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REGARDING THE DYNAMICS OF ROBOTIC STRUCTURES WORKING AT HIGH SPEEDS

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Abstract: *The paper presents the dynamic analysis for a translation coupling within a robot cinematic element. Based on the schematic of the elastic structure comprised of the coupling M and tool S and on the external links of the coupling it is emphasized the equation of the frictionless vibration movement. It is presented the dynamic response of the system under the conditions of subcritical damping. This depends on the shape of the pushing force variation at the tool bearer level. Thus are provides the premises for structural optimization of the robot size.*

Key words: *translation coupling, dynamic analysis, high speed cutting.*

1. INTRODUCTION

The role of dynamic processes within complex cutting – welding operations performed by robots has increased significantly lately due to increased demands regarding the dimensional and shape precision of work surfaces and also due to appearance of new materials with special properties. This is further magnified under the conditions of introduction of large scale automation of such technological processes by using robots.

Solving certain problems related to the dynamic phenomena is required both for design and for construction and use of industrial robots. First of all it is necessary to ensure conditions for relative movements between the robot arm with the tool mounted on its end and the work surface [1]. The stability of such relative movements consists in: the absence of vibrations in the technological system comprised of work surface – cutting/welding tool - robot arm, maintaining constant contact between tool and workpiece and precise movement without skipping of the mobile components of the robot.

2. ESTABLISHING THE DYNAMIC ANALYSIS MODEL

It is considered the mobile point M as current point that generates the trajectory (C). Corresponding to fig. 1, M belongs to the mechanical structure of the robot and it is characterized by parameters (r, θ) with respect to a fixed reference system (x0y).

The point M is considered as a translation coupling belonging to the robot which performs a relative translation movement alongside guide G (Fig. 2).

It is also considered the elastic system comprised of the translation coupling (M) and the work tool (S) with the structure form Fig. 3a and external links corresponding to the schematic from Fig. 3b.

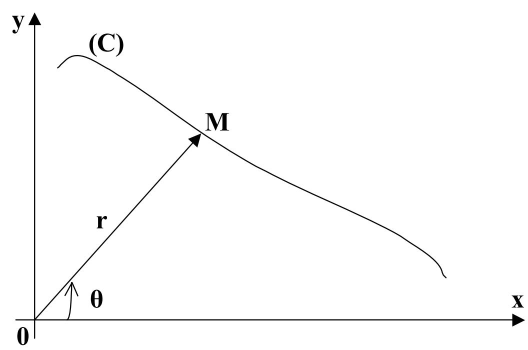

Fig. 1. Geometric parameters of point M.

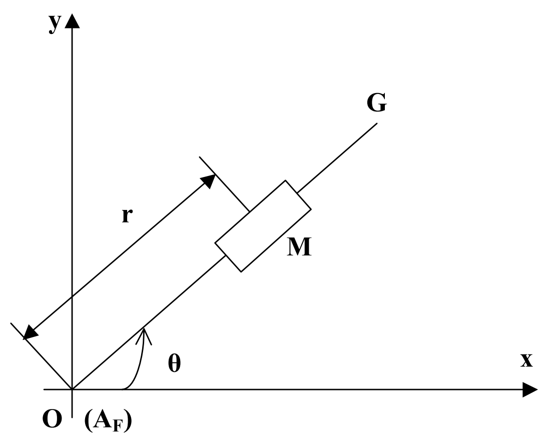

Fig. 2. M translation coupling within a cinematic element belonging to the robot.

The forces are:

F_1 – pushing force component in longitudinal direction of the tool (S)

F_2 – pushing force component in transversal direction of the tool (S)

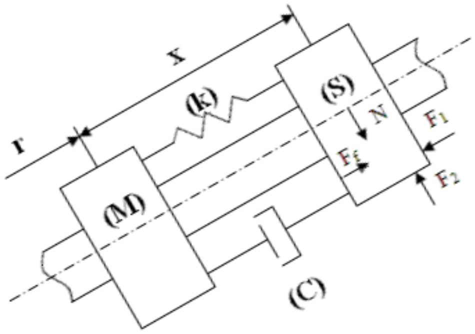

Fig. 3a. Schematics of the elastic system comprised of coupling M and tool S.

