

## NEW THEORETICAL ASPECTS CONCERNING THE GENERATION OF THE COMPLEX SURFACES

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**Abstract:** The paper constitutes a new original study about the generation of the complex surfaces. In that sense, there are presented the kinematic generation laws of the  $D$  and  $G$  curves at the generation of the spherical and toroidal surfaces that are complex surfaces of the second specie. The central problem at the generation of these surfaces is represented by the realization of the movement  $M^G(\vec{v}_g)$  for the generation of the generating curve  $G$ . The paper shows new aspects concerning the realization of this movement at the turning proceeding, by a pseudo program support different by the classic one.

**Key words:** surface generation, generatrix, directrix, spherical surfaces, toroidal surfaces.

### 1. INTRODUCTION

In an anterior paper [2], there were presented original aspects about the complex surfaces, concerning the definition, the cinematic principle of their generation and a classification of them. Considering that classification, in function of the  $G$  and  $D$  curves' generation modes, we have three categories: a – complex surfaces of the first specie, b – complex surfaces of the second specie, c – complex surfaces of the third specie. In the same paper, we presented the cinematic laws for the realization of the movement  $M^D(\vec{v}_d)$  and of the movement  $M^G(\vec{v}_g)$  at the generation of same complex surfaces of the first and second specie, largely used in technique.

As it was presented in [2], the complex surfaces of the second specie are those geometrical surfaces in which:

1. The  $G$  curve has such geometrical form that impose its generation by compositions of single movements kinematical coordinated by  $R_{CCIN}^G$ .

2. The  $G$  curve has such a geometrical form that can be generated by the main generating movement or by a feed movement (rectilinear or circular), if the generation is made by milling or by grinding.

In the category of the complex surfaces of the second specie, near the profile rotation surfaces and conical rotation right surfaces having cinematic generation particularities exposed in the paper [2], there are the geometrical surfaces with spherical and toroidal geometrical surfaces, existing as real surfaces at many pieces from the industry. The present paper brings original aspects concerning the generation of these surfaces of the second specie.

### 2. THE GENERATION OF THE SPHERICAL AND THOROIDAL SURFACES

These surfaces form a distinct group in the family of the rotation surfaces. The fundamental surface of the group is the thoroidal surface, and the spherical and globoid surfaces are obtained as derived surfaces.

The spherical and globoid surfaces are the generatrix  $G$  a circle that prop up on an infinite multitude of directrix  $D_i$  which are circles too. The generation of these surfaces imposes the generation of the  $G$  curve that can be realized in two ways:

1. By materialization on a half of the length, by the generating edge of the generating tool;
2. By generation by cinematic way as a trajectory of the movement of a point in a plane, by programmed generatrix.

Speaking about the curve  $D$ , it is realized as a trajectory of the movement of the point in a plane that is of the main movement in the case of the generation by turning. In generation by milling and grinding processes, the curve  $D$  is realized as an envelope of the positions of a cinematic curve, by a circular feed movement.

The spherical and globoid surfaces generated with a materialized generatrix have small dimensions, which permits the materialization of the curve  $G$  on a half on

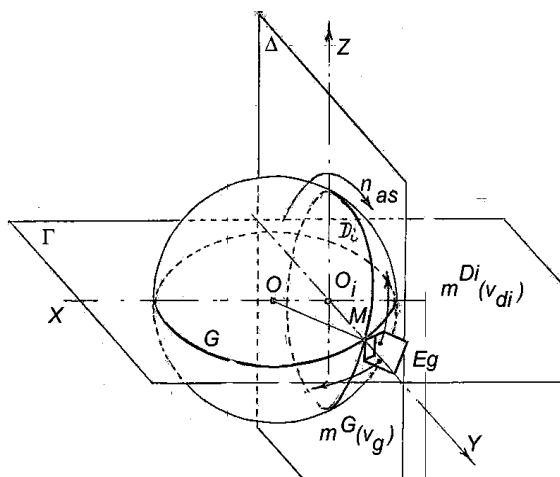


Fig. 1. Generation of a spherical surface by turning.

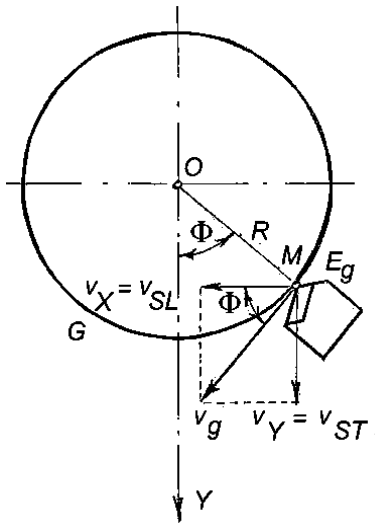


Fig. 2. The cinematics of the movement  $M^G(\vec{v}_g)$ .

