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“MEDIAL MACHINING” A NEW PARADIGM IN GEOMETRICALLY COMPLEX SURFACES MACHINING

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Abstract: *The paper presents a new approach in the generation of geometrically complex surfaces called Medial Machining. This is a new, complex, rational and high productivity way to machine surfaces during which the center of the real-tool moves on medial tool paths. The medial tool paths are curves equidistant to the associated medial surface and to the model surface. The existing contour machining being only a three-dimensional transposition of the bi-dimensional contouring without observing the additional features offered by the three-dimensionality become thus a particular case of the new paradigm. Several new concepts are being introduced such as medial machining, virtual tool, side and center-medial tool paths. The new concepts are derived from milling but can be applied to any other type of machining using cutting tools. A particular case of the medial machining is expected to offer unprecedented productivity and quality of the machined surface.*

Key words: *virtual tool**,** medial surface, geometric complexity, NC machines, NURBS, subdivision.*

1. INTRODUCTION

An object is created from a material and has a shape as *the appearance of something, especially its outline* [1]. If this shape is stable enough we can call it a solid. We will not discuss the shape complexity *per se*, because it contains the risk of becoming too philosophical without actually defining anything but we must observe that the *geometrical complexity* [2] is a common factor among the different kinds of complexity of a product and is fundamental for technical objects manufacturing [3].

The complexity of the obtained shapes is somehow synchronized with the history of the mankind. The tools “Olduwan” are 2.5 million years old and are the first objects ever manufactured in the history and trace the boundary between the genus “Homo” and genus “Apes” (Fig. 1) [4]. Even if those shapes are rather complicated obviously do not pose difficult problems to obtain them because are convex. Therefore the term complexity can be associated only to concave shapes.

The *geometrically complex surfaces* (GCS) carried names such as sculptural surfaces, free-form surfaces, impossible shapes [5], arbitrary topology shapes [6], subdivision surfaces [7] or hybrid surfaces [8] but a definitive name was not yet found. Therefore as a restriction of our next statements we will denote by geometrically complex surfaces the surfaces composed of parametric patches (including NURBS) and the subdivision surfaces

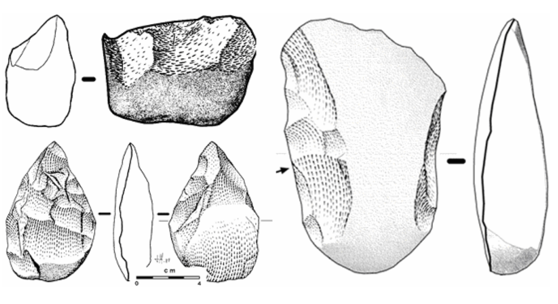

Fig. 1. The convex shaped “Olduwan” tools.

2. DIFFICULTIES IN THE MACHINING OF THE GEOMETRICALLY COMPLEX SURFACES

Despite the huge advances of this domain there still are many difficulties that arise in the milling of GCS.

To solve those difficulties models are required, which take into account different attributes of the elements that are implied into the machining process. The nature of attributes that have to be taken into account in order to *reproduce, recognize and solve* the difficulties can be used to classify these into: geometrical, technological and economical difficulties. These attributes are also concurrent and an industry-robustness solution should be able to meet them more or less all [9].

The most known of the difficulties encountered in machining the geometrically complex surfaces are:

2.1. Geometrical Difficulties

- Offset and tessellation requires offset-stable primitives (Fig 2,c).
- Uncut and gouging requires that the offset notion should be split into offset surface (Fig. 2,a) and distance surface or equidistance (Fig. 2,b).

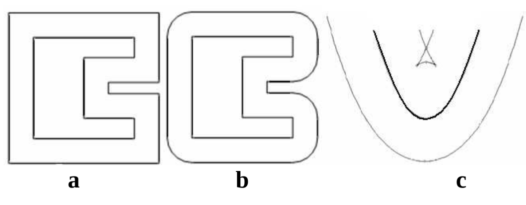

Fig. 2. Geometrical difficulties:

a – offset; b - equidistance/distance surface; c - offset unstable rational curve (parabola).

