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AN OPTIMIZATION OF TYPE WILDHABER GEAR SETS SYNTHESIS BASED ON LOADING CAPACITY. A SOFTWARE ESTIMATE OF THE HYDRODYNAMIC LOADING CAPACITY

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Abstract: *In Part 1 a quality analysis has been performed based on solving the isotherm static contact hydrodynamic problem. On this basic criteria for quality estimate of the loading capacity of hyperbolic gears of type Wildhaber are worked out. They are used in a program for optimizing synthesis of Wildhaber gearings w. r. t. loading capacity. The paper is an element of a mathematical model, algorithm and program for synthesis and design of skew-axes gears of type Wildhaber.*

Key words: *Wildhaber worm gear drive, mathematical model, tooth contact synthesis, Olivier's principles.*

1. INTRODUCTION

High present-day requirements to the characteristics that refer to accuracy and reliability of the machines, devices and contrivances applied in industry and transport, to a great extent depend on scientific methods for technological synthesis, design and manufacture. In the processes of synthesis and design of different types of gears, the designer should resolve exclusively complex problems, which considered all together define the optimal construction. In this case, the optimal construction means a gear transmission that could insure preliminary given kinematic and/or strength requirements in condition of minimum expenses for design, manufacture and exploitation (including expenses for repair in the exploitation process). These requirements are related to different quality characteristics of the gears, namely [1]:

- geometric ones which control the kinematic exactness, the smoothness in working process, the character of the contact (*i.e.* the position of the spot of contact), etc.;
- dynamic ones by which one may influence on the noise and vibrations of the gear drive, on the conditions for appearance of resonance phenomena, etc.;
- strength ones which determine the durability and the reliability of the gear set, etc.;
- economic ones that define the expenses for manufacture of a unity of power, the loss of energy in rotations transformation, etc.

However, the realization of an optimal variant of construction of a gear set becomes difficult because of the modern technological possibilities of gear set manufacture. This is the reason for a limited introducing in industry of gear transmissions with new exploitation characteristics.

Therefore, having in mind the large number of requirements to the quality characteristics of the gears and in relation with really existing technological and exploitation limitations, it is evident that the solution of a real problem for creating an optimal gear set construction is not exact – it is close to the theoretical solution.

Creating real gear set requires take into account desired quality characteristics of the synthesized gears, existing technological possibilities and a manufacture equipment in the gear producing.

2. ASPECTS OF THE GEAR SETS COMPUTER DESIGN

Different kinds of gear sets, applied in industry and transport, as reduction drives as well as the unceasing scientific ambition for investigate new and more modern gearing on one hand, and on the other hand – the different and varying approaches to mathematical modeling of synthesis and design make impossible to construct an universal CAD system. It should be specially mentioned that the hardware and software improve very fast.

For scientific researchers in the field of the gearing theory as well as for performing an adequate scientific support of this kind of manufacture, the computer design has improved forming three types software:

First type. Here are attached programs intended to examine different constructive, technological and exploitation parameters of the studied gear pairs. This type of software does not obey a definite strategy, related to the CAD systems construction. The elaborated mathematical models, algorithms and computer programs aim at settling the influence of ones or others real existing parameters over the quality characteristics of concrete gearings. The created programs could be used as program modules representing elements of criteria systems, which will control the quality of the synthesized gear pairs.

Second type. This group consists of computer programs based on algorithms including in standards [2], in firm methodics or in handbooks [3]. Here are attached programs constructed on algorithms for geometric and strength calculation of traditional types of gear pairs as cylindrical involute or wormgear ones. It should be specially mentioned that suitable for use algorithms don't insure an optimization when synthesizing and designing gear pairs. For example, the well-known algorithms for synthesis of cylindrical involute gears, conical gears and cylindrical wormgear pairs are able to insure only the technological possibilities of their manufacture by using standard modules (pitches) and by eliminating the appearance of undercutting of the active tooth surfaces. On second place, here may be attached computer programs for verifying the strength characteristics of already synthesized in geometric and technological aspects gear pairs. Thus, these computer programs can be used as gear pairs analysis.

Third type. Here are attached computer programs that are based on mathematical models in result of scientific researches. The modern gear transmissions require the construction of new mathematical approaches to their geometric, technological and strength synthesis. The process of optimizing synthesis and design is based on adequate iteration procedures to find out an optimal solution by varying definite parameters.

