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MATHEMATICAL MODEL TO FORMALIZE ORIENTATION ERROR

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Abstract: *Orientation error specifies the ranges of permissible variations of the relative position between the basis systems so that a manufactured part will be correctly oriented. The purpose of this paper is to continue creating a mathematical model of orientation error. This model is based on the mathematical coordinates transformation theory and can be used into a CAD system. The results from our earlier work show that the proposed math model is compatible with the standards, and that it provides three-dimensional relations for orientation error. The project includes the implementation of the error analysis system into a CAD system.*

Key words: *accuracy, CAD, design optimization, fixtures, precision engineering.*

1. INTRODUCTION

Existing methods for analyzing orientation error today are based strongly on ad-hoc conventions from engineering practice and less on mathematical principles. Consequently, full three-dimensional analysis of error in the orientation process is not done today, and contemporary design software [4, 5, 6] is only partially compatible with existing orientation schemes libraries. The outcome of this project will provide the tools to complete full three-dimensional analyses of orientation error, and, thereby, it will improve quality and lower cost.

We anticipate that the math model will provide improvements in a designer's ability to assess the following two issues that are important in the fixture device design: to express and to capture conceptual designs and design intents.

This deployment raises two major issues. The first one is to define a data model that provides for exchange capabilities between heterogeneous information and for archive. The second one is to integrate the database into a specific CAD system.

2. ORIENTATION ERROR

As one of the deterministic components of a machining error, orientation error is primarily caused by size and position variations both from locating elements on a fixture side and from locating features on a workpiece side. Because of the versatile of part shape, locating feature form and tolerance specification, there is no comprehensive solution to estimate orientation error.

The accuracy of the piece orientation into the device is determined by the variation of the relative position of

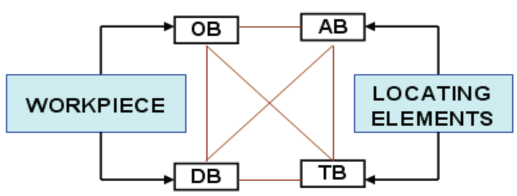


Fig. 1. Basis system.

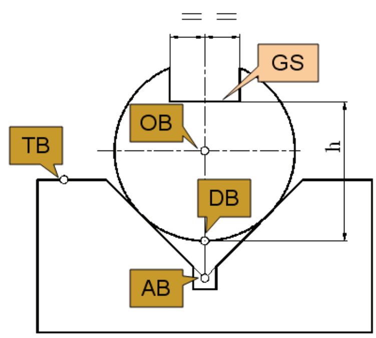


Fig. 2. Workpiece oriented on V block.

the basis systems, belonging to the workpiece and of the support element [1, 2] – Fig. 1 (OB – orientation base, DB – dimensioning base, AB – active base, TB – tuning base, GS – generating surface). Fig. 2 shows the basis system for a cylindrical workpiece, oriented on V block.

3. THE GENERAL MODEL

There are three levels involved in the model we have proposed:

- **The current instance.** Features, geometric relations and tolerances are modeled according to the specifications. In the geometric model, the type, the number and the relative position of the orientation bases are imposed. Features include faces and the center, axis, or mid-plane. We can further decompose features into basic entities, i.e., point, line, plane and combinations of these. A cylindrical feature can be represented by a line and a cylindrical radius; a sphere can be represented by its center point and a spherical radius; a slot can be decomposed into a center plane and its width; an arbitrary curve or an arbitrary non-planar face can be defined by points with an equation.
- **The parametric specification.** Geometric relations are used to locate features relative to one another. They include angle, distance and radius/diameter, as well as geometric relations (orientation, coincidence, intersection and tangency). The parametric specification is modeled by adding in the geometric model the

information that defines the type and direction for the dimension affected by the orientation error.

- **The dynamic context** is an in-between level that provides the components of the orientation error, based on the coordinates transformation method [2, 3]. This major issue consists in modelling the relationships between the references as they appear in the parametric specifications and the values as they appear in the current instance.

In order to represent in the same framework the values of the current instance, the variables to which they correspond (components of the orientation error) and the expressions where these variables are involved, we propose to capture in the data model the abstract syntax tree of these expressions and to use the entity relationships to model the usual interpretation function that associates values to variables.

The resulting model is completely defined and can be transferred to CAD for the design process of the orientation scheme.

