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University POLITEHNICA of Bucharest, Machine and Manufacturing Systems Department
Bucharest, Romania

QUALITY LOSS FUNCTION BY ROBUST TOLERANCE DESIGN AT THE OPTIMAL COST

George DRAGOI, Daniel BRISSAUD, Miha JUNKAR, Costel Emil COTET

Abstract: *The paper addresses the issue of the achievement of the quality by robust tolerance allocation at the minimum cost. This model proposes an optimization model taking into account the process capability by determining the optimal tolerance combination considering dependent variables for tolerance design. The basic idea is that product functions are combinations of part characteristics and the client view is a function of the overall product. This model enables the process design not only to predict scrap rate in accordance with the tolerances allotted, but also to minimize the total quality loss due to the dependence of parts on the life-cycle of the product.*

Key words: *quality loss function, dependant variables, robust tolerance design, optimal cost.*

1. INTRODUCTION

The paper addresses the issue of the achievement of the quality by robust tolerance allocation at the minimum cost. The basic idea is that product functions are combinations of part characteristics and the client view is a function of the overall product (not of the part characteristics in isolation). The complex interactions between tolerances force to individually analyze the relationships between functional requirements and tolerances. A systematic procedure should define the permissible dimensional and geometrical deviations, and expert knowledge and experience in tolerancing should help. The paper features the following contributions. First, QLF (Quality Loss Function) is defined in the multidimensional case taking into consideration dependent variables. Secondly, the QLF is decomposed into a sum of variances and cross products of the deviations from the arithmetical averages, which are obtained at each target feature for each constitutive part. Data modelling is based on now well-known mechanisms. Part models are appropriated to the type of tolerance. The manufacturing results are simulated stochastically on the basis of deviations of the machine-tools, coming from basic standards. Thirdly, results enable to extract capable processes and to select among alternatives. A comparison of the part tolerance zones to resulting calculated deviations from the process chain provides first criteria for acceptance or rejection of the matching part tolerance - process. Alternative processes can be compared by an integrated estimation of the manufacturing effort. And fourthly, the proposed system is basically used to connect tolerancing modules to the functional, manufacturing, inspection or utilization requirements. Taguchi defines quality as «*the quality of the product is the minimum loss imparted by the product to the society from the time product is shipped*» [1]. This economic loss is associated with losses due to rework, waste of resources during manufacturing, warranty cost, customer complaints and dissatisfaction, time and money spent by customers on failing products, and eventual loss of market share. The principle is very simple. When a performance is targeted, a variation from this target means a loss of quality of the system. Quality simply means no variability or only a very little variation on target performances [4] when a quality characteristic deviates from the target value, it causes a loss. Taguchi proposed to formalize the loss by a quality loss function.

The concept of the quality loss function (QLF) uses the principle of the electrical engineering signal/noise ratio used to maximize the ratio of useful energy to wasted energy. In the production process there exist certain variability. We try to have no difference between the actual process means and the nominal values, and the smallest variances when the number of items is relatively big, i.e. a robust design. The ideal function of a design represents the theoretically perfect relationship between the expected performance of the product and the performance that can be achieved. Unfortunately, many sources of variations increase the discrepancy between those two values. The quality loss function is the function characterizing this discrepancy and based on a formulation of the loss for the client due to the non perfect realization of the product. The best design should minimize this function. This minimization function is constrained by the cost of the manufacturing processes to be operated to achieve the tolerance; deviations must be compatible with the requested quality. In the absence of constraints, minimizing the Taguchi quality loss function would result in tolerance values equal to zero, which is not technological feasible. The tolerance allocation problem involves optimizing the manufacturing cost in addition to the quality cost subject to a tolerance stack up and other constraints

2. PREVIOUS STDUDIES

A lot of work has been already done in the field. Zhang and Huq (1993) made a review on tolerance design, after Chase (1991) focused on tolerance analysis on design. Krishnaswani & Mayne (1994) proposed to optimize tolerance allocation based on manufacturing and quality costs. Kusiak addressed the issue of the achievement of quality by robust tolerance design in several papers. It extended the Taguchi QLF to multi-dimensional chains and considered discrete tolerances rather than continuous. He proposed the stochastic integer programming to model the relationship between manufacturing cost, manufacturing yield and discrete tolerances. He also proposed to use the design of experiments approach to minimize sensitivity of tolerances to manufacturing process variations [5]. The assumptions needed to be made in his approach are:

- The process independence law. The manufacturing processes used to generate each tolerance are independent.
- The normal distribution law. The processes used to generate each tolerance follow the normal (Gaussian) distribution for a huge number of identical items. This assumption is only due to the mathematical techniques and does not come from the physics of the production even if it is generally true.
- The value of the standard deviation Δ of each process can be known previously the tolerance allocation made.