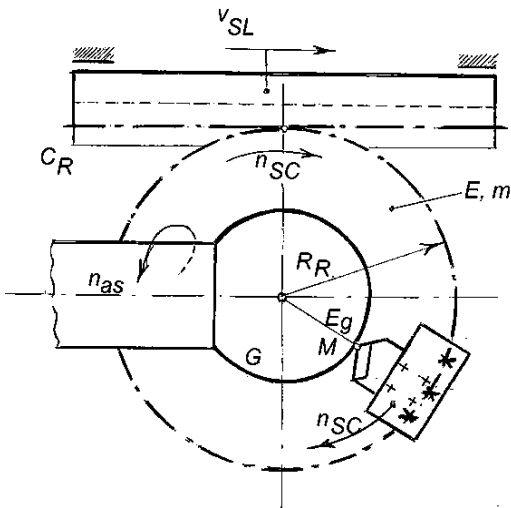


Fig. 3. Generation device DG.

the length. These curves, generated by turning, are not complex surfaces, because their curves \$G\$ and \$D\$ are not generated as trajectories by compositions of simple movements, kinematical coordinated. By consequence, there are simple surfaces, and their generation represents a restraint activity field.

The largest field of the generation of the spherical and thoroidal surfaces is the field at which the curve \$G\$ is cinematically generated.

For the generation of a spherical surface by turning (Fig 1), the generating element \$E\_g(M)\$ (representing the generation cutting tool), makes simultaneously the movement \$m^G(\vec{v}\_g)\$ along the some circular generatrix \$G\$, and the movement \$m^{D\_i}(\vec{v}\_{di})\$ along the some circular directrix \$D\_i\$, as main generating movement, with the \$n\_{as}\$ frequency.

Cinematically speaking, the movement \$m^G(\vec{v}\_g)\$ on the circular curve \$G\$ is realized by composition of the

transversal feed movement \$E\_g(M)\$ with the variable speed \$\vec{v}\_{ST}\$, which is the complementary generating movement of the generatrix \$G\$ (so, \$\vec{v}\_{cg}^G = \vec{v}\_{ST} + const.\$), with the longitudinal feed movement, with a constant speed \$\vec{v}\_{ST}\$, which is the feed movement for the generation of \$G\$ (so, \$\vec{v}\_s^G = \vec{v}\_{ST} + const.\$) (Fig. 2).

These movements are cinematically coordinated by \$R\_{CCIN}^G\$, for the reason that \$E\_g(M)\$ generates the circular trajectory \$G\$. The size of this ratio is:

$$R_{CCIN}^G = \frac{v_{ST}}{v_{SL}} = \tan \Phi, \quad (1)$$

in which the angle \$\Phi \neq const\$ during the generation of the curve \$G\$.

The speed \$\vec{v}\_g\$ of the element \$E\_g(M)\$ for generating of the circular curve \$G\$ is given by the kinematic law of the movement \$m^G(\vec{v}\_g)\$, which has the expression:

$$\vec{v}_g = \vec{v}_s^G + \vec{v}_{cg}^G = \vec{v}_{SL} + \vec{v}_{ST}. \quad (2)$$

By consequence, for the generation of the circular generatrix \$G\$ as a trajectory of the movement of the generating element \$E\_g(M)\$ in plane (the cinematic plane of base \$XOY\$) obtained by composing of two feed movements, one rectilinear and the other rectangular cinematically coordinated by \$R\_{CCIN}^G\$, it is necessary that the theoretical \$G\$ to be generated as a scheduled generatrix, in that case \$R\_{CCIN}^G\$ being assured by the program support.

If the simplest program support is considered, namely the template, it contains the generatrix \$G\_s\$ with the form of a concave circle arc, which is followed by the palpator with the longitudinal feed movement \$\vec{v}\_{SL}\$. In that case, behind it the transversal feed movement \$\vec{v}\_{ST}\$ appears, and the element \$E\_g(M)\$ generates the circular generatrix \$G\$.

For the generation of the directrix \$D\_i\$, and also implicitly for the generation of the spherical surface, the movement \$M^{D\_i}(\vec{v}\_{di})\$ of the element \$E\_g(M)\$ is realized by the main rotation movement with the frequency \$n\_{as}\$ made by the semi-finished product (thus \$\vec{v}\_{D\_i} = \vec{v}\_{as}^{D\_i}\$).

