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OPTIMAL DESIGN OF THE WORM-GEARING WITH CIRCULAR PROFILE USING MEDIUM RIGIDITY CRITERION

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Abstract: *It is known that the kinematic chains include worm-gear drives, screw-nut mechanisms and pinion-rack drives. During the operation, these gear drives and mechanisms of the machine-tool deform under the load, having as result the manufacturing errors. The errors can't be eliminated entirely, but their maximum values must be limited depending on the proposed aims. The paper presents some aspects regarding the achievement of a worm-gearing with improved performances, depending on the optimization criteria: medium rigidity criterion and rigidity variation criterion. In the case when the worm-gearing tooth has a medium rigidity as high as possible, the goal is to ensure a deformation as small as possible and constant.*

Key words: *optimal design, worm-gearing, numerical method, amplitude, rigidity, computer simulation.*

1. INTRODUCTION

This study is based on:

- a numerical method to evaluate the rigidity of the worm-gearing tooth;
- the elasticity characteristic of the worm-gearing tooth;
- the influence of the geometrical parameters on rigidity of the worm-gearing tooth.

The software, with numerical set-up and graphic display, is an original and special program for determining the rigidity [1]. It was developed by the authors and could be adopted for any kind of cylindrical worm-gear drives, as well as for spur-gear drives and bevel-gear drives [2, 4]. The main steps of the computation are presented in Fig. 1.

The elasticity characteristic represents the variation of rigidity of the worm-gearing tooth depending on the rolling angle ($j \cdot \Delta\varphi$), where j is the rolling angular parameter [3]. It is cvasisinusoidal curve with the high jumps when a tooth binds or recesses (Fig. 2).

The investigation of the elasticity characteristic is very important for the study of an elastic system, such as: gearing, linkage, machine-tool. Hence, the introduction of this concept contributes to the completion of the used gearing study and it leads to increase of the gearing tooth rigidity.

The influence of the geometrical parameters on the rigidity was obtained by means of the computerized simulation [1, 5]. It was applied to 150 worm-gear drives and we can present the following conclusions:

1) the rigidity of worm-gearing tooth increases if:

- diametral quotient q increases (Table 1, Fig. 3);
- radius of profile curvature R increases (Table 2, Fig. 4).

2) the rigidity of worm-gearing tooth reduces if:

- profile angle increases (Table 3, Fig. 5);
- number of the gear teeth increases (Table 4, Fig. 6).

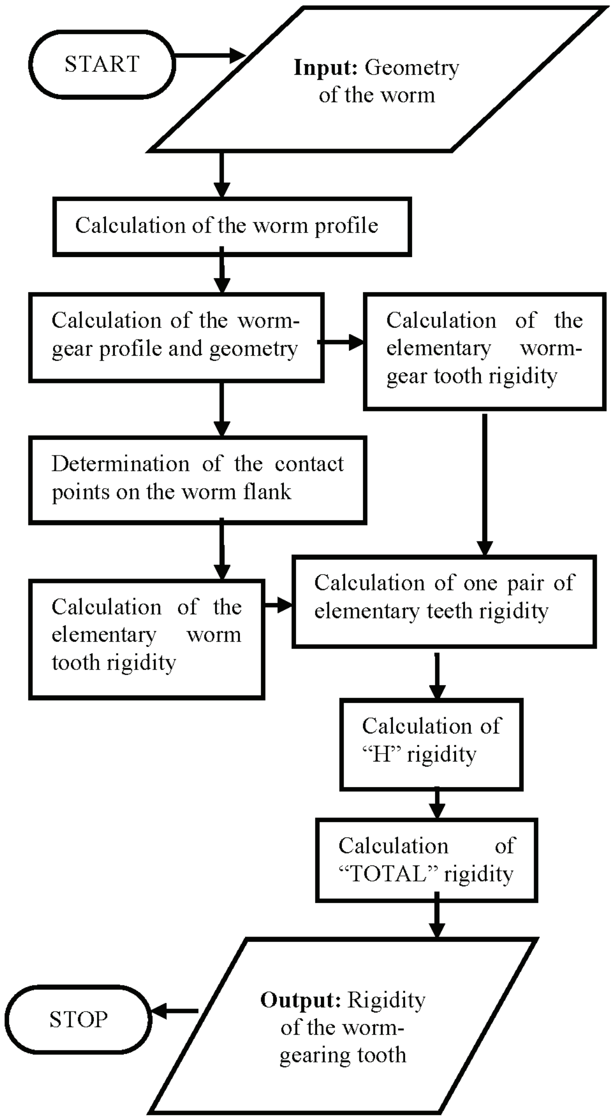

Fig. 1. The main steps of the computation.