- Exact determination of the pencil curves [9] is a non trivial problem.
- 5 axis machining creates difficulties of interpolation by simultaneous control of the 5 corresponding joints and joint ranges.
- Collision and access, cannot be modeled by using C-space [9], tool-center cannot cut due to zero cutting speed.

2.2. Technological & Economical Difficulties

- High speed machining, precision and thin walls generate long finishing paths and huge data flows.
- NC Simulation cannot be avoided because it remains a part of *directly involved human processing*, prone to errors.
- For the dynamic process the determination of chip-load accordingly to the geometry is difficult.
- 5 axes machines require special maintenance, high qualified operators and expensive high precision tools.

3. IMPLICIT MODELS

3.1. Implicit Volumes

An implicit function f is a continuous real function on $\Re^3$. Using f we can define an *implicit surface* S_f

$$S_f = \left\{x \in \Re^3 \mid f(x) = c, c \in \Re\right\}. \tag{1}$$

and an *implicit volume* V_f

$$V_f = \left\{x \in \Re^3 \mid f(x) \le c\right\}. \tag{2}$$

For each set $A \subset \Re^3$ we can define an implicit distance function d_A associated to the implicit volume

$$d_A(q) = \left\{q \in V_f \mid d_A = \inf\left\{\|p - q\| \mid p \in A\right\}\right\} \tag{3}$$

where $\|p - q\|$ is the Euclidean distance between two points p and q. The function d_A is called the unsigned distance function to A [2].

Although the distance constraints can be expressed *analytically* the defining function can be *procedural* [2]. The surrounding space of A can be modeled as a three-dimensional matrix that contains in each cell the distance from the cell centre to the object. We will call the matrix (by notation abuse) *implicit volume* (Fig. 3,a).

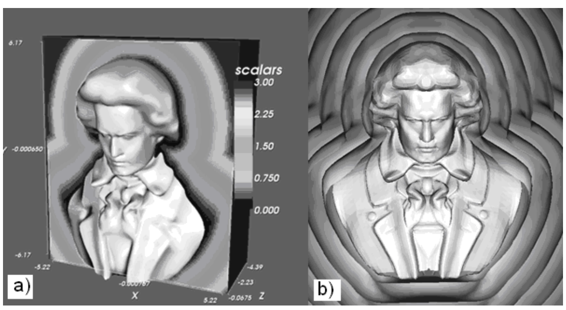

Fig. 3. Distance constraints: a – implicit volume; b – distance surface isocontours.

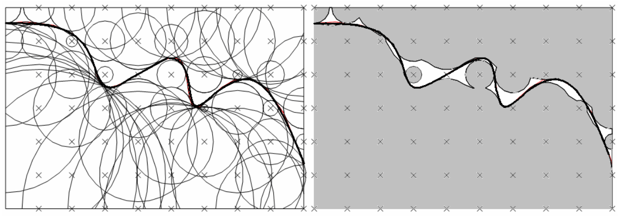

Fig. 4. Rough approximation (grey) of a parametric curve (black) using a low res. implicit volume slice.

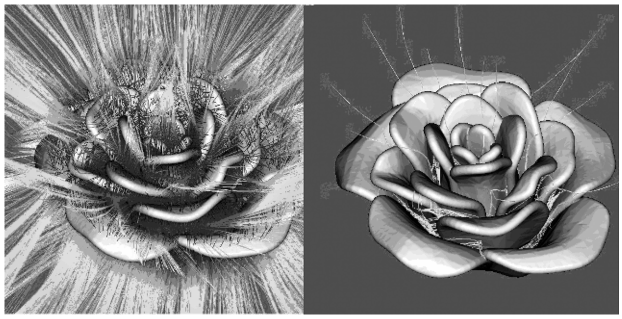

Fig. 5. Stream lines of the gradient field.

3.2. Implicit volume applications

The implicit volume approach has/can have a number of applications such as:

Milling model for complex surfaces. The implicit surfaces may also be seen as the union of spheres centered in each cell centre of the implicit volume matrix and having as radius the corresponding distance to the implicit surface. Therefore another definition of the implicit surfaces may be seen also as *limit-milling with continuous variable radius mill.* [10] (Fig 4)

Computing the equidistance. In order to compute the offset surface – distance surface of A, we have to contour the implicit volume with a non-zero value (corresponding to the offset distance) (Fig. 3,b).