A variant of the programmed generatrix for the generation of the circular \$G\$ is brought by the paper [6], in which the author replaces the two rectilinear feed movements cinematically coordinated by the classic program support with a rotation movement realized using a generating device \$DG\$ (Fig. 3).

By means of the device \$DG\$, it is obtained the transfer of the feed longitudinal movement made by the longitudinal slide of the lathe, at the rack \$C\_R\$ of the device, carried by that slide. Appropriately, the linear movement \$\vec{v}\_{SL}\$ of the rack is transformed in a rotation movement of the wheel teeth that put into gear with the rack, having \$z\$ teeth and the module \$m\$. That gear is fixed on a rotating plate of the \$DG\$ that contains the generating cutting tool \$E\_g(M)\$.

In this way, the circular feed movement  $m^G(\bar{v}_g)$  of the element  $E_g(M)$  is obtained with the speed of circular feed  $n_{sc}$ ,  $E_g(M)$  generating the circular generatrix  $G$ .

It is possible to make a similitude with the classically programmed generatrix, in the sense that the gear  $z$  through its rolling circle  $R_R$  represents the programmed circular generatrix  $G$  that is followed by some pseudo palpators – the teeth of the rack  $C_R$  that are meshing with the teeth of the gear  $z$ .

In this way, it results that the generating device  $DG$  is an original model of the authors, realizing the circular generatrix  $G$  of a spherical surface as a programmed generatrix. By consequence, the circular generatrix  $G$  is realized as a circular trajectory of the movement of a point in a plane by a pseudo program support with programmed generatrix.

It is important to show that this pseudo program support – the gear  $z$  – does not assure a  $R_{CCIN}^G$ , because the movement  $m^G(\bar{v}_g)$  is not obtained by composing simple movements, being itself a simple rotation movement (circular feed movement).

However, by this pseudo program support it is assured a ratio between the cinematic parameters of the two generating movements, namely the frequency  $n_{as}$  of the main movement and respectively the frequency  $n_{sc}$  of the circular feed movement.

We named this ratio *the ratio of the roughness*  $R_{RUG}$  that is a linear variable value with the longitudinal feed  $s_l$  of the longitudinal slide, respectively of the rack  $C_R$  in its rectilinear movement with the speed  $\bar{v}_{SL}$ . For the determination of its expression, we start from the cinematic generation of the circular generatrix  $G$  with the device  $DG$ .

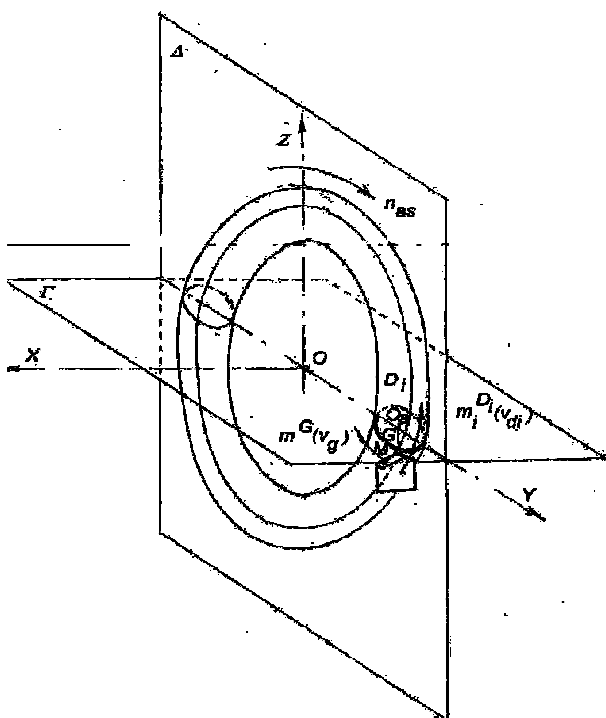


Fig. 4. Generation of a toroidal surface by turning.

In this way, the size of the longitudinal feed  $\bar{v}_{SL}$  of the longitudinal slide of the lathe has the expression:

$$v_{SL} = s_L \cdot n_{as} \text{ [mm/min]}, \tag{3}$$

in which:  $s_L$  is the size of the longitudinal size [mm/rot];  $n_{as}$  – frequency of the main movement, [rot/min].

On the other side, the speed  $\bar{v}_{SL}$  is also the speed of the rack  $C_R$ , resulting its size, that will be:

$$v_{SL} = R_R \cdot \omega_{SC} \text{ [m/min]}, \tag{4}$$

in which:  $R_R$  is the rolling radius having the expression

$$R_R = \frac{1}{2} m \cdot z; \tag{5}$$

$\omega_{SC}$  – angular speed

$$\omega_{SC} = 2\pi n_{sc} \text{ [rot/min]}, \tag{6}$$

$n_{sc}$  – frequency of the circular feed of the gear  $z$ , in rot/min.