This is a process, which appears simple on the first view, but can be laden with the potential errors of the real surfaces. Geometric relations between features are theoretically perfect dimensions, which cannot be achieved on the real workpiece. Tolerances relax these theoretical dimensions. They are specified on the features intrinsically or on features with respect to other features.

4. THE MATHEMATICAL MODEL

Let $(Oxyz)$ be the reference system, connected to the locating elements that release the piece orientation and let $(O'x'y'z')$ be the reference system connected to the workpiece – Fig. 3.

Transformations may conveniently be performed using matrix arithmetic. For each transformation, there is a transformation matrix **T** such that $\mathbf{Q}_b = \mathbf{T}\mathbf{Q}_a$, where $\mathbf{Q}_a$ is a vector in the first coordinate system, and $\mathbf{Q}_b$ is the same vector in the second coordinate system.

The relationship between the rectangular coordinates (a, b, c) and $(a'b'c')$ of a point in the two systems, is established by the formula:

$$\begin{bmatrix} a' \\ b' \\ c' \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} a'_0 \\ b_0 \\ c_0 \end{bmatrix}, \qquad (1)$$

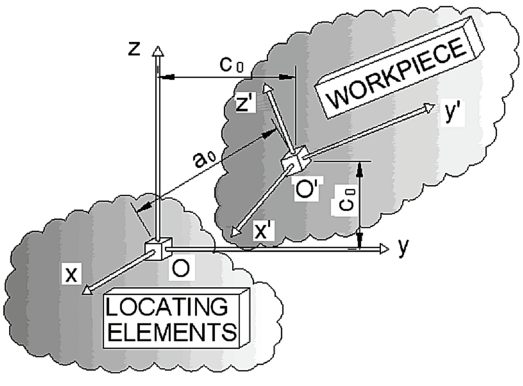


Fig. 3. The coordinates transformation method for rectangular dimensions.

where (a'_0, b_0, c_0) are the coordinates of the origin of the $(O'x'y'z')$ system in the $(Oxyz)$ system.

The orientation error represents the extreme values of the coordinates variations:

$$\begin{aligned} v_a &= i(a' - a), \\ v_b &= i(b' - b), \\ v_c &= i(c' - c). \end{aligned} \qquad (2)$$

The coefficient i will be 1, or 2, when the workpiece can move on the support elements in one, respectively two senses, on the error's direction.

From the analytical equations [2] of the functions (2) there results the orientation error formula.

Let $\nu(c_1, c_2, c_3)$ be a function, considered continuous together with its partial derivates of the first and second order. ν can be v_a, v_b or v_c. Let $P_0(c_1^0, c_2^0, c_3^0)$ be a stationary point of the function, that is:

$$\left(\frac{\partial v}{\partial c_1}\right)_0 = 0; \ \left(\frac{\partial v}{\partial c_2}\right)_0 = 0; \ \left(\frac{\partial v}{\partial c_3}\right)_0 = 0. \qquad (3)$$

Let **D** be the bounded closed domain of definitions for the variables c_1, c_2 and c_3.

If the function ν has a positive maximum or a negative minimum $v_0 = v(c_1^0, c_2^0, c_3^0)$ attained inside the domain **D**, then the orientation error formula is:

$$\varepsilon_0 = |v_0|. \qquad (4)$$

If the function ν has a positive maximum and a negative minimum $v_1 = v(c_1^{01}, c_2^{01}, c_3^{01})$, respectively $v_2 = v(c_1^{02}, c_2^{02}, c_3^{02})$, then the orientation error is defined as follows:

$$\varepsilon_0 = \max(|v_1|, |v_2|). \qquad (5)$$

If the extremum point P_0 turn out of **D**, the greatest or the smallest value of the function ν is attained at a point belonging to the boundary of the domain. The greatest absolute value of these numbers is the sought value for orientation error.

For example, in plane, for rectangular coordinates, (1) can be written as:

$$\begin{aligned} a' &= a\cos\alpha + b\sin\alpha - r\cos(\beta - \alpha), \\ b' &= -a\sin\alpha + b\cos\alpha - r\sin(\beta - \alpha), \end{aligned} \qquad (6)$$

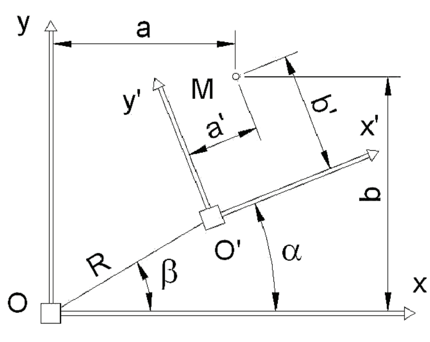


Fig. 4. The coordinates transformation method for rectangular dimensions.

where (a, b), respectively (a', b') are the coordinates of a point M, α is the angle between Ox and $O'x'$ axis, (R, β) define the position of the origin of the $(O'x'y')$ system in the (Oxy) system and r is the variation of the R variable – Fig. 4.