Measuring of a complex surface. The measuring of a complex surface is ultimately the construction of the implicit volume with a bigger or smaller procedural resolution [11].

Distance field gradient. The implicit volume can be seen as a scalar field $\boldsymbol{F}$ (4) of the scalar distance function [12] and we can associate, to each cell of the implicit volume matrix, a gradient $\boldsymbol{G}$ as a vector that point in the direction of maximum distance function increase.

$$\vec{G} = \nabla F . \tag{4}$$

The gradient can be visualized using stream lines. We see that their density is maximal in zones situated near the concave edges as they converge. We can filter the streams and obtain the main access directions (Fig 5).

4. MEDIAL OBJECTS

In many applications we need information such as: how flat is the object how many branches it has, how we could define a symmetry axis (Fig. 6) for an object that is not completely symmetric, how we can define a similarity and apply group technologies. These are things that cannot be found out by representation of boundaries, but by structural shape properties which go beyond the

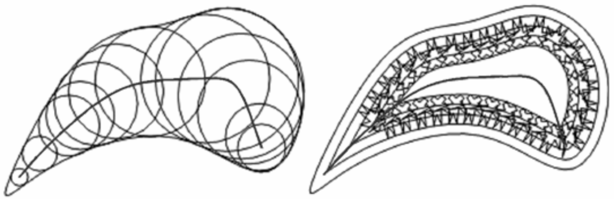

Fig. 6. Medial Axis as maximal discs and offsets self- intersections locus (grassfire).

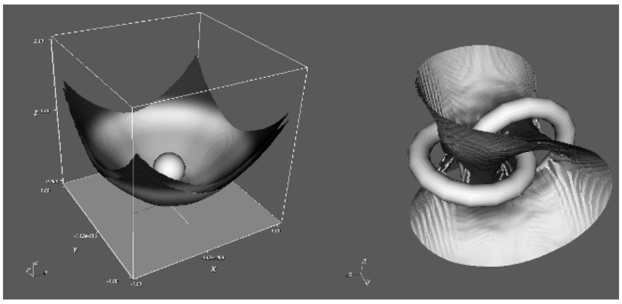

Fig. 7. Medial surface between a point and a plane is a paraboloid and two thori have one medial surface.

simple visible limits. However those can be addressed by introducing the *medial object* (Fig. 7) [13]:

Definition. The medial surface/axis $\mathcal{MS}$ of O is the locus of centers of maximal spheres/discs in O. (we say that a sphere is maximal if no other sphere contains it completely) [14].

The medial object was originally proposed many years ago [14] and has been the subject of research in different fields. Interest for the medial surface in machining comes from a number of useful properties [15]:

1. **is a homotopy to the original shape** thus *can be continuously deformed into the model surface*;
2. **is a thin set**, i.e., it contains no interior points therefore *does not need another model*;
3. **is invariant under Euclidean transformations** of the volume (rotations, translations);
4. given the radius of the maximal inscribed sphere associated with each medial surface point, **the volumetric object can be reconstructed exactly.**

Properties 1, 2, 3 are needed for the modeling of the complex surface and the property 4 allows us to have a reconstruction of the object starting from its medial surface (e.g. by machining with spherical tools).

4.1. Obtaining the medial surfaces

Despite its popularity, the numerical computation remains non-trivial. Most algorithms are not stable to small boundary perturbations, and heuristic measures for simplification [15] are often introduced: The most known methods to obtain the medial surface are:

- **Thinning** [14], peeling away layers from an object, retaining special points where offset is not smooth.
- **Voronoi diagram / skeleton** the vertices of the Voronoi diagram of a set of boundary points converge to the exact skeleton as the sampling rate increases.
- **Distance Functions** [15] as the locus of skeletal or medial surface points coincides with the singularities of the *Euclidean distance function* to the boundary. In the following (Fig. 8) the medial surfaces are obtained using