Choi, Park and Salisbury (2000) developed a model for the allocation of statistical tolerances to part features to minimize the sum of machining cost and quality loss under alternate processes with different cost models. Assumption is made that the loss is an incremental cost from the centre of the tolerance to the tolerance limits since the functionality is worst at the tolerance limits than at the tolerance centre. It is entirely true when parts are addressed in isolation. But the assembly of the parts in a product can fit the best results even when some inclusive parts are tolerance-limit. It is due to the interactions between parts and variables cannot reasonably be considered independent. Kim et al. (1999) proposed a heuristic algorithm to optimize the tolerance allocation based on the two criteria tolerance and cost. Liu and Wei (2000) proposed a non-linear formulation to minimize manufacturing loss due to non conform parts. Robust process tolerance can be generated based on a mix of both manufacturing and quality costs. Our formulation of the problem leans on Kusiak's work to define the quality loss function in a multi-variables case and on Choi's work for the models of the manufacturing costs. It rids of the assumptions on independent variables and symmetrical distribution to address dependences among variables and loose distributions.

3. A FORMULATION OF THE QLF WITH DEPENDENT VARIABLES TO PERFORM TOLERANCING

Taguchi's Loss Function is a method of measuring quality central to Taguchi's approach to design. It establishes a financial measure of user dissatisfaction on a product performance deviating from a target value. Thus, both average performance and variation are critical measures of quality. The use of the QLF with dependent variables in the multidimensional case was proposed and justified [10]. The following assumptions are made as usual:

a) By definition, the Quality Loss Function is zero at the point $(a_1,\ldots, a_n)$.

$$L\ (a_1,\ldots,a_n) = 0; \tag{1}$$

b) At the target point, the function L has a minimum, so it's all first order partial derivatives at this nominal value vanish,

$$\frac{\partial \mathrm{L}}{\partial \mathrm{x_i}}(\mathrm{a_1}, \mathrm{a_2}, \ldots, \mathrm{a_n}) = 0, \qquad \forall \mathrm{i} \in \overline{1,\mathrm{n}}\ ; \tag{2}$$

c) It can also be assumed that the argument x is close enough to the target point, $x \approx a$, and consequently it can be considered that terms of orders higher than two, are zero because the rest of the Taylor formula tends towards zero. In these conditions, the approximate formula is got:

$$L(x_1, x_2, \ldots, x_n) \approx \frac{1}{2!}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2 L}{\partial x_i \partial x_j}(a_1, a_2, \ldots, a_n)(x_i - a_i)(x_j - a_j)\ . \tag{3}$$

Then it is natural to take the following quadratic form as a model for the Quality Loss Function:

$$L(\mathrm{x}_1, \ldots, x_n) = \sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}(x_i - a_i)(x_j - a_j)\ . \tag{4}$$

Here (k_{ij}) denote the costs that can be determined for each specific case.

Remarks:

- If $k_{ij} = 0$ for each pair (i, j), $i \neq j$, the model of Multi-Component Tolerances [5] is obtained.
- For a given pair (i, j), the cost k_{ij} is determined by estimating the loss when x_i deviates from a_i by Δ_i and x_j deviates from a_j by Δ_j.
- It is possible that losses caused by deviations have unequal values for the lower and upper limits respectively. In other words, for a given pair of indices, k_{ij} can take a single value or two values, depending on the position of the variable in regard to the target point.

