary contact line $D_{12}^{(j)}$ in the stationary frame $S(O, x, y, z)$. For each parameter φ_2, we will represent $D_{12}^{(j)}$ as a common straight-line of two planes: the tooth surface $\Sigma_2^{(j)}$ and the plane given with the equation of meshing. Taking into account the set of equations (5) and (6) we represent the region of mesh in the form:

$$\begin{aligned}
&\cos\alpha_{x,\Sigma_2}^{(j)} \cdot x^{(j)} + \cos\alpha_{y,\Sigma_2}^{(j)} \cdot y^{(j)} + \cos\alpha_{z,\Sigma_2}^{(j)} \cdot z^{(j)} - \\
&\quad - p_{\Sigma_2}^{(j)} = 0,\\
&\cos\alpha_{x,f}^{(j)} \cdot x^{(j)} + \cos\alpha_{y,f}^{(j)} \cdot y^{(j)} + \cos\alpha_{z,f}^{(j)} \cdot z^{(j)} \\
&\quad - p_f^{(j)} = 0,
\end{aligned} \tag{10}$$

where:

$$A^{(j)} = (\alpha^{(j)} + \varphi_2),$$

$$\cos\alpha_{x,\Sigma_2}^{(j)} = \pm\cos A^{(j)}, \quad \cos\alpha_{y,\Sigma_2}^{(j)} = \mp\sin A^{(j)},$$

$$\cos\alpha_{z,\Sigma_2}^{(j)} = 0,$$

$$\cos\alpha_{x,f}^{(j)} = -\frac{(\cos\delta - i_{21}) \cdot \sin A^{(j)}}{\sqrt{(\cos\delta - i_{21})^2 + \sin^2\delta \cdot \sin^2 A^{(j)}}},$$

$$\cos\alpha_{y,f}^{(j)} = -\frac{(\cos\delta - i_{21})\cdot\cos A^{(j)}}{\sqrt{(\cos\delta - i_{21})^2 + \sin^2\delta\cdot\sin^2 A^{(j)}}},$$

$$\cos\alpha_{z,f}^{(j)} = \frac{\sin\delta\cdot\sin A^{(j)}}{\sqrt{(\cos\delta - i_{21})^2 + \sin^2\delta\cdot\sin^2 A^{(j)}}},$$

$$p_{\Sigma_2}^{(j)} = r_2\cdot\sin\alpha^{(j)} > 0,$$

$$p_f^{(j)} = \frac{a_w\cdot\cos\delta\cdot\cos A^{(j)}}{\sqrt{(\cos\delta - i_{21})^2 + \sin^2\delta\cdot\sin^2 A^{(j)}}} > 0.$$

The first equation in (10) is the normal equation of the plane $\Sigma_2^{(j)}$, and the second one is the equation $f(x^{(j)}, y^{(j)}, z^{(j)}) = 0$. Their normal vectors are:

$$\overline{e}_2^{(j)} \equiv \overline{n}_2^{(j)}(\cos\alpha_{x,\Sigma_2}^{(j)}, \cos\alpha_{y,\Sigma_2}^{(j)}, \cos\alpha_{z,\Sigma_2}^{(j)}),$$

$$\overline{e}_f^{(j)}(\cos\alpha_{x,f}^{(j)}, \cos\alpha_{y,f}^{(j)}, \cos\alpha_{z,f}^{(j)}).$$

Then the coordinates of the unit vector $\overline{e}_D^{(j)} = \overline{e}_2^{(j)} \times \overline{e}_f^{(j)}$ of the contact line $D_{12}^{(j)}$ are

$$A^{(j)} = (\alpha^{(j)} + \varphi_2),$$

$$\cos\alpha_{x,D}^{(j)} = \mp\frac{\sin\delta\cdot\sin^2 A^{(j)}}{\sqrt{(\cos\delta - i_{21})^2 + \sin^2\delta\cdot\sin^2 A^{(j)}}},$$

$$\cos\alpha_{y,D}^{(j)} = \mp\frac{\sin\delta\cdot\sin A^{(j)}\cdot\cos A^{(j)}}{\sqrt{(\cos\delta - i_{21})^2 + \sin^2\delta\cdot\sin^2 A^{(j)}}},$$

$$\cos\alpha_{z,D}^{(j)} = \mp\frac{\cos\delta - i_{21}}{\sqrt{(\cos\delta - i_{21})^2 + \sin^2\delta\cdot\sin^2 A^{(j)}}}.$$

The upper (lower) signs in all equations refer to the case when are mated $\Sigma_1^{(j)}$ and $\Sigma_2^{(j)}$, when $j = 1$ ($j = 2$).

Let express the total circumferential velocity vector using the notations in Fig. 1:

$$\overline{V}_\Sigma^{(j)} = \overline{\omega}_1 \times (\overline{\rho}_2^{(j)} - \overline{a}_w) + \overline{\omega}_1 \times \overline{\rho}_2^{(j)} =$$
$$= V_{\Sigma,x}^{(j)}\cdot\overline{i} + V_{\Sigma,y}^{(j)}\cdot\overline{j} + V_{\Sigma,z}^{(j)}\cdot\overline{k}.$$

Its coordinates are:

$$V_{\Sigma,x}^{(j)} = (i_{21} + \cos\delta)\cdot y^{(j)} - a_w\cdot\cos\delta,$$

$$V_{\Sigma,y}^{(j)} = z^{(j)}\cdot\sin\delta - (i_{21} + \cos\delta)\cdot x^{(j)},$$

$$V_{\Sigma,z}^{(j)} = (a_w - y^{(j)})\cdot\sin\delta.$$

Then the angle $\Omega_\varphi^{(j)}$ that the vector $\overline{V}_\Sigma^{(j)}$ forms with the contact line of the gear-set of type Wildhaber is calculated by:

$$\cos\Omega_\varphi^{(j)} = \frac{V_{\Sigma,x}^{(j)}\cdot\cos\alpha_{x,D}^{(j)} + V_{\Sigma,y}^{(j)}\cdot\cos\alpha_{y,D}^{(j)} + V_{\Sigma,z}^{(j)}\cdot\cos\alpha_{z,D}^{(j)}}{\sqrt{(V_{\Sigma,x}^{(j)})^2 + (V_{\Sigma,y}^{(j)})^2 + (V_{\Sigma,z}^{(j)})^2}}. \quad (11)$$

Improved conditions for hydrodynamic lubrication, an increased hydrodynamic loading capacity respectively are clearly observed if $\Omega_\varphi^{(j)}$ has a value close to 90°.

4. CONCLUSION

In the article are illustrated the most essential and specific geometric and technological characteristics and related with them exploitation special features of the globoid gear-set of type Wildhaber. The chosen approach to mathematical modeling for the synthesis of the examined hyperbolic three-links mechanisms when the rotation axes of their moving links are arbitrarily skewed has been grounded. An algorithm oriented to the analysis and synthesis of these gear pairs is worked out: the active tooth surfaces and the region of mesh are defined analytically, the singularity of the gear-set is studied (the positions of the ordinary nodes are determined); algorithm for determining the orientations of the total circumferential velocity with respect to an arbitrary contact line is elaborated. Computer program for their study and optimizing synthesis on the basis of loading capacity is written. Concrete patterns of these gears are calculated, design and manufactured.

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