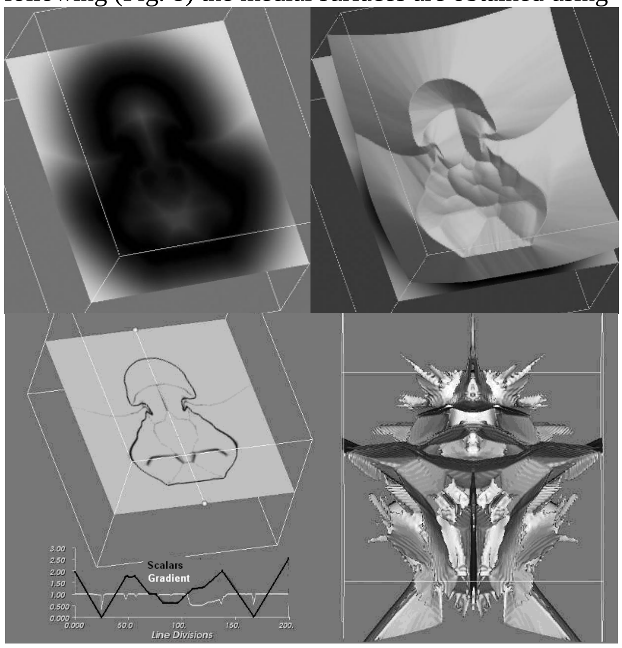

Fig. 8. A slice through an implicit volume : the distance function as greymap (a), distance function plot as crests and valleys (b), plot of the distance function vs. gradient magnitude (c) and the medial surface as contour of the points having gradient magnitude 0.98 (d).

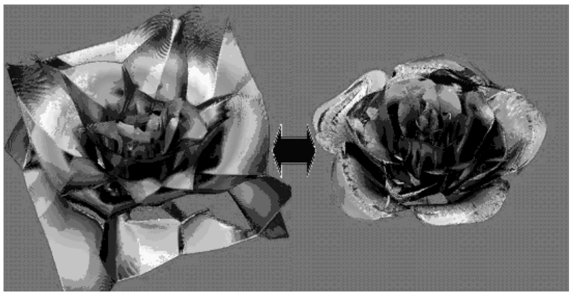

Fig. 9. Warping is allowed by the medial surface homotopy.

a variation of the distance function singularities method as contour of the points that have a gradient magnitude of the distance field different than 1.

4.2. Medial Surface Applications

The applicability of this kind of approach range from pattern recognition, robotic motion planning, finite element mesh generation, surface reconstruction from unstructured point clouds, analysis and quantification of the shape in medical images to the fields of mechanical and materials engineering. In manufacturing in particular this has many applications such as: A warp transform of the medial surface on the direction of the gradient (normal to the machined surface) allows the obtaining of the tool contact path from the tool centre paths, can show us which radii of the tool are needed to machine each point on the surface and also which areas of the tool will be machining each area of the model (Fig. 9).

5. THE MEDIAL MACHINING

In manufacturing the medial surfaces have two important applications: the Virtual Tool and the Medial Machining.

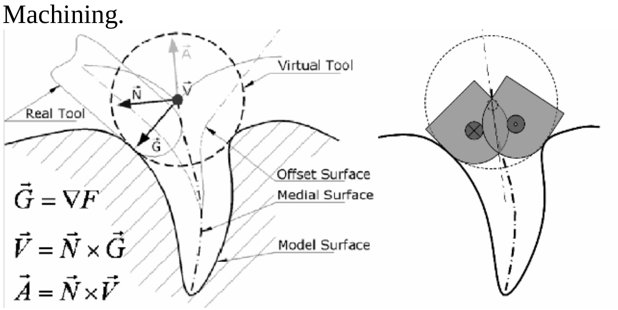

Fig. 10. The virtual tool, virtual tool trihedron and virtual tool decomposition into left and right medial paths.

5.1. The virtual tool

Definition. The *virtual tool* is defined as a hypothetical sphere that touches the machined surface in the point that is being machined, and has its centre located on the associated medial surface (Fig. 10)**.**

Let F be the scalar field associated to the implicit distance function. We can write the gradient *G* as (4). We define the *unit speed vector V* as being the vector product between the medial surface normal *N* and gradient *G*. Similarly *unit vector A* is the axis of the virtual tool:

$$\vec{V} = \vec{N} \times \vec{G} \text{ and } \vec{A} = \vec{N} \times \vec{V} . \qquad (5)$$

This defines *the virtual tool trihedron* that allows establishing the position of the real tool relative to the virtual tool (Fig 10). A machining procedure can thus be sufficiently described by the medial paths of the tool center and the tool axis inclination relative to the virtual tool trihedron.