Let us suppose now that m observations of identical products with multiple dimensions are taken. The expected loss given by the expression of the Quality Loss Function for a sample of m items is defined as the arithmetic mean of the loss,

$$\overline{L}(x_1, x_2, \ldots, x_n) = \frac{1}{m}\sum_{\lambda=1}^{m} L(x_1^{\lambda}, \ldots, x_n^{\lambda}) \tag{5}$$

Considering the observed values as outcomes of a random vector, risk can be computed in terms of mathematical statistics. The arithmetical Quality Loss Function can be decomposed into the following factors: the sum of variances of the arithmetic mean value and the cross products of deviations of the empirical mean from the target value for every constitutive part:

$$\begin{aligned}\overline{L}(x_1, \ldots, x_n) &= \frac{1}{m}\sum_{\lambda=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}(x_i^{\lambda} - a_i)(x_j^{\lambda} - a_j) \\ &= \frac{1}{m}\sum_{\lambda=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}\left[(x_i^{\lambda} - \bar{x}_i) + (\bar{x}_i - a_i)\right] \\ &\quad \left[(x_j^{\lambda} - \bar{x}_j) + (\bar{x}_j - a_j)\right]\end{aligned}\ . \tag{6}$$

So:

$$\overline{L}(x_1,...,x_n) = \sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}\left[s_{ij}^2 + (\overline{x}_i - a_i)(\overline{x}_j - a_j)\right]$$

$$\text{with} \quad \overline{x}_i = \frac{1}{m}\sum_{\lambda=1}^{m} x_i^{\lambda}$$

$$\text{and} \quad s_{ij} = \frac{1}{m-1}\sum_{\lambda=1}^{m}(x_i^{\lambda} - \overline{x}_i)(x_j^{\lambda} - \overline{x}_j), \quad . \qquad (7)$$

$$i \in \overline{1,n}\,;\; j \in \overline{1,n}$$

Value k_{ii} is determined by estimating the loss when x_i deviates from a by Δ_i. Value k_{ij} ($i \neq j$; i, j=1, 2, 3) is determined by estimating the loss when x_i and x_j deviates from ai_i and a_j by Δ_i and Δ_j . In the bi-dimensional case, some previous results [10] were proposed (see equation 8)

$$\begin{aligned}\overline{L}(x,y) &= k_{11}\left[s_{11}^2 + (\overline{x}-a)^2\right] + k_{22}\left[s_{22}^2 + (\overline{y}-b)^2\right] \\ &+ k_{12}\left[s_{12}^2 + (\overline{x}-a)(\overline{y}-b)\right] + k_{21}\left[s_{21}^2 + (\overline{x}-a)(\overline{y}-b)\right]\end{aligned}. \quad (8)$$

4. CASE STUDY

Figure 1 shows the classical example of Fortini's (1997) overrunning clutch [6] studied by many authors [8, 9, 6, 7].

Let us consider that the functional condition y is the contact angle. The results from a functional analysis coupled with the engineers' know-how give a tolerance value on y. Design variables are dimensions x_1, x_2 et x_3 as shown on figure 1. The design problem consists in determining the tolerance requirements Δ_i on a dimension x_i.

Let us also consider the figures for the example:

1) The nominal value and tolerances of angle y are 0.144 ± 0.02 rad based on the results from the functional analysis and engineers' know-how.

2) The target vector (a_1, a_2, a_3) = $(2.17706, 0.9000, 4.000)$ in inches, is considered as the designer's wish on the three dimensions x_i respectively.

3) The tolerance requirements for dimensions i are: Δ_1 for a_1, Δ_2 for a_2, Δ_3 for a_3. Those are the unknown of our synthesis problem.

Now, the following calculation model is proposed: let us consider the «optimal» dimensions for each component (that implies «optimal» costs), devices with different sizes (and different costs implicitly) are taken into consideration. Considering the values observed as outcomes of a random vector, risk can be computed in terms of mathematical statistics. Deviations around the «optimum» can be used to define k_{ij}. Obviously, there exist many other possibilities for evaluating k_{ij}. Loss caused by respectively unacceptable hub, roller, cage, hub and roller, hub and cage, cage and roller, is estimated by statistical methods [0.12, 0.2, 0.7, 0.1, 0.3, 0.4] × C, where C is the repairing cost (i.e. service cost, manufacturing cost and assembly cost, customer's dissatisfaction). When value Y is not in the tolerances for given (x_1, x_2, x_3), the devices is rejected. When value Y is in the tolerances, the given (x_1, x_2, x_3) values are used for the arithmetical mean.