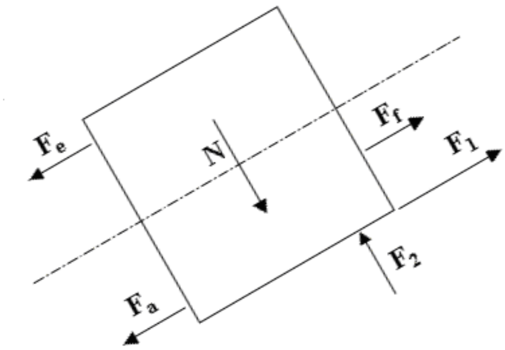

Fig. 3b. External links of schematic.

F_f – friction force inside guiding (G)
N – guiding reaction
F_e – elastic force
F_a – damping force

Corresponding to the fundamental law of dynamics we have:

$$\vec{R} = m\vec{a}\,. \tag{1}$$

The current position of mobile point M is given by:

$$\vec{r} = (r+x)\cos\theta\vec{i} + (r+x)\sin\theta\vec{j} \tag{2}$$

Also:

$$\begin{cases} \rho = r + x \\ \theta = \theta \\ z = 0 \end{cases} \tag{3}$$

represents the position of M in polar coordinates.

The speed and acceleration of the study point can be determined with (4), (5):

$$\begin{cases} V_\rho = \dot{\rho} = \dot{r} + \dot{x} \\ V_\theta = r\dot{\theta} \\ V_z = 0 \end{cases}, \tag{4}$$

$$\begin{cases} a_\rho = \ddot{\rho} - \rho\dot{\theta}^2 = \ddot{r} + \ddot{x} - (r+x)\dot{\theta}^2 \\ a_\theta = 2\dot{\rho}\dot{\theta} + \rho\ddot{\theta} = 2\left(\dot{r} + \dot{x}\right) + (r+x)\ddot{\theta}\,. \\ a_z = 0 \end{cases} \tag{5}$$

The movement equation in polar coordinates is:

$$\begin{cases} ma_\rho = F_\rho \\ ma_\theta = F_\theta \end{cases}. \tag{6}$$

Or more detailed:

$$\begin{cases} m\left[\ddot{r} + \ddot{x} - (r+x)\dot{\theta}^2\right] = F_1 + F_f + F_e + F_a \\ m\left[2\left(\dot{r} + \dot{x}\right) + (r+x)\ddot{\theta}\right] = F_2 + N \end{cases}. \tag{7}$$

We also have:

$$\begin{aligned} F_e &= -K \cdot x, \\ F_a &= -c \cdot \dot{x}, \\ F_f &= -\mu N \cdot \operatorname{sgn}\left(\dot{r} + \dot{x}\right), \end{aligned} \tag{8}$$

where:
m – tool mass,
c – damping coefficient.
K – elastic constant.
μ – friction coefficient between tool and guiding.
$\left(\dot{r} + \dot{x}\right)$ - relative speed of tool with respect to guide,

$$N = m\left[2\left(\dot{r} + \dot{x}\right) + (r+x)\ddot{\theta}\right] - F_2\,. \tag{9}$$

Under these conditions, the general equation of vibration movement in this case is:

$$\begin{aligned} m\left[\ddot{r} + \ddot{x} - (r+x)\dot{\theta}^2\right] &= F_1 - \\ -\mu\left\{m\left[2\left(\dot{r} + \dot{x}\right) + (r+x)\ddot{\theta}\right] - F_2\right\} \\ \operatorname{sgn}\left(\dot{r} + \dot{x}\right) - Kx - c\dot{x}\,. \end{aligned} \tag{10}$$

The frictionless vibration movement is a second order differential equation [2]:

$$m\ddot{x} + c\dot{x} + \left(K - m\dot{\theta}^2\right)x = F_1 + r\dot{\theta}^2 m - m\ddot{r}\,. \tag{11}$$

The solution of the differential equation under the assumption of sub-critical damping can be determined by arranging the equation (1) under the form of a first order linear differential system:

$$\begin{cases} \dot{x} = \dot{x} \\ \ddot{x} = \left(\dot{\theta}^2 - \frac{k}{m}\right)x - \frac{c}{m}\dot{x} + r\dot{\theta}^2 + \frac{1}{m}F_1 \end{cases}. \tag{12}$$

with:
$$\begin{cases} \frac{k}{m} = \omega^2 \\ \frac{c}{m} = 2\alpha \end{cases}. \tag{13}$$

where:

ω – natural frequency;

α – damping factor.