The radius of the virtual tool is greater or equal to the radius of the real tool. Each tangency of the virtual tool transforms into a cutting edge of a real tool. We must also observe that *the virtual tool model can be applied also for all other types of machining using cutting edges because the virtual tool is associated to the model surface and not to the tool.*

5.2. The Medial Machining

Motivation. Using a large radius BEM *cutting directly at the model surface* would solve the problems of: tool bending, obtaining the HSM using standard machines, roughness of the surface and also considerably reduce the length of the tool paths. Following the larger tool, smaller tools can be used but only for the areas where the larger diameter BEM, due to interference, was unable to cut (Fig. 11).

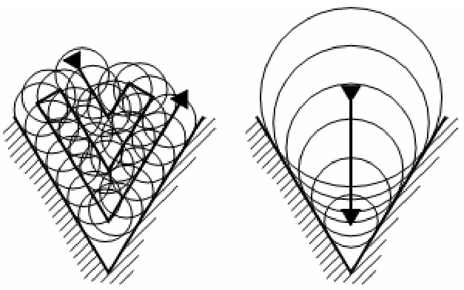

Fig. 11. Medial machining: a – contouring machining principle; b – medial machining principle.

Definitions

The Medial Machining is a complex, rational and high productivity way of machining surfaces during which the center of the real tool moves on medial tool paths, being always tangent to the final surface of the model.

The Medial Tool paths can be center-medial or side-medial tool paths.

The center-medial tool paths are composed of curves belonging to the medial surface associated to the machined model located at a distance equal to the associated radius of the *virtual tool*. The *real tool* has the size of the *virtual tool* and is therefore tangent in minimum two points to the machined model surface.

Center-medial machining is defined as the position of the real tool axis relatively to the virtual tool trihedron (of course limited by the access conditions [10]):

Normal medial machining – the axis of the tool is the virtual tool axis A.

Tangential medial machining – the axis of the real tool is perpendicular to the gradient vector being therefore tangent to the machined surface. This tool position generates high quality surfaces (Fig. 12).

The side-medial tool paths are composed of curves parallel to the medial surface and located on a surface equidistant to the model surface. The equidistance (offset) of this surface is equal to the radius of the **real tool** that is cutting the model surface. These can be side-medial left or side-medial right tool paths (Fig 13).

Side-medial machining real tool has a radius smaller than the associated virtual tool. Its displacement, radius and orientation in the plane *A*, *G* (relative to the virtual tool trihedron) define the particularities of the process. This is the procedure used to reduce the number of tools needed for medial machining.

Contact surface vs. real tool. If the real tool is in contact with the surface of the model this will be *contact full* else the machining is *contact less* (Fig. 14).

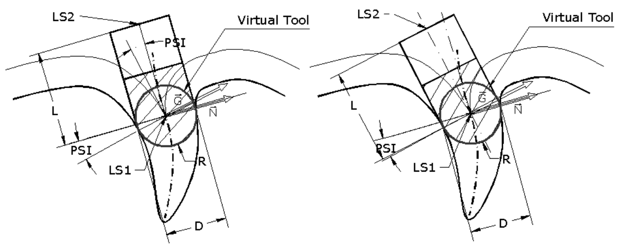

Fig. 12. Normal and tangential center-medial toolpaths.

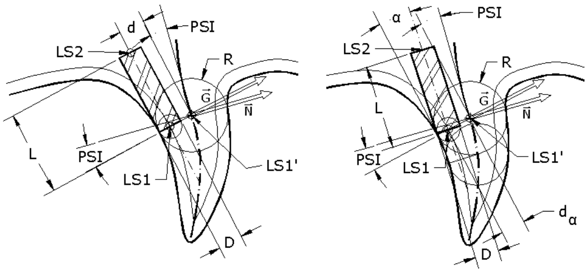

Fig. 13. Tangential and inclined side-medial machining.