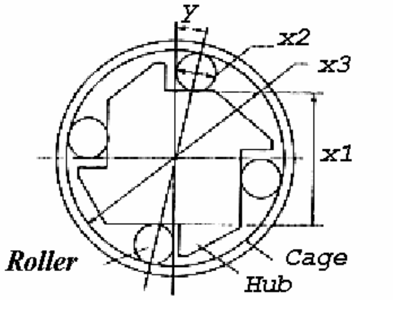

Fig. 1. The Fortini's overrunning clutch.

Value k_{ii} is determined by estimating the loss when x_i deviates from a by Δ_i. Value k_{ij} ($i \neq j$; i, j = 1, 2, 3) is determined by estimating the loss when x_i and x_j deviates from a_i and a_j by Δ_i and Δ_j respectively.

$$\begin{aligned} k_{11} &= \frac{0.12C}{\Delta_1^2};\; k_{22} = \frac{0.2C}{\Delta_2^2};\; k_{33} = \frac{0.7C}{\Delta_3^2}; \\ k_{12} &= \frac{0.1C}{\Delta_1\Delta_2};\; k_{13} = \frac{0.3C}{\Delta_1\Delta_3};\; k_{23} = \frac{0.4C}{\Delta_2\Delta_3} \end{aligned} \qquad (9)$$

In this case, the arithmetical mean of values of Quality Loss Function is (21) using $L(x_1, x_2, .., x_n,)$ proposed for three-variable function:

$$\overline{L}(x_1, x_2, x_3) = \sum_{i=1}^{3} k_{ij}\left[s_{ij}^2 + (\overline{x_i} - a_i)(\overline{x_j} - a_j)\right], \quad (10)$$

where k_{ij} is determined by equation (9).

Results in Table 1 are obtained from dependent variables and symmetric distribution. The overall problem can now be solved. What are the best manufacturing processes for the three independent parts of the overrunning clutch minimizing the global cost (manufacturing cost + loss cost)?

This model proposes an optimization model taking into account the process capability; it allows design process tolerances to minimize the total cost due to both the manufacturing process and the global loss cost. In practice, a work piece flowing through process operations implies that the work piece must be conformably produced by all preceding operations.

Table 1

QLF for Fortini's overrunning clutch with dependent variables

y-y_t system output	0.008756	0.009889	0.0128	0.013
Δ_1 (in inches)	0.00425	0.00425	0.00485	0.006
Δ_2 (in inches)	0.0006	0.0006	0.0006	0.0005
Δ_3 (in inches)	0.00256	0.0025	0.0022	0.003
Manufacturing cost ($)	6.582112	6.82033	6.90069	6.6661
L (x_1, x_2, x_3) Dependent Variables	0.016	0.0162	0.02074	0.0256
Total cost ($)	6.5981	6.8365	6.92629	6.6917

Obviously, it would like both the area under the cumulative standard normal probability curve between the range, and the conformance rate of each part, to be maximized. It is widely known that the larger the tolerance, the lower the manufacturing cost. But, the overall objective of the problem becomes to design tolerances minimizing the total expected loss of the whole process (usage, maintenance, service, client satisfaction). Criteria of dependence between parts throughout the product life-cycle are very significant.

Traditionally, process tolerances are allocated by individual engineers based on personal expertise. Consequently, tolerances are frequently underestimated or overestimated from the manufacturing cost point of view. In addition, the cost of the loss due to tolerances combinations (dependence of parts) is often neglected. Consequently, a lot of production costs with respect to tolerances designed are unnecessarily wasted.

Our model enables the process design not only to predict scrap rate in accordance with the tolerances allotted, but also to minimize the total quality loss due to the dependence of parts on the product life-cycle. Studying customer, product utilization and service inputs, the product design group based on technical and manufacturing constraints, derives compromise values for tolerances of many parts of the product.

5. CONCLUSIONS

Two other approaches were illustrated based on the same Fortini's example. Even if it cannot be exactly compared, the results are discussed here. Feng and Kusiak (1997) indicated that the quality cost only lightly impacts on the total cost, despite the fact that any shift of the process mean in relation to the design mean was penalized by a quadratic term.

