# ACTIVITIES PLANNING AND SCHEDULING USING TIT METHOD IN PRODUCTION WORKSHOPS 

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#### Abstract

For each production task or activity carried out in production workshop the decisions are made on two characteristic tips: time characteristic, that requires to allocate time for task execution, considering the time constraints and resource characteristic, that requires to allocate one or more resources, considering the resource constraints. Therefore, the decision is the result of constraints given by time and resource limits. The method of time-intensity-task triad, TIT, permits to take both time and resource aspects into consideration, being suited for a hierarchical approach to decision making in production workshops.


Key words: time, intensity, task, constraints, time horizon, time segmentation, task cell.

## 1. INTRODUCTION

In planning and scheduling of activities in production workshops, the decision over tasks is the result of reasoning about constraints determined, guided by time limits - that suppose the allocation of necessary time for execution; and resource limits - that suppose to take into account the availability and the capacity of resources.

Therefore, papers, $[1,2,4]$ show that the problems which arise in planning phase are determined by time allocation and resources allocation for each task. Then, in decision phase, it must be taken into account the constraints related to tasks for temporally phases and availability of resources at this moment.

Other authors $[5,6]$ take into account the constraints of "unexpected loading with tasks in time" of resources, without a detailed evaluation of resources capacity.

With all that, [4] put into account the problem that is necessary to use some instruments which will help decision making (this decision is made by workshop chief). This thing is an intermediary step between planning and scheduling and the decisions are carried but to a superior level and this decisions must be respected and fulfilled.

The current software makes a distribution of loaded tasks using loading curves, which show the tasks' evolution in time.

Because the activities aren't precise in time, the tasks' distribution will be made for taking into account the earliest start or the latest finish times of activities. But currently, the plan unexpected activities and he doesn't have the possibility to allocate resources for that. For that, the planner must use all the information about these unexpected activities and associated tasks for maximum use of time and availability of resources. For this, there are proposed some loading curves that show the tasks loading flexibility which arise from temporal flexibility of activities. In this mode, it is presented the mode in which these curves are attended using task loading flexibility. These modifications can be used for task reduction process, considering the constraints of capacity, availability and profitability of resources.

The triad method of time - intensity - task (TIT), permits the use of time aspects and the resource aspects
and it is good hierarchical approach of decisions in production workshops.

## 2. CONCEPTUAL BASES OF THE METHOD

### 2.1. Method definition

The three entities, time $T$, intensity $I$, and task $S$, make a triad (TIT) that is defined by the following relationship: $S=D \cdot I$. The method is defined by the next characteristics in accordance with problem type that will be studied and solved:

- time characteristics (time limits): the earliest start of activity $i\left(\mathrm{ES}_{\mathrm{i}}\right)$ and latest finish of $i$ activity $\left(\mathrm{LS}_{\mathrm{i}}\right)$;
- intensity functions of resources, $J$, defined for each time $t$ in $\left[\mathrm{ES}_{i}, \mathrm{LS}_{i}\right]$ and any associated resource, $k$; in case that these functions are constant in time, it can tell that $J$ is uniform;
- virtual time (virtual duration) of activity, $D_{i}$, defined as the difference between $\mathrm{LF}_{i}$ and $\mathrm{ES}_{i}$;
- effective duration of activity, $D_{e i}$, being the time needed to complete the activity $i$ (processing time);
- resource capacity or the maximum quantity of available resource during the time buckets of the planning horizon;
- resource loading (resource task) is the percentage of the allocated resource capacity for processing one or more activities.


### 2.2. Specific problems of the method

The decisions studied refer to planned activities set that are limited by ES and LF dates, and share some common resources.

We suppose that the intensity of shared resources is 1 ( $100 \%$ ), which means a full load on resources is possible.

This analysis is accomplished through hierarchical approach, the decisions are broken at production system level on successive levels and then there will be delivered to inferior levels in terms of goals or constraints.

Time, intensity and task (TIT) are inputs, so they are information about time and resource for users.

Thus, there have been made the following notations: $t_{i}$ - effective start moment for $i$ activity; $D_{e i}$ - processing time for $i$ activity; $S$ - resource loading which is the percentage from allocated resource capacity of $i$ activity. This can be equal with the processing time of $i$ activity, if the intensity is 1 .

Windows are associated to the planned activities TITs are time intervals, $\left[t_{j}, t_{j+1}\right]$, delimited by ES and LF of each activity $i$.

The time characteristics are established at high lead level (frequently, at the planning level). These characteristics are treated as time constraints and must be respected at workshop level.

The time interval $\left[t_{j}, t_{j+1}\right]$ and the afferent task will be designed as task cells that consume time and resources.