By substituting (6) in (2) we finally get the orientation errors for the dimensions a and b:

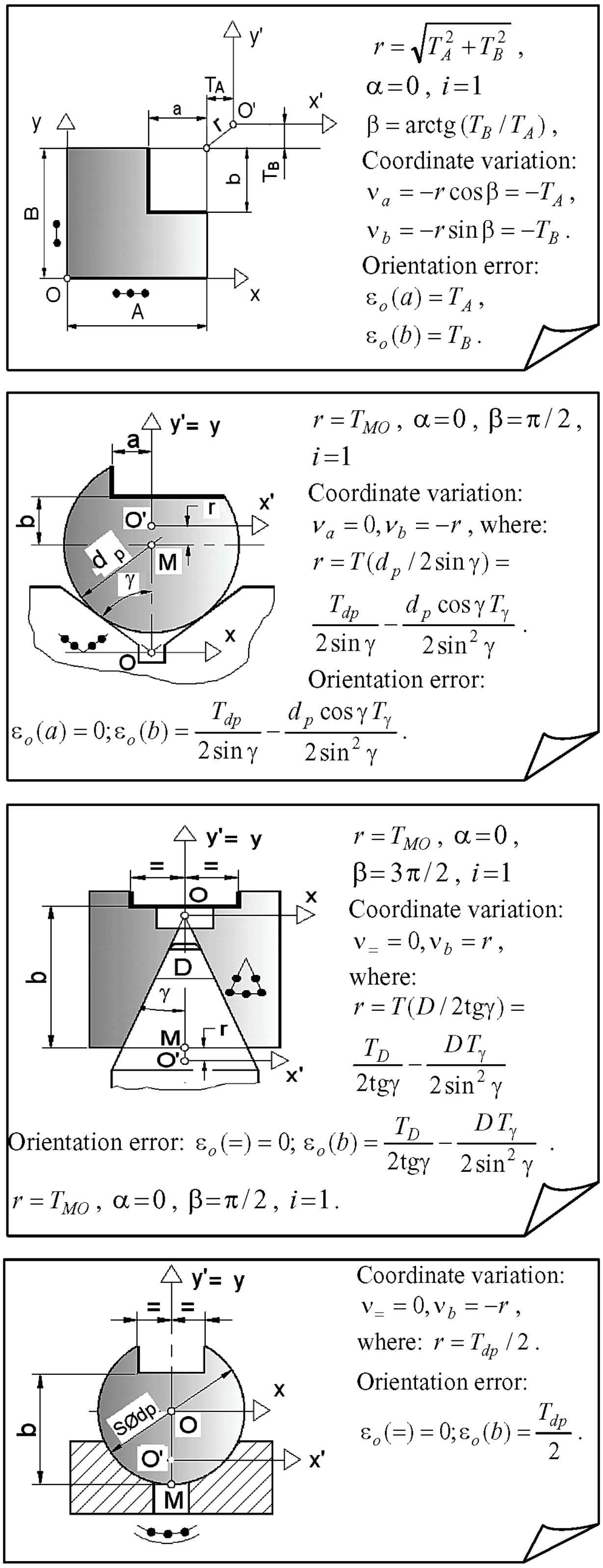


Fig. 5. Orientation schemes based on single surfaces.

$$\begin{aligned}\varepsilon_o(a)&=\max\{i[a(\cos\alpha-1)+b\sin\alpha-r\cos(\beta-\alpha)]\}\\ \varepsilon_o(b)&=\max\{i[-a\sin\alpha+b(\cos\alpha-1)-r\sin(\beta-\alpha)]\}.\end{aligned}\quad(7)$$

Fig. 5 shows some samples with orientation error for orientation schemes determined by using the coordinates transformation method. The schemes are build on a single orientation surface.

In Fig. 6 and Fig. 7 are illustrated two orientation errors for a complex scheme, based on two, respectively three orientation surfaces. The error is also determined by using the proposed model.