Choi, Park and Salisbury (2000) purposed to allocate optimal tolerance to each individual feature at a minimum cost, considering the Taguchi loss function and incorporating multiple potential manufacturing processes. They also indicated that the quality loss cost is only a small portion of the total cost. We have postulated, and demonstrated in section 4, that variables dependence highly impacts on the estimated loss. Feng and Kusiak's (1997) quality loss function is applied to the single and multicomponents tolerance synthesis with independent variables. In our model with dependent variables, global cost results from a global optimization based on both manufacturing process and quality loss. Loss is estimated with the sensitivity of that particular change based on product life-cycle and customer requirements. The higher the level of dependence, the more it's the effect on the user satisfaction function. The gap between the current tolerance design solution and the purposed solution widens the sensitivity of that particular dependence changes, based on product utilization and customer reflection. Consequently, a potential user may decline the product although that particular attribute do not have a high importance level rating in the traditional approach. A compromise between conflicting interests occurs and dependent variable parts of the product (the user view) may purchase a product that does necessarily meet the prerequisites. In reality the customer may purchase the product due to functionality and the perfection of design due to the dependence of the constitutive parts. Tolerancing aims to maintain functionality and it is done in this approach.

This paper has defined a quality loss function in the multidimensional case taking into consideration dependent variables. The method provides an efficient and systematic way to optimize product design in quality, cost and performance points of view.

REFERENCES

[1] Bryne and Taguchi, (1986). T*he Taguchi approach to Parameter Design,* ASQC Quality Congress Transactions, Anahaim, CA, p. 168.

[2] Di Lorenzo (1990), *The Monetary Loss Function or Why We need TQM*, Proceedings of The International Society of Parametric Analysts, 12th Annual conference. San Diego, CA, pp. 16-24.

[3] Feng, C., X., Kusiak, *A.,* (1997). *Robust Tolerance Design with the Integer Programming Approach*, Journal of Manufacturing Science and Engineering, vol.119, pp 603-610.

[4] Feng. C.-X., Kusiak, A., (2000), *Robust Tolerance Synthesis with the Design of Experiments Approach*, Journal of Manufacturing Science and Engineering, vol.122, pp. 521-528.

[5] Choi, R., Park M-H., Salisbury, E. (2000). *Optimal Tolerance Allocation with Loss Functions*, Journal of Manufacturing Science and Engineering, Vol. 122, pp.529-535.

[6] Fortini, E. (1967). *Dimensioning for Interchangeable Manufacture*, Industrial Press, New York.

[7] Krishnaswami, M. Mayne, R. (1994). *Optimising Tolerance Allocation Based on Manufacturing Cost and Quality Loss,* Proceedings of the ASME on Advances in Design Automation, DE-Vol. 69-1, ASME, New York.

[8] Dragoi, G., Tarcolea C., S., Draghicescu, D., Tichkiewitch S. (1999). *Multidimensional Taguchi's model with dependent variables in quality system*, in Integrated design and Manufacturing in Mechanichal Engineering'98, edited by de Jean Louis Batoz, Patrick Chedmail, Gerard Cognet, Clément Fortin, (in Kluwer Academic Publisher Dordrecht/ Boston/London), pp. 635-642, ISBN 0-7923-6024-9.

[9] Chase K. and Parkinson A. (1991). *A survey of research in the application of tolerance analysis to the design of mechanical assemblies*, Research in Engineering Design, Vol.1, n°3, pp. 23-37.

Authors:

PhD, George DRAGOI, Professor, University Politehnica of Bucharest, Romania,
E-mail: gdragoi@mix.mmi.pub.ro
PhD, Daniel BRISSAUD, Professor, Institut National Polytechnic of Grenoble, France,
E-mail: Daniel.Brissaud@hmg.inpg.fr
PhD, Miha JUNKAR, Professor, University of Ljubljana, Slovenia,
E-mail: miha.junkar@fs.uni-lj.si
Professor, Costel Emil COTET, Lecturer, University Politehnica of Bucharest, Romania,
E-mail: costel@mix.mmi.pub.ro