### 2.3. Time segmentation

Planning complexity for a set of operations can be reduced using segmentation methods [1]. For time segmentation, the loading consists in associated interval of time displaying for planned operations that are in a common time slot [3].

The difficulty often consists in total overlapping of planned activities, and it is not possible to find independent time intervals in which to be processed successive operation to be processed without overlapping of different classes.

But, not always it is possible to apply segmentations of time horizon then the overlapping process is in only two successive time intervals (successive task cells).

These structures divide the time horizon in successive time intervals (task cells), with the following fundamental property: for each $i$ activity, does not exist a j interval with $t_{j-1}$ moment lower than ES and $t_{j+1}$ moment higher than LS of activity $i$ :

$$
\forall i, \quad t_{j-1} \leq \mathrm{ES}_{i} \leq L F_{i} \leq t_{j+1}
$$

That means no activity overlaps more than two successive time intervals. It results that any translation of the $i$ activity task within the interval $\left[t_{j}, t_{j+1}\right]$ can affect only two successive task cells.

The following algorithm is an example for time limit $t_{0}$. For each step $n$ there are processed ,context $n-1 "$ classes of successive planned activities, that affects the selection of a couple of temporal limits $\left[t_{-n}, t_{n}\right]$ in the time horizon.

Step 1.

1) context $\left(t_{0}\right)$ is a set of activity windows $i$ for which $\mathrm{ES}_{i}<t_{o}<\mathrm{LF}_{i}$.
2) for windows $i$ in context $\left(t_{0}\right)$, a temporal limit $t_{1}$, with $t_{1} \geq \max \left\{\mathrm{LF}_{i}\right\}$, and a limit $t_{-1}$, with $t_{-1} \leq \min \left\{\mathrm{ES}_{i}\right\}$ are chosen (Fig. 1).

Step $n$.

1) context $\left(t_{n}\right)$ is a set of activity windows $i$ for which $t_{-(n-1)}<L F_{i}<t_{-(n-2)}$ or $t_{n-2}<\mathrm{ES}_{i}<t_{n-1}$.
2) for windows $i$ in $\operatorname{context}\left(t_{n}\right)$, a temporal limit $t_{n}$, with $t_{n} \geq \max \left\{\mathrm{LF}_{i}\right\}$, and a limit $t_{-n}$, with $t_{-n} \leq \min \left\{\mathrm{ES}_{i}\right\}$ are chosen.

a)
b)

Fig. 1. The time segmentation around $t_{0}$ moment.

## Properties:

1. The "actualization" implies a reduction of the length of $\left[t_{j}, \mathrm{t}_{j+1}\right]$ windows. Any "actualization" of an ES date (increase) or LF date (decrease) doesn't have any effect on the neighbouring property of time segmentation. The decreased window continues to include two successive task cells. The temporal structure is built in conformity with the actualization of correspondent windows.
2. Let $C^{*}=\left\{t_{o}, \ldots t_{j}, t_{j+1}, \ldots\right\}$ be the associate set for a time adjacent segmentation. Every subset $C$ of $C^{*}$ is associated also to a structure of this type (the neighbouring property is maintained). The time intervals of $C$ are equal to the intervals of $C^{*}$.
3. The previous property involves the definition of order relations between the sets of temporal limits that correspond to different adjacent time segmentation, so $C \subset C^{*}$. These relations create a partial ordering in the structure of adjacent time segmentation, where the smallest elements are the more detailed ones.

Fig. 1 indicates that the adjacent time segmentation obtained by choosing the initial moment $t_{0}$ at each step $n$ respects the following conditions:

$$
\begin{gather*}
t_{n} \geq \max \left\{\mathrm{LF}_{i}\right\},  \tag{1}\\
t_{-n} \leq \min \left\{\mathrm{ES}_{i}\right\} \tag{2}
\end{gather*}
$$

for each activity window $i$ that is the smallest element of the context $\left(t_{0}\right)$.

The efficacy of this time segmentations is strongly connected with the virtual times of activities. For example, if a planned operation spreads between $t_{\mathrm{o}}$ and $t_{n}$, it is not possible to have more intervals. Thus, this method assumes that the lengths of task cells are limited to a value much lesser than the planned time horizon.

### 2.4. The associated constraints of task cell $\left[\boldsymbol{t}_{\boldsymbol{j}}, \boldsymbol{t}_{\boldsymbol{j}+1}\right]$

The task cells can be considered in TIT context constraints, when there are available a limited set of resources or a maximum quantity of task for team workers. The TIT tasks that are determinate can be considered external constraints and these constraints are imposed by a high level, in generally by planning level.

The local constraints of resource availability are:

- $N_{R \max }\left(t_{j}, t_{j+1}\right)$ are the maximum number of available resources in $\left[t_{j}, t_{j+1}\right]$, period, that is existent capacity;
- $N_{R \min }\left(t_{j}, t_{j+1}\right)$ are the minimum number of resources in $\left[t_{j}, t_{j+1}\right]$, period, that is profitability.