The system becomes:

$$\begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ \dot{\theta}^2 - \omega^2 & -2\alpha \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} + \begin{Bmatrix} 0 \\ r\dot{\theta}^2 + \frac{1}{m}F_1 - \ddot{r} \end{Bmatrix}. \tag{14}$$

We denote:

$$[A] = \begin{bmatrix} 0 & 1 \\ \dot{\theta}^2 - \omega^2 & -2\alpha \end{bmatrix}; \{B\} = \begin{Bmatrix} 0 \\ r\dot{\theta}^2 + \frac{1}{m}F_1 - \ddot{r} \end{Bmatrix}. \tag{15}$$

The system is:

$$\left\{\dot{q}\right\} = [A]\cdot\{q\} + \{B\} \tag{16}$$

The general solution of the system is:

$$\{q\} = e^{[A]t}\{q_0\} + \int_0^t e^{[A](t-\sigma)}\{B(\sigma)\}d\sigma. \tag{17}$$

In order to determine the function, the following must be performed [3]:

- compute $[A]t$
-

$$[A]t = \begin{bmatrix} 0 & t \\ \left(\dot{\theta}^2 - \omega^2\right)t & -2\alpha t \end{bmatrix}; \tag{18}$$

- compute the chareacteristic polynomial:

$$|sI - At| = \begin{bmatrix} s & -t \\ -\left(\dot{\theta}^2 - \omega^2\right)t & s + 2\alpha t \end{bmatrix} = s^2 + 2\alpha ts - -\left(\dot{\theta}^2 - \omega^2\right)t^2. \tag{19}$$

In case of subcritical damping the solution is obtained by solving the characteristic equation:

$$s_{1,2} = \frac{-2\alpha t \pm 2it\sqrt{\omega^2 - \alpha^2 - \dot{\theta}^2}}{2}, \tag{20}$$

in which:

$$p_1 = -\alpha - \sqrt{\alpha^2 + \dot{\theta}^2 - \omega^2}, \quad p_2 = -\alpha + \sqrt{\alpha^2 + \dot{\theta}^2 - \omega^2}. \tag{21}$$

We have:

$$s_1 = -\alpha t - it\sqrt{\omega^2 - \alpha^2 - \dot{\theta}^2} = p_1 t, \quad s_2 = -\alpha t + it\sqrt{\omega^2 - \alpha^2 - \dot{\theta}^2} = p_2 t. \tag{22}$$

The characteristic polynomial is:

$$R(x) = ax + b. \tag{23}$$

For determining coefficients a and b, the following system must be solved:

$$\begin{cases} R(s_1) = e^{s_1} \\ R(s_2) = e^{s_2} \end{cases}. \tag{24}$$

We get:

$$a = \frac{e^{p_1 t} - e^{p_2 t}}{(p_1 - p_2)}, \quad b = \frac{p_1 e^{p_2 t} - p_2 e^{p_1 t}}{p_1 - p_2}. \tag{25}$$

The matrix $e^{[A]t}$ becomes:

$$e^{[A]t} = R([A]t) = \frac{e^{p_1 t} - e^{p_2 t}}{p_1 - p_2}\begin{bmatrix} 0 & 1 \\ \dot{\theta}^2 - \omega^2 & -2\alpha \end{bmatrix} + \frac{p_1 e^{p_2 t} - p_2 e^{p_1 t}}{p_1 - p_2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{p_1 e^{p_2 t} - p_2 e^{p_1 t}}{p_1 - p_2} & \frac{e^{p_1 t} - e^{p_2 t}}{p_1 - p_2} \\ \frac{e^{p_1 t} - p_2 e^{p_2 t}}{p_1 - p_2}\left(\dot{\theta}^2 - \omega^2\right) & \frac{p_1 e^{p_2 t} - p_2 e^{p_1 t}}{p_1 - p_2} - 2\alpha\frac{e^{p_1 t} - e^{p_2 t}}{p_1 - p_2} \end{bmatrix}. \tag{26}$$