3. APPROACH TO CONSTRUCTING COMPUTER PROGRAM FOR TYPE WILDHABER GEARS CALCULATION

Computer program that is used in the Wildhaber gear synthesis and design belong to the third type of software discussed above. Taking into account some methodological aspects when constructing this software, the authors of the present article follow a definite sequence in Wildhaber gear pairs computer design.

3.1. Definition of basic principles and approaches to mathematical modeling of Wildhaber gear pairs

When profiling kinematically related surfaces by which is realized a transformation of rotations between skewed axes, the basic principles that have to be observed are known as Olivier's principles in the meshing theory literature [4].

When forming the surfaces Σ_1 and Σ_2 firmly connected with the bodies T_1 and T_2 by the generating (instrumental) surface Σ_{1J} Σ_J firmly connected with T_J, the relative motion T_1/T_J and T_2/T_J may be arbitrary. From a practical point of view it is of interest to consider theses cases when the absolute motions of T_1, T_2, and T_J are rotations. Let the surface Σ_2 be one parameter envelope of the generating surface Σ_J and the conditions $\Sigma_J \equiv \Sigma_1$, $T_J \equiv T_1$, $T_J/T_2 \equiv T_1/T_J$ be fulfilled. This method of generation of kinematically connected surfaces Σ_1 and Σ_2 is known as Olivier's second principle. Thus, a linear contact between Σ_1 and Σ_2 is always present.

When defining the above conditions, the technologic features of conjugate surfaces generation (characterizing by some deviations of Σ_J geometric form from Σ_1 one) are not treated. The technologic motion of Σ_J which starts in the initial generation process moment and finishes in the final one is not considered too.

3.2. Mathematical modeling related to the gear pair quality characteristic in the whole mesh region

The authors' opinion about the mathematical modeling of skew-axes gears of type Wildhaber whose tooth synthesis obeys the Olivier's second principle is presented here. In this case is synthesized a skew-axes gear pair with guaranteed quality characteristics in one point of contact, and in its vicinity, respectively. This is achieved by a choice of a suitable point in the stationary space as a point belonging to the region of mesh aiming at insuring the optimal set of quality characteristics, such as: accuracy of realization of the necessary motions transformation function, conditions for effective driving energy utilization, etc. These characteristics which control the gearing quality in only one contact point, define the technologic strategy for a manufacture of the whole gear set, and its price, respectively.

The mathematical modeling method is based on the following. Some contact point of a pair of conjugate surfaces is a common point of two circles whose centers lie on the axes between which is performed the rotations transformation and whose planes are perpendicular to the described axes. These circles have been called by the authors *pitch circles* because of the fact that their diameters and mutual position in the space are in relation with gear dimensions and gears blanks, respectively, but with the helix angle in the contact point and with the pitch of the pitch helix. In that reason, in the synthesis problem the basic contact point is known as a pitch point [5]. The simplest surfaces of revolution (cone or cylinder) that are uniquely defined by these circles are known as primary or pitch surfaces.

The mathematical modeling using one contact point is most widely applied when synthesizing skew-axes gear pairs. But its application requires an information about specific kinematic, dynamic and strength characteristics of the examined gears. The satisfaction of the desired quality indices depends on the optimal pitch point place choice.

4. ELEMENTS OF COMPUTER DESIGN OF THE TYPE WILDHABER GEAR SETS

4.1. Basic problems of computer design

In Part 1 are presented some of the theoretical researches that we have accomplished regarding the skew-axes gears of type Wildhaber with arbitrarily skewed axes of rotation. On this base, a program system has been worked out. In general, the following two problems have to be solved by the computer program:

- Study of the matted pairs of tooth surfaces of type Wildhaber in order to find out their basic geometric and kinematic characteristics that influence on their exploitation characteristics including hydrodynamic loading capacity
- Geometric and technological synthesis of gears of type Wildhaber.

4.2. Description of the program

The program consists of modules. It is written in C++, system Windows.

Input data are:

- speed ratio u;
- a set of standard offsets a_w;

- angle δ between the axes of rotation (*i.e.* the angle between angular velocity vectors of the moving links);
- coefficient of a radial backlash in meshing c_z;
- coefficient of the tooth addendum h_{ak} (k = 1–3).