### 2.5. Temporal distribution of task

The task represents the necessar number of resources for achievement of planned operations.

The elements that must be identified are:

- maximum task, $T_{\text {max }}$ is the maximum number of resources used for all $i$ activities, in all time $\left[t_{j}, t_{j+1}\right]$ as in relation:
$T_{\text {max }}\left(t_{j}, t_{j+1}\right)=\sum_{i=1}^{n}\left(\max \left(0, \min \left(L F_{i}-t_{j}, D_{i}, t_{j+1}-E S_{i}\right)\right)\right)(3)$
That is obtained by distribution of all task in framework of $\left[t_{j}, t_{j+1}\right]$ interval as in Fig. 2.
- minimum task, $T_{\text {min }}$ is minimum quantity of resources necessary $\left[t_{j}, t_{j+1}\right]$ period and it is determined using the relation:

$$
\begin{align*}
& T_{\min }\left(t_{j}, t_{j+1}\right)= \\
& =\sum_{i=1}^{n}\left(\max \left(0, \min \left(E S_{i}+D_{i}-t_{j}, t_{j+1}-L F_{i}+D_{i}, D_{i}\right)\right)\right) \tag{4}
\end{align*}
$$

that is obtained through task distribution in outside of $\left[t_{j}, t_{j+1}\right]$, period as it is presented in Fig. 3.

## Decisions:

The decisions must be in connection with $T_{\text {max }}$ and/or $T_{\text {mim }}$ adjustments and these adjustments must to be compatible with maximum number of resources, $N_{\text {Rmax }}$, or minimum number of resources $N_{R \text { min }}$.

Thus, $T_{\max }<N_{R \max }$ or $T_{\min }>N_{R \min }$.
If $T_{\max }>N_{R \max }$ and/or it is imposed the decreasement of and/or increasement of $T_{\text {min }}<N_{R \text { min }}$.

The decisions of task quantity can be taken by increasement of ES date or by decreasement of LF date.

For overloading risks if, maximum task is $T_{\text {max }}\left(t_{j}, t_{j+1}\right)>N_{R \text { max }}\left(t_{j}, t_{j+1}\right)$, it results that maximum task $T_{\text {max }}\left(t_{j}, t_{j+1}\right)$ must be increased with a quantity, noted $\Delta T$, where


Fig. 2. Task distribution associated with maximum task.


Fig. 3. Task distribution associated with minimum task.


Fig. 4. Maximum task decreasement through increasement of ES date or decreasement of LF date.

$$
\Delta T\left(t_{j}, t_{j+1}\right) \geq T_{\max }\left(t_{j}, t_{j+1}\right)-N_{R \max }\left(t_{j}, t_{j+1}\right) .
$$

This action can be made through increasement of ES date and/or through decreasement of LF date as it is shown in Fig. 4.

The maximum task $T_{\max }\left(t_{j}, t_{j+1}\right)$ is decreased with quantity $\Delta T\left(t_{j}, t_{j+1}\right)$ determined with the relation

$$
\Delta T\left(t_{j}, t_{j+1}\right)=\Delta T L\left(t_{j}, t_{j+1}\right)+\Delta T R\left(t_{j}, t_{j+1}\right),
$$

where: $\Delta T L\left(t_{j}, t_{j+1}\right)$ is task quantity that corresponds to decreasement of LF date (left moving) for $i$ activity with $\mathrm{ES}_{i}$ and $\mathrm{LF}_{i}$ dates are in $\left[t_{j-1}, t_{j}\right]$ and $\left[t_{j}, t_{j+1}\right]$ task cells (area with dot hatch it is the task quantity allocated to adjacent task cell $\left.\left[t_{j-1}, t_{j}\right]\right) ; \Delta T R\left(t_{j}, t_{j+1}\right)$ is quantity task that corresponds increasement of ES date (right moving) for $i$ activity with $\mathrm{ES}_{i}$ and $\mathrm{LF}_{i}$ dates are in $\left[t_{j}, t_{j+1}\right]$ şi $\left[t_{j+1}, t_{j+2}\right]$.

The excess task $\Delta T$ is allocated to adjacent task cells $\left[t_{j-1}, t_{j}\right]$ and $\left[t_{j+1}, \mathrm{t}_{j+2}\right]$ as it is presented in Fig. 5.

The maximum task, in period $\left[t_{j}, t_{j+1}\right]$ is greater than the maximum number of resource $N_{R \max }$ (Fig. 5a), then the excess task is allocated to adjacent periods, (Fig. 5b), $\left[t_{j-1}, t_{j}\right]$ şi $\left[t_{j+1}, t_{j+2}\right]$ (hatched area is equal with double hatches areas sum).