The system solution in this case is:

$$x(t) = \frac{p_1 e^{p_2 t} - p_2 e^{p_1 t}}{p_1 - p_2}x(0) + \frac{e^{p_1 t} - e^{p_2 t}}{p_1 - p_2}\dot{x}(0) + \int_0^t \frac{e^{p_1(t-\sigma)} - e^{p_2(t-\sigma)}}{p_1 - p_2}\cdot\left(r(\sigma)\dot{\theta}^2(\sigma) + \frac{1}{m}F_1(\sigma) - \ddot{r}(\sigma)\right)d\sigma. \tag{27}$$

In case of subcritical damping we have:

$$x(t) = \frac{e^{-\alpha t}}{p}(p\cos pt - \alpha\sin pt)x(0) + \frac{e^{-\alpha t}}{p}\sin pt\,\dot{x}(0) + \frac{1}{p}\int_0^t e^{-\alpha(t-\sigma)}\sin(p(t-\sigma))\left[\frac{1}{m}F(\sigma)\dot{\theta}^2(\sigma) - \ddot{r}(\sigma)\right]d\sigma. \tag{28}$$

For uniform rectilinear translation movement:

$$x(t)=(A)x(0)+(B)\dot{x}(0)+ \frac{1}{mp}\int_0^t e^{-\alpha(t-\sigma)}\sin(p(t-\sigma))F(\sigma)d\sigma. \quad (29)$$

with:

$$p=\sqrt{\omega^2-\dot{\theta}^2-\alpha^2}\ . \quad (30)$$

Equation (29) represents the dynamic response of the system in the given conditions.

If it is considered the variation of the force as:

$$F=K_1-K_2e^{-\frac{t}{T_p}}, \quad (31)$$

where:

K_1, K_2 – constants that depend on the thickness variation of the work material;

T_p – time constant

The dynamic response of the system in the case of alternative uniform rectilinear translation frictionless movement will be [4]:

$$x(t)=(A)x(0)+(B)\dot{x}(0)+\frac{K_1p}{\alpha^2+p^2}-\frac{K_1e^{-\alpha t}}{\alpha^2+p^2}(\alpha\sin pt+p\cos pt)-\frac{K_2}{\left(\alpha-\frac{1}{T_p}\right)^2+p^2}\cdot e^{-\frac{t}{T_p}}+ +\frac{K_2e^{-\alpha t}}{\left(\alpha-\frac{1}{T_p}\right)^2+p^2}\left[\left(\alpha-\frac{1}{T_p}\right)\sin pt+p\cos pt\right], \quad (32)$$

where:

$$A=\frac{e^{-\alpha t}}{p}(p\cos pt-\alpha\sin pt), \quad B=\frac{e^{-\alpha t}}{p}\sin pt. \quad (33)$$

In case of uniform rotation movement $\left(r=R=ct;\dot{\theta}=ct\right)$ we have:

$$x(t)=(A)x(0)+(B)\dot{x}(0)+\frac{\left(R\dot{\theta}^2+K_1\right)p}{\alpha^2+p^2}+ +\frac{-\left(R\dot{\theta}^2+K_1\right)e^{-\alpha t}}{\alpha^2+p^2}\cdot(\alpha\sin pt+p\cos pt)- -\frac{K_2e^{-\frac{t}{T_p}}}{\left(\alpha-\frac{1}{T_p}\right)^2+p^2}+\frac{K_2e^{-\alpha t}}{\left(\alpha-\frac{1}{T_p}\right)^2+p^2} \left[\left(\alpha-\frac{1}{T_p}\right)\sin pt+p\cos pt\right]. \quad (34)$$

3. CONCLUSIONS

The dynamic response of the system in the presented case was obtained for a translation and rotation movement in the absence of friction

The differential system solution depends on the shape of the pushing force at the tool-bearer level

Thus are created the premises for structural optimization of the robot size

Based on the dynamic response obtained, it can be performed stability analysis in case of subcritical damping

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