By similitude with the relations (6), for polar coordinates (ρ, θ), (7) can be rewritten as:

$$\begin{aligned}\varepsilon_o(\rho)&=i\left|\sqrt{\rho^2+r^2-2\rho r\cos(\theta-\beta)}-\rho\right|,\\ \varepsilon_0(t)&=i\left|\mathrm{arctg}\frac{\rho\sin(\theta-\alpha)-r\sin(\beta-\alpha)}{\rho\cos(\theta-\alpha)+r\cos(\beta-\alpha)}-\theta\right|.\end{aligned}\quad(8)$$

Fig. 8 shows one orientation error determined in polar coordinates. The workpiece is oriented on three surfaces. Because the angular displacements are in bosth directions for the locator 1 but in only one direction for the V block, we consider $i = 2$ and $\beta = \pi/2$. The r variable, due to the orientation on V block is:

$$r=\frac{T_{dp}}{2\sin\gamma}-\frac{d_p\cos\gamma T_\gamma}{2\sin_\gamma^2},\quad(9)$$

where d_p and T_{dp} are the actual value and the tolerance for the workpiece's diameter, respectively γ and $T\gamma$ are the actual value and the tolerance for the V block's angle. From the $O'AM$ and $O'AC$ triangles – Fig. 9, we obtain the expression of α parameter:

$$\alpha_{\max}\cong 2\arcsin\frac{J_{\max}}{2R_1}+\mathrm{arctg}\frac{r}{R_1}.\quad(10)$$

Finally, from the relations (8), we finally get the orientation errors for the dimensions R and t:

$$\begin{aligned}\varepsilon_0(\rho)&=2\left|\sqrt{R^2+r^2-2Rr\sin t}-\rho\right|\\ \varepsilon_0(t)&=2\left|\mathrm{arctg}\frac{R\sin(t-\alpha_{\max})-r\cos\alpha_{\max}}{R\cos(t-\alpha_{\max})+r\sin\alpha_{\max}}-t\right|\end{aligned}\quad(11)$$

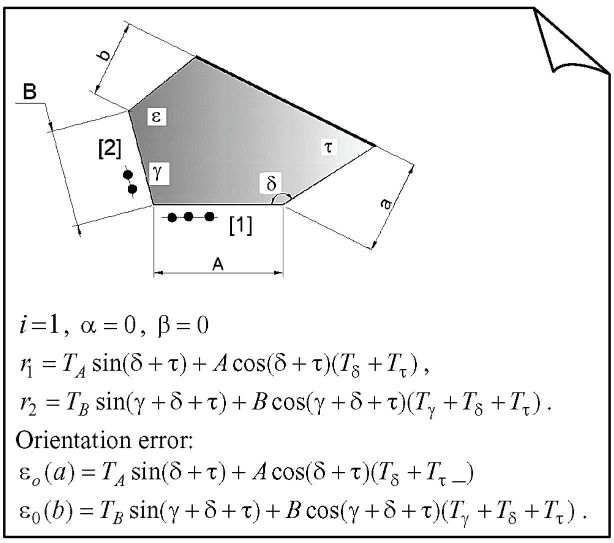
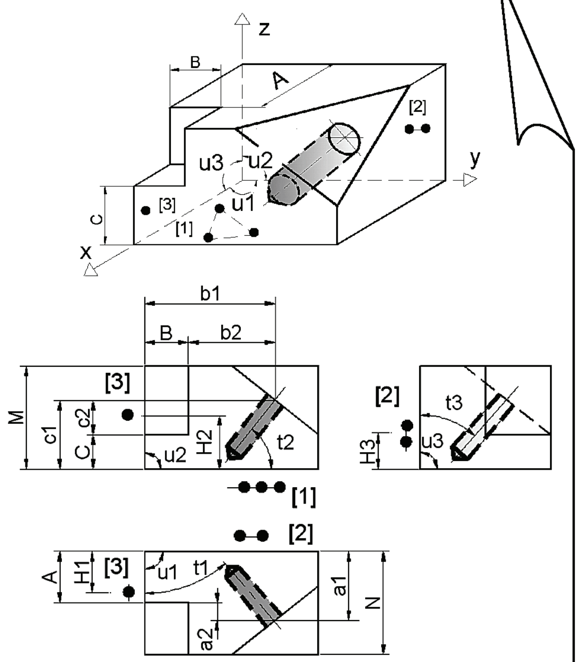