For under loading task if the minimum task in $\left[t_{j}, t_{j+1}\right]$, $T_{\min }\left(t_{j}, t_{j+1}\right)$, period is lesser as minimum resource number, $\quad N_{R \text { min }}\left(t_{j}, t_{j+1}\right), \quad T_{\text {min }}\left(t_{j}, t_{j+1}\right)<N_{R \text { min }}\left(t_{j}, t_{j+1}\right)$, the minimum task, $T_{\text {mim }}\left(t_{j}, t_{j+1}\right)$, must be increased with a quantity, noted $\delta T$, associate $\left[t_{j}, t_{j+1}\right]$ period, where:

$$
\begin{align*}
\delta T\left(t_{j}, t_{j+1}\right) & =\delta T R\left(t_{j}, t_{j+1}\right)+\delta T L\left(t_{j}, t_{j+1}\right) \geq \\
& \geq N_{R \max }\left(t_{j}, t_{j+1}\right)-T_{\min }\left(t_{j}, t_{j+1}\right) . \tag{5}
\end{align*}
$$

$\delta T R\left(t_{j}, t_{j+1}\right)$ is the task quantity that corresponds to increasement of ES date (right moving) for activity which is processed in $\left[t_{j-1}, t_{j}\right]$ period.
$\delta S S\left(t_{j}, t_{j+1}\right)$ is the task quantity that corresponds to decreasement of LF date (left moving) for activity which is processed in $\left[t_{j+1}, t_{j+2}\right]$ period.

The neighbour propriety of adjacent segmentation in under loading risk cases is presented in Fig. 6.

These analyses of adjacent time segmentation associated to curves give information about minimum and maximum production tasks, about resource number, about capacity and profitability.


Fig. 5. Decrease decision of maximum task.


Fig. 6. Increasement decision of minimum task.


Fig. 7. Resources task diagram.
An example of described method is presented in Fig. 7 where the grey areas are limited by maximum task levels $T_{\text {max }}$ and minimum task levels $T_{\text {min }}$ in $\left[t_{j}, t_{j+1}\right]$ period. The grey areas, that arise above of thick line that limits maximum quantity of resources $N_{R \max }$, means that $T_{\text {max }}$ is greater as minimum number or resource $N_{R \text { min }}$, and will arise an overloading risk $\Delta T$ and in this case it must decreased maximum task, $T_{\max }$, in $\left[t_{0}, t_{1}\right]$ period.

In period $\left[t_{2}, t_{3}\right], T_{\max }<N_{R \max }$, it appears a free capacity and this free capacity is equal with area between $S_{\max }$ and $N_{R \max }$.

This free capacity can be used for additional task or exists the possibility to distribute a part of resources in other periods with respect to capacity constraints.

The grey areas that arise under the line that limits minimum number of resources $N_{\text {Rmin }}$, means that minimum task $T_{\text {min }}$ is lesser as minimum number of resources $N_{R \text { min }}$, in this case it must increased minimum task $T_{\text {min }}$. If the minimum task $T_{\text {min }}$ is lesser as minimum number or resources $N_{R \min }$, task quantity $N_{r \min }-T_{\text {min }}$, can be considered free task and it is equal with area boundaries in analysed period $\left[t_{j}, t_{j+1}\right]$ and $T_{\min }$ and $N_{R \min }$. This free
task shows that production task is underevaluated or the numbers of resources are in excess.

A white area in framework surface between $N_{R \max }$ and $N_{R \min }$, shows an external flexibility, that means in this period it can be added task or it can be reduced production resources.

If after applying decisions in conformity with communicating vase principle, the maximum task $T_{\text {max }}$ and minimum task $T_{\min }$ are in correspondence with area $N_{R \max }$ and $N_{R \min }$, the differences between $T_{\text {max }}$ and $N_{R \max }$ and $T_{\text {max }}$ and $N_{R \max }$, represent internal flexibility, IF.

## 3. CONCLUSIONS

The TIT method, used for activities planning and scheduling of product project, offers some information like: Critical time interval of planning in framework of loading time where can arise the risk of overloading or under loading task. It is a clear panel of constraints or necessary conditions for taking one decision in operations planning and respecting allocated resources constraints. Assign feasibility of decisions moving the ES and LS dates in neighbour's task cell, thus the identification of underloading and overloading can be allocated in period with overloading capacity or free capacity. The decisions in framework of time interval are made based on communicating vase principle. All these information are independent and it is not necessary a detailed planning as in traditional curve of task with limited capacity, where the decision is taken by chief of workshop or by planner. In associating with cost elements, the use of the model TIT can lead to an optimum strategy for process in workshop.

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