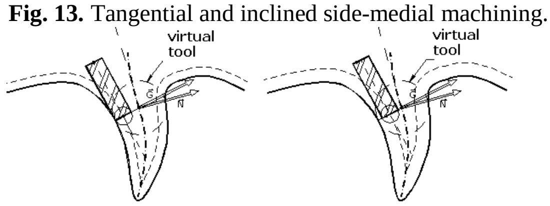

Fig. 14. Contactfull and contactless side-medial machining.

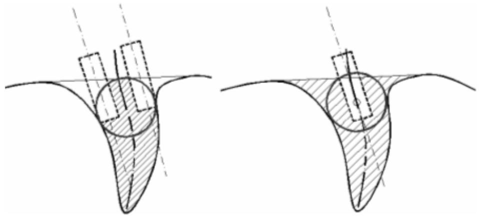

Fig. 15. Clipping and punch side-medial machining.

The classical contouring is therefore a contact less case of medial machining as result of the three-dimensional transposition of the bidirectional contouring without fully exploiting the additional features offered by the three-dimensionality.

Clipping occurs during contact full medial machining if the diameter of the real tool is smaller than the radius of the virtual tool (Fig. 15).

The side medial tool paths can be obtained by cutting the surface equidistant to the model (obtained for the tool radius) with several surfaces equidistant to the medial surface. The result consists in curves that are able to avoid interference and „parallel” to each other to enable scallop overlapping (Fig. 16).

Observations

Transforming the center-medial paths into sets of side-medial paths decreases the size and number of tools used but increases the tool path length (Fig. 17). Thus an optimization time–quality-cost [9] is needed.

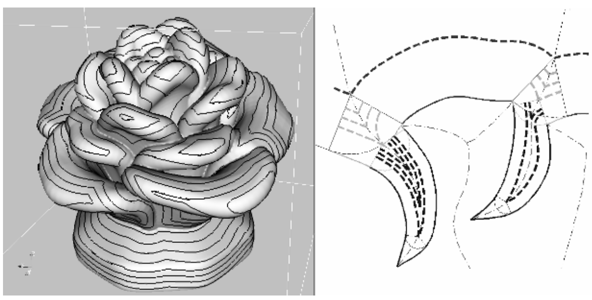

Fig. 16. Side medial toolpaths for a complex model and the side medial toolpaths distribution scheme.

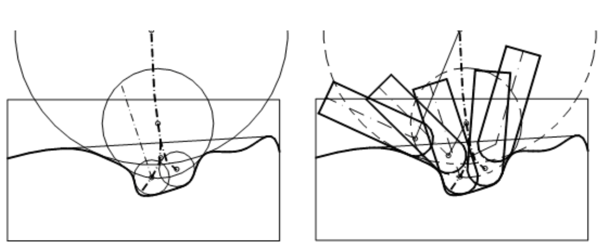

Fig. 17. Transforming large radius tool center- medial tool paths into affordable radius tool side-medial toolpaths.

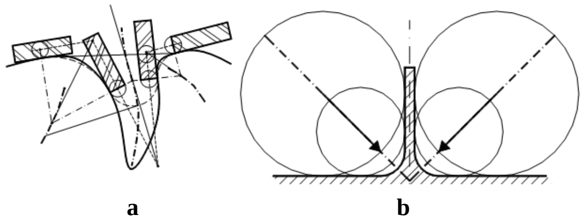

Fig. 18. Machining: a – exterior side-medial machining; b – thin walls machining.

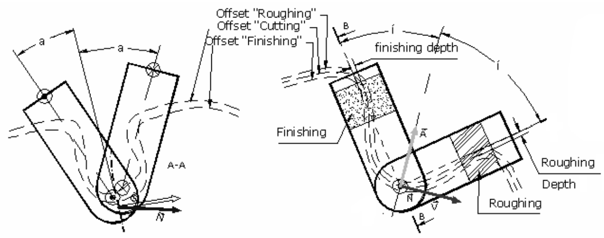

Fig. 19. The double tangential side-medial machining.

The surfaces in $\Re^3$ have two sides (obviously) and everything (with some restrictions) that was said until now on the medial machining of concave surfaces can be extended to convex surfaces (Fig 18,a).

Machining is being done starting with big radius tools and progressively decreasing their diameter therefore medial machining has no difficulty in the machining of the thin walls (Fig 18,b).