If the relations (5) and (6) are replaced in the relation (3), we obtain the expression of  $R_{RUG}$ :

$$R_{RUG} = \frac{n_{sc}}{n_{as}} = \frac{s_L}{\pi m z} = k^{const} \cdot s_L, \tag{7}$$

in which:  $k^{const} = 1/\pi m z$  is a constant value of the generation device  $DG$ .

From the expression (7) of the ratio  $R_{RUG}$  it is obtained the conclusion that for the generation of a spherical surface by turning with a lower roughness (with a superior quality of the machined surface), it is necessary that the size of the longitudinal generation feed having the value  $s_L^{min}$  to be possible on the respective classic lathe. Thus, we obtain a ratio  $R_{RUG}^{min}$ , which imposes to

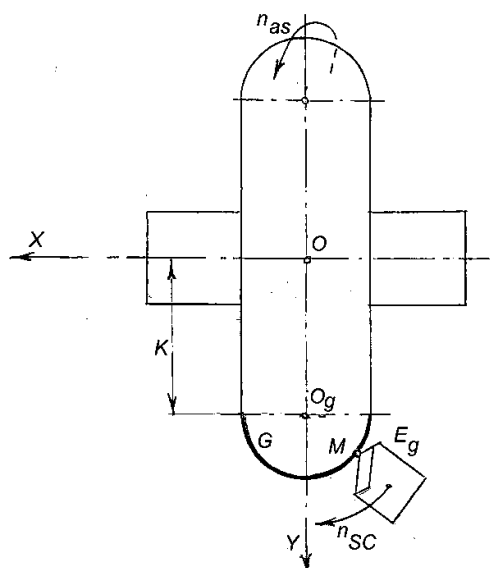


Fig. 5. The  $m^G(\bar{v}_g)$  movement at the toroidal surfaces.

acquire in the generation processes a lowest frequency  $n_{as}^{\min}$  of the main movement, from which it results implicitly  $n_{sc}^{\min}$  for the circular feed movement of the element  $E_g(M)$ .

In the case of the generation of a toroidal surface by turning (Fig. 4), the cinematic of the processes is the same as for the spherical surfaces. Therefore, the generating element  $E_g(M)$  realizes the movement  $m^{D_i}(\vec{v}_{di})$  along the some circular directrix  $D_i$ , as main movement with the frequency  $n_{as}$ . Simultaneously, the element  $E_g(M)$  makes the movement  $m^G(\vec{v}_g)$  along the circular generatrix  $G$ .

From a cinematic point of view, as in the case of spherical surfaces, the movement  $m^G(\vec{v}_g)$  can be realized by the composition of the longitudinal feed movement of the element  $E_g(M)$  with its transversal feed movement, cinematically coordinated by a classical program support.

At the toroidal surfaces, these feed rectilinear movements can be replaced by the rotation movement of the pseudo port program – the wheel gear  $z$  – from the device  $DG$  (Fig. 5).

In that case, the rotation centre  $O_g$  of the circular feed movement  $n_{sc}$  of the element  $E_g(M)$ , that is also the center of the gear  $z$ , can be adjusted at the distance  $K$  in regard with the rotation axis  $OX$ .

Thus, the element  $E_g(M)$  generates the circular convexes generatrix  $G$  through the movement of circular feed  $n_{sc}$ , and by consequence of its main movement  $n_{as}$ , it generates the toroidal surface.

If by the movement of the circular feed  $n_{sc}$  the element  $E_g(M)$  generates a circular concave generatrix  $G$ , then by this main movement  $n_{as}$ , the element generates a globoid surface.

### 3. CONCLUSIONS

The generation of the spherical and toroidal surfaces is produced by respecting the kinematic laws of the generation of the curves  $G$  and  $D$  specific for the complex surfaces of the second specie, to which these surfaces are belonging. In that sense, the central problem of the generation is constituted by the realization of the movement

$m^G(\vec{v}_g)$  for the realization of the circular curve  $G$ , depending on the adopted generating process. So, in the case of the generation by turning, the circular generatrix  $G$  is generated as a programmed generatrix, by the composition of two feed rectilinear movements cinematically coordinated by  $R_{CCIN}^G$  assured by a classical program support.

The generation of the circular generatrix  $G$  can be realized by turning in another way, replacing the classical program support with a pseudo one. The presented paper makes evident new aspects in the generation of the complex surfaces of the second specie.

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