The computer program consists of 10 parts. In present paper we will discuss those of them that are based on the considered in Part 1 kinematic criteria serving for estimating the hydrodynamic loading capacity of the examined transmissions. The rest of parts will be mentioned only because of completeness.

Module 1. In the fist part either the offset a_w is given or the torque $M_{2,\max}$ acting on the driving gear is given. In the second case, a_w is calculated. The obtained value compares with the standard offsets (of the given set) and the closer one chooses.

Module 2. The aim of the second part is the choice of the number z_1 of the helical teeth (threads) of the worm of type Wildhaber. This number depends on the offset a_w and on the speed ratio u. Then the number z_2 of the teeth of the cylindrical gear with plane teeth is evaluated.

Module 3. The choice of the coefficient q of the worm diameter realizes here. In the most common case, its value chooses depending on the calculated number z_2 of the cylindrical gear of type Wildhaber. This coefficient takes part in the analytical relations for a calculation of the geometric dimensions of the gear pair: reference diameters d_1 of the throat of the worm and d_2 of the cylindrical gear; worm face width l_1 and the gear face width b_2 of the cylindrical gear of type Wildhaber; the module m (respectively the depth of the tooth and the tooth proportions).

Module 4. The goal of this module is the determining of the profile angle $\alpha^{(j)}$ of the tooth of the cylindrical gear with plane straight-line teeth $j = 1$ refers to that flank of the cylindrical gear matted with the corresponding tooth surface of the globoid worm when the gear-pair "works" in the low-side driving $(|\alpha^{(1)}| < |\alpha^{(2)}|)$. When $j = 2$ the opposite active tooth surfaces are matted (it means that gear set operates in high-side driving). When $|\alpha^{(1)}| = |\alpha^{(2)}|$ we do not differ low-side and high-side driving.

This part of the computer program has an essential significance in the process of the technological synthesis of gears of type Wildhaber. The introduced key *ialpha* obtains one of the following characters:

- *s* when the straight-line plane teeth of the gear are manufactured by one standard side milling cutter (with a standard profile angle);
- *p* if the gear is cut by two half-side milling cutter (a rectangular in a normal section);
- *c* when the plane teeth are generated by a specialized side milling cutter (with a trapezoid form of the normal section).

The calculation of $\alpha^{(j)}$ depending on the cutting method is illustrated in Fig. 1.

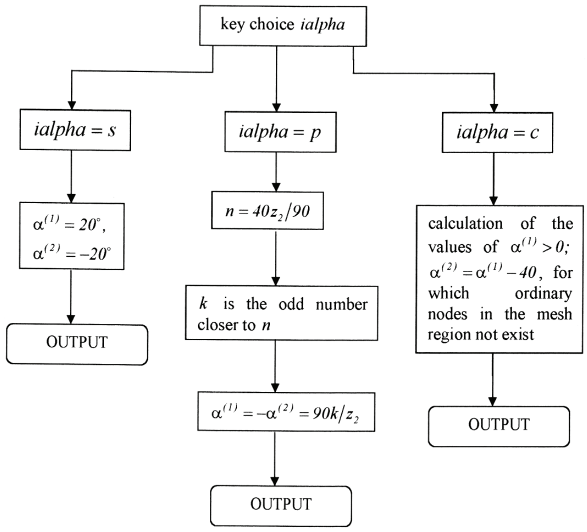

Fig. 1. Block-scheme of Part 4.

The first two cases are considered one more time, when analyzing whether there are ordinary nodes in mesh region.

Modules 5 and 6. The logic of these modules is one and the same and is illustrated in Fig. 2. There, the gear-pair analyses for a presence of singular points of first and second order, respectively. In the common case the singularity is relevant to the loading capacity of the synthesized gears. The singular points of first and second order are the so-called points of undercutting. Their presence of the flanks on the threads of the globoid worms of type Wildhaber leads to a decrease of their strength of bending.

In the present research, we are interested in the reasons for a decrease of the hydrodynamic loading capacity. Especially, we consider those kinematic and geometric characteristics of the gears of type Wildhaber that reflect on the quality of the oil film by means of which the loading between matted surfaces is transmitted.

The singular points of first order are the common points of the contact lines of the matted tooth surfaces. As a rule, in these points the oil film is broken because of increasing specific friction between the contacting surfaces. This makes the oiling and the heat transfer from the tooth surfaces to the lubrication fluid worse. The result is the decrease of the hydrodynamic loading capacity.