The inclining angle towards (in the plane *A*,*V*, the so called sturz/plunge angle is an important variable of the medial machining. Its limit values (when the tool has minimum two points of tangency to the machined surface) allow huge increases in the speed of chip removal and the obtention of a good quality surface. If the tool is conical then we could combine more operations on the same tool (eg. an HSM medial machining with a grinding). This type of medial machining is expected to be *the most efficient machining ever attained* both as productivity and as quality of the obtained surface.

6. EXPERIMENTAL CONSIDERATIONS

Our goal was to find out how many of the difficulties arising in the machining of the complex surfaces can be solved by using medial machining. The problems that have been addressed by this experiment are the following: offset, uncut, access, modeling, surface quality problem, thin walls machining and productivity. The model chosen is a rose model (Fig. 20) downloaded from [16] and carries all these difficulties

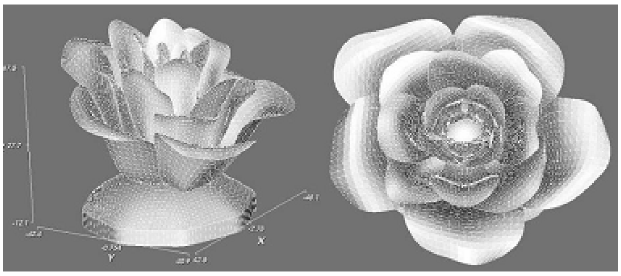

Fig. 20. The rose model.

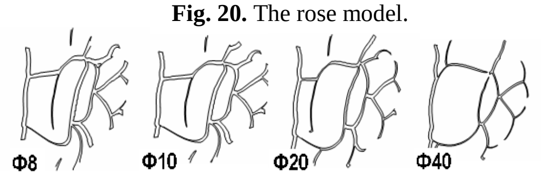

Fig. 2. Some center medial toolpaths used for the lateral orientation (*XY* plane projection).

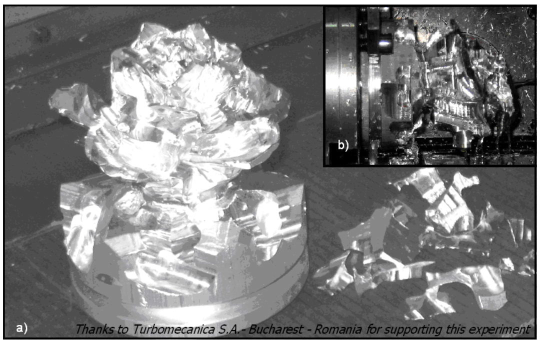

Fig. 22. Machining results: a – result of the first medial machining experiment; b – result of the lateral machining phase on the right side chunks of material cut out by clipping.

A discrete orientation center-medial machining for 4 lateral and 10 frontal orientations was used

The machining was done on a 5 axis machining center type Steinel BZ30-1982, having an ECN type ECN AEG System III, connected to 486PC needed to transfer the CN programs (Figs. 21 and 22). Total length of the programs executed in DNC was of about 50000 lines.

7. CONCLUSIONS

Advantages. High quality of the machined surfaces, paths are shorter with magnitude orders, possibility of machining for thin walls, rational wear of the tools, complete machining automation, reduction of the chip volume and easy removal and even bad formed surfaces can be machined.

Drawbacks. If using only centre-medial machining the number of tools is not small, is impossible to compute the paths „by hand” and even if advanced computing means are used high speed and huge amounts of memory are needed.

The future. The medial machining is now only in its beginnings but even from now we can see radical changes that are going to take place in the future landscape of the complex surfaces machining.

The shape features have no meaning in the context of medial machining because there remain only two kinds of shapes: concave and convex. Operations such as pocketing, contouring etc. have no meaning for medial machining.

Phases such as roughing, finishing, clean-up have no longer a meaning in medial machining. Planning the machining process using shape features, heuristic tool path topologies (such as zigzag, parallel, strip parallel etc..), repeated entry and exit of the tool, tool path linking optimization are no longer needed because the medial tool paths are simply optimal.

This new view on the generation of the surfaces enables new approaches to the old problems and at another level, adds problems such as HSM for big radius tools, deep cutting, access problem, new type of tools and finally new types of completely automated machines able to really help in the machining of complex surfaces.

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