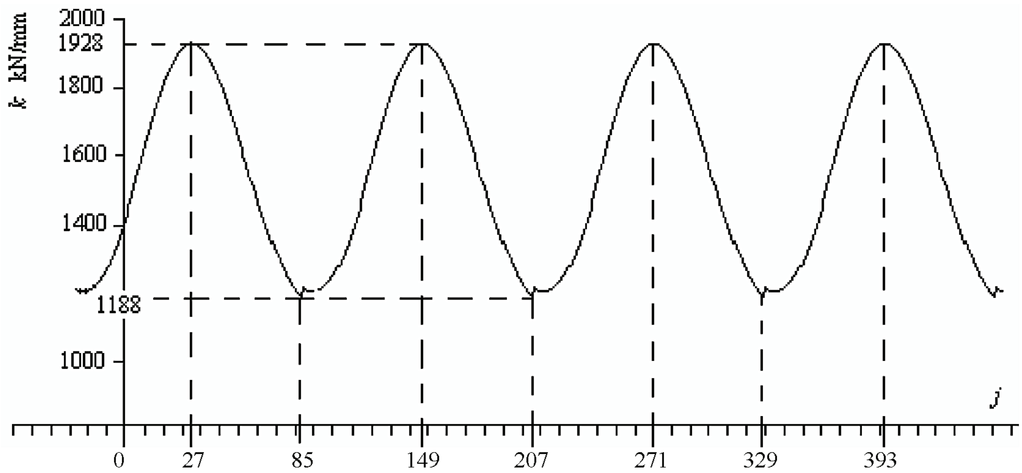

Fig. 2. Elastic characteristic of the worm-gearing tooth.

Table 1

Influence of diametral quotient on rigidity

q	$R = 3 \cdot m_x$ [mm]	**Maximum rigidity [kN/mm] for α = 10°**	**Minimum rigidity [kN/mm] for α = 10°**	**Medium rigidity [kN/mm] for α = 10°**
7	15.61	1087.027	647.259	867.143
8	15.49	1268.945	805.068	1037.007
9	15.36	1453.498	977.226	1125.362
10	15.24	1627.640	1114.246	1387.443
11	15.12	1792.178	1303.839	1548.008
12	15.00	1974.295	1452.423	1713.359
13	14.88	2136.667	1604.346	1870.006
14	14.76	2290.487	1759.527	2025.007
15	14.65	2414.695	1913.697	2164.196
16	14.53	2552.447	2059.235	2305.841

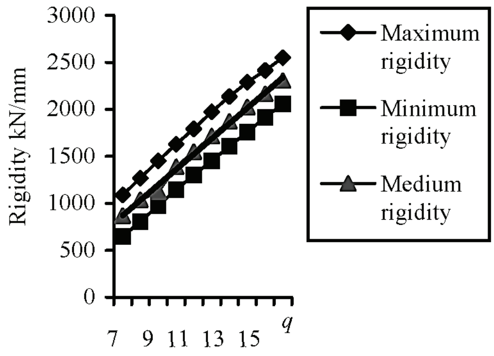

Fig. 3. Rigidity depending on diametral quotient *q*.

Table 2

Influence of radius of profile curvature on rigidity

q	m_x [mm]	**Maximum rigidity [kN/mm] for $R = 3 \cdot m_x$, α = 10°**	**Maximum rigidity [kN/mm] for $R = 4 \cdot m_x$, α = 10°**
7	5.20	1087.027	1231.259
8	5.16	1268.945	1454.918
9	5.12	1453.498	1660.191
10	5.08	1627.640	1874.163
11	5.04	1792.178	2052.678
12	5.00	1974.295	2246.004
13	4.96	2136.667	2421.314
14	4.92	2290.487	2602.595

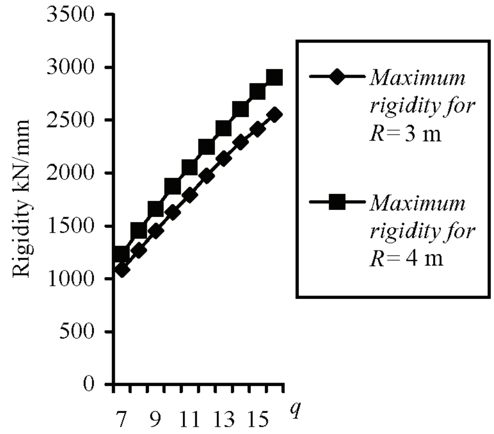

Fig. 4. Rigidity depending on radius of profile curvature *R*.