In Part 5, the computer program verifies whether there are ordinary nodes in the mesh region of the synthesized gear-set of type Wildhaber. If such points exist, it is possible to reduce the active tooth surfaces so that those points to be eliminated.

In Fig. 2, φ_0' and φ_0'' are initial and final values of the parameter of meshing φ_2; k_f shows how many times the tooth will be restricted so that the contact point be not an ordinary node; $u^{(j)}$ and $\tau^{(j)}$ are the coordinates of the contact point as a point of the tooth surface $\Sigma_2^{(j)}$ of the cylindrical gear; h_k, h_k' are coefficients which shows how the tooth to be restricted.

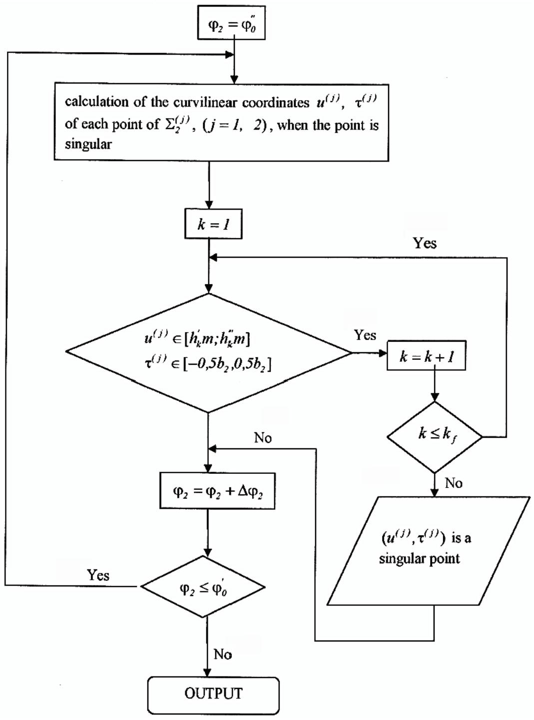

Fig. 2. Block-scheme of Parts 5 and 6.

Module 7 includes an optimization of the synthesized gear-pair according to one kinematic feature relevant to the hydrodynamic attitude of the spatial gear mechanism. The criterion in this part refers to the angle $\Omega_\varphi^{(j)}$ between the total circumferential velocity vector and the contact line in each point of contact. We have proposed two criteria. The first one means this angle to belong to an interval $(90^\circ - \Delta\omega,\ 90^\circ + \Delta\omega)$ where $\Delta\omega$ is given. The second one is of the form:

$$\frac{\sqrt{\sum_{N_\varphi}\left(\Omega_\varphi^{(j)}\right)^2}}{N_\varphi} \to \max,$$

where N_φ is the number of the calculated angles $\Omega_\varphi^{(j)}$. In the process of optimization the program changes the parameter of meshing in the interval $[-\varphi_0'',\ \varphi_0']$, the coordinate $\tau^{(j)}$ in the interval $[-0.5\cdot b_2,\ 0.5\cdot b_2]$ and the shaft angle δ in $[\delta_0 - \Delta\delta,\ \delta_0 + \Delta\delta]$. We have no experience to precise which one of the both criteria is more suitable in different cases.

Part 7 can be applied when analyzing already calculated variants of gear pairs from a point of view of hydrodynamic loading capacity estimated by the values of $\Omega_\varphi^{(j)}$ in different zones of the mesh region.

Module 8 serves for evaluating the contact strength of the synthesized gear-pairs in the process of their analysis. It performs also when making an optimization synthesis of the gears. The criterion is related with the values of the relative curvature in different points of the mesh region.

Module 9 refers to the optimization synthesis connected with specific friction in the mesh region treated as a locus of the contact lines.

Module 10 is a print as a table of the basic geometric parameters of the gear pair of type Wildhaber as the geometric dimensions of each gear that form the gear set.

5. CONCLUSION

The presented here program system belongs to the third type of computer programs concerning typical features. The program has a possibility to solve problems of the analysis of the considered hyperbolic gears. Thus, it can be considered as software of first type. As the authors' have not finished their researches, the computer program for an optimization synthesis of gear pairs of type Wildhaber is not the final one.

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