Fig. 6. Orientation schemes based on two surfaces.



$$\alpha = 0 , \beta = \mathrm{arctg}\frac{r_x}{r_y} , r_x = \sqrt{r_{1x}^2 + r_{2x}^2} , \; r_y = \sqrt{r_{1y}^2 + r_{2y}^2}$$

For a_1 and b_1 dimensions, the orientation errors are:

$$\varepsilon_o(a_1) = r\sqrt{r_{1x}^2 + r_{2x}^2} , \; \varepsilon_o(b_1) = r\sqrt{r_{1y}^2 + r_{2y}^2}$$

where: $r_{1x} = (M - H_3)\,\mathrm{tg}T_{u3}$, $r_{1y} = (M - H_3)\mathrm{tg}T_{u2}$,

$r_{2x} = r_{2y} = (N - H_2)\mathrm{tg}T_{u1}$.

For a_2 and b_2 dimensions, the orientation errors are:

$$\varepsilon_o(a_1) = r\sqrt{r_{1x}^2 + r_{2x}^2} , \; \varepsilon_o(b_1) = r\sqrt{r_{1y}^2 + r_{2y}^2} \text{ s}$$

where: $r_{1x} = \sqrt{(M - H_3)^2\, tg^2 T_{u3} + T_A^2}$,

$r_{1y} = \sqrt{(M - H_2)^2\, \mathrm{tg}^2 T_{u2} + T_B^2}$,

$r_{2x} = r_{2y} = (N - H_2)\mathrm{tg}T_{u1}$.

For c_1 and c_2 dimensions, $\beta = \pi / 2$ and the orientation errors are:

$\varepsilon_o(c_1) = 0, \varepsilon_o(c_2) = T_C$.

For the angular dimensions, $r = 0$ and the orientation errors are:

$\varepsilon_o(u_1) = T_{u1}$, $\varepsilon_o(u_2) = T_{u2,}, \varepsilon_o(u_3) = T_{u3}$.

Fig. 7. Orientation schemes based on two surfaces.

5. CONCLUSION

The results of this research will impact both on the state of the art for orientation scheme design and on the productivity of designers who use the results. We anticipate that successful completion of the project will provide the designers with a tool for identifying trade-offs and for making better decisions about optimizing the fixture design. The impact should be an accurate work, shorter design time, fewer iterations and hence lower cost.

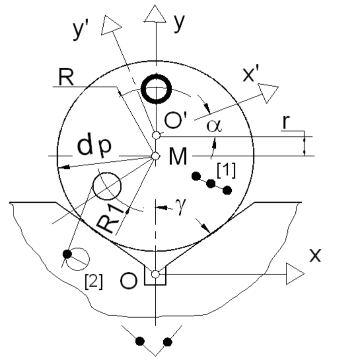


Fig. 8. Orientation scheme in polar coordinates.

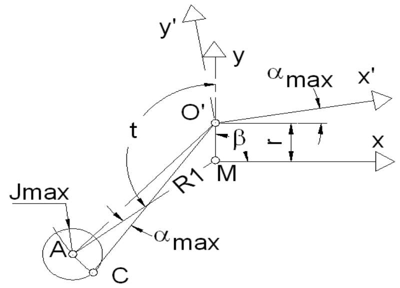


Fig. 9. The α parameter.

REFERENCES

[1] Brăgaru, A. (1998). *Proiectarea dispozitivelor*, Edit. Tehnică, ISBN 973-31-0717-4, Bucharest.

[2] Simion, I. (1995). *Research concerning the precision of the orientation schemes*, Ph.D. Thesis, "Politehnica" University of Bucharest.

[3] Simion, I.. (2000). *New approaches on the orientation error*, The Scientific Bulletin of "Politehnica" University of Bucharest, vol. 62, no. 2, pp. 45–52.

[4] Simion, I. (2003). *Infographic on the fixture design*, Proceedings of 3rd International Conference „Research and Development in Mechanical Industry – RaDMI 2003", pp. 1445–1450, Herceg Novi, Serbia and Montenegro.

[5] Simion, I. (2003). *Computer graphics for computer aided design*, Proceedings of International Conference "Advanced Manufacturing Technologies - CITAF 2003", in: Machine Building, vol. 55, no. 11–12, pp. 124–128, Bucharest.

[6] Simion, I. (2005). *AutoCAD for engineers*, TEORA USA LLC, ISBN 1-59496-033-X, Wisconsin, USA.

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