Table 3

Influence of profile angle on rigidity

α [°]	**Maximum rigidity [kN/mm] for $R = 3m_x$, $q = 7$**	**Minimum rigidity [kN/mm] for $R = 3m_x$, $q = 7$**	**Medium rigidity [kN/mm] for $R = 3m_x$, $q = 7$**
10	1087.027	647.259	867.143
15	930.087	442.790	686.438
20	817.641	280.151	548.896
25	747.323	157.58	402.651
30	710.374	77.53	393.952

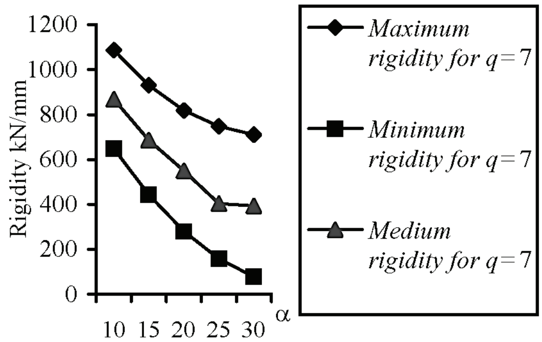

Fig. 5. Rigidity depending on profile angle α.

Table 4

Influence of number of gear teeth on rigidity

z_2	Maximum rigidity [kN/mm]	Minimum rigidity [kN/mm]	Medium rigidity [kN/mm]
53	2267.385	1215.140	1741.262
80	1727.633	1132.201	1429.917
90	1581.896	1079.696	1330.796
169	1055.990	853.826	954.908

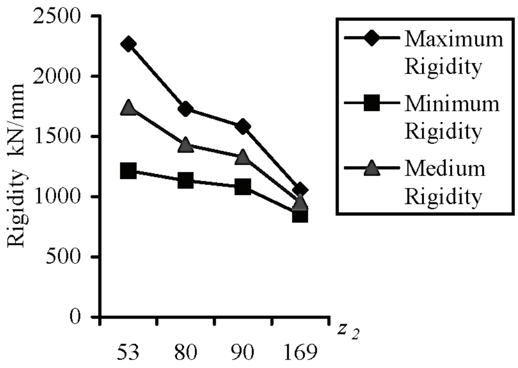

Fig. 6. Rigidity depending on number of gear teeth z_2.

2. OPTIMIZATION USING MEDIUM RIGIDITY CRITERION

For the design phase, taking into account the influence of each geometrical parameter on the rigidity of worm-gearing tooth with circular profile, we present the mode to obtain a higher rigidity. Generally, the methodology regarding the achievement of rigidity as high as possible is:

- selection of a high value for diametral quotient q;
- increasing radius of profile curvature R;
- adoption of low value for worm profile angle α;
- reduction of number of gear teeth z_2.

2.1. Case of the requested number of gear teeth z_2

In this case, the parameter with the most influence on the rigidity of worm-gearing teeth is diametral quotient, q. The investigation of rigidity of worm-gearing teeth [1] shows us that if diametral quotient increases twice, medium rigidity increases two and a half times.

The second important parameter for the increase of the rigidity is radius of profile curvature R.

The worm profile angle has the lowest influence on the modification of the rigidity.

In conclusion, from the viewpoint of the influences on the increase of medium rigidity, the hierarchy of the geometrical parameters is the following:

1) diametral quotient q;
2) radius of profile curvature R;
3) profile angle α.

2.2. Case of the chosen number of the gear teeth z_2

The number of gear teeth is adopted as low as possible when the designer may choose it . There are two situations:

a) radius of profile curvature is independent. This means that the radius doesn't depend on another parameter.

Table 5

Influence radius of profile curvature, which depends on axial module, on rigidity

z_2	m_x [mm]	$R = 3m_x$, $q = 7$, $\alpha = 20°$		$R = 4m_x$, $q = 7$, $\alpha = 20°$	
		Max. rigidity	Min. rigidity	Max rigidity	Min. rigidity
53	10.5	1533.2	470.56	1608.6	721.91
114	5.2	817.64	280.15	861.19	411.68

In this case, the hierarchy of geometrical parameters is the following:

1) diametral quotient q;
2) radius of profile curvature R;
3) profile angle α;
4) number of gear teeth z_2.

b) radius of profile curvature depends on axial module m_x ($R = am_x$, where a is constructive parameter).

The value for axial module increases, if number of gear teeth reduces, and result larger radius of profile curvature and higher rigidity. In the Table 5, we present the values for rigidity of two worm-gear sets which are different as number of gear teeth and their radius of profile curvature depend on axial module.

The obtained results of simulation of meshing show the following hierarchy:

1) diametral quotient q;
2) radius of profile curvature R;
3) number of gear teeth z_2;
4) worm profile angle α.

Our recommendation, if it is possible, is to apply the last case, where radius of profile curvature depends on axial module.

2.3. Design algorithm

In conclusion, in the design phase, having as goal achievement of the medium rigidity as high as possible for the worm-gearing tooth with circular profile (Fig. 7), we recommend the algorithm presented in Fig. 8.

Fig. 7 presents the axial section of the worm with constant pitch, having a circular arch profile with center O_1 for the right flank and O_2 for the left flank, where:

$$\begin{aligned} &u = 1,25 \cdot m / \cos\alpha, \\ &p = m/2, \\ &b = \pi \cdot m/4 - 1,25 \cdot \mathrm{tg}\,\alpha, \\ &R = \sqrt{a^2 + u^2}. \end{aligned} \tag{1}$$

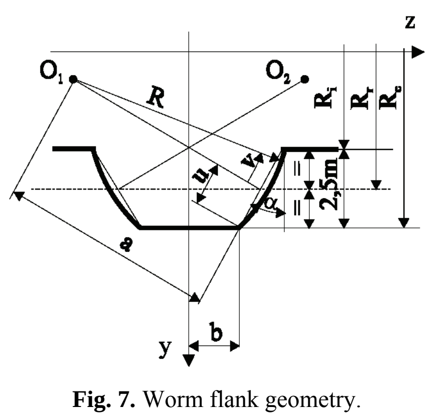

Fig. 7. Worm flank geometry.

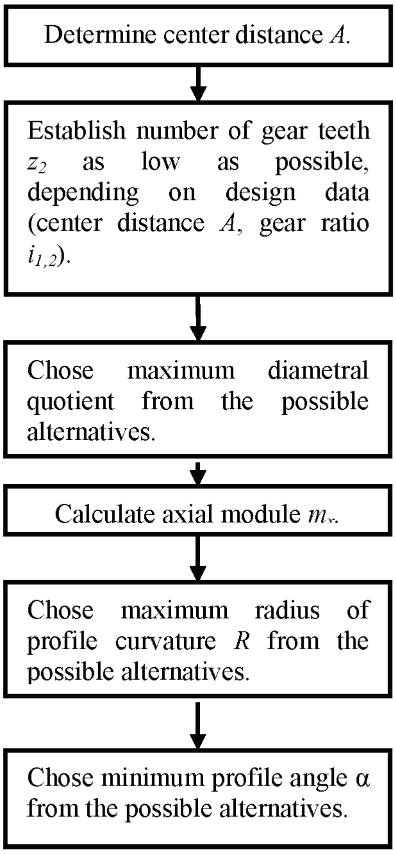

Fig. 8. Design algorithm.

3. CALCULUS EXEMPLE FOR OPTIMAL DESIGN USING MEDIUM RIGIDITY

The known design data are:
– center distance $A = 315$ mm;
– gear ratio $i_{1,2} = 114/1$.

So, results $z_2 = 114$.

We follow the algorithm established at **2.3**:
Chose diametral quotient as high as possible $q = 16$;
Calculate axial module with the formula:

$$m_x = \frac{2 \cdot A}{q + z_2}, \tag{2}$$

$$m_x = \frac{2 \cdot 315}{16 + 114} = 4.846 \text{ mm.}$$

The axial module is low enough, what doesn't help us to increasing rigidity. The design of worm-gearing imposes the adoption of standardized axial module. So, $m_x = 5$ mm.

Referring to the design data can't modify, we adopt $q = 12$, what will attract a reduction of medium rigidity, from 2305.841 kN/mm, what is in the case of $q = 16$, to 1713.359 kN/mm, the deference being 592.484 kN/mm (Table 1).

Another possibility would be, after the adopting of axial module $m_x = 5$ mm and diametral quotient $q = 16$, the reduction of number of gear teeth z_2 ($A = 315$ mm), but the gear ratio deviation must be less than 3% ($\Delta i_{1,2} < 3\%$). So, if: $m_x = 5$ mm, $q = 16$, $z_2 = 110 \Rightarrow \Rightarrow \Delta i_{1,2} = 3.5\%$ (doesn't agree); $m_x = 5$ mm, $q = 15$, $z_2 = 111 \Rightarrow \Delta i_{1,2} = 2.63\%$.

The last solution would allow to obtain higher rigidity than in the case of $q = 12$ and $z_2 = 114$.

Don't forget about the possibility of the design of x-toothing gear drive (with negative or positive addendum modification).

Radius of profile curvature must be as large as possible, especially the value for axial module is low: $R = 4m_x = 4 \cdot 4.846 = 19.38$ mm.

That is way, we recommend to adopt a larger radius of profile curvature, limited by the technological procedures of the circular profile manufacturing.

The value for profile angle must be as low as possible, $\alpha = 10°$.

3. CONCLUSION

Based on the computerized simulation of the meshing and the influence of geometrical parameters on the rigidity of worm-gearing tooth, a new approach has been developed for design of the worm gear drives;

The proposed methodology improves the performances of worm-gearing, what is very important for the accuracy of the machine-tool or robot linkages.

The basic idea of the approach to obtain high rigidity, is to take into account the medium rigidity criterion in the design phase.

The study presents the main steps of the proposed algorithm and a calculus example.

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