

## THE LIFETIME OF MECHANICAL STRUCTURES WITH ERRORS OF CONSTRUCTION UNDER FATIGUE CONDITIONS

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**Abstract:** In the present paper is presented a theoretical research about the lifetime determination of cylindrical pressured vessels with errors of construction. In the same time is take account of the cracks presence, which can influence noticeably the resistance of the construction material in the fatigue work conditions.

**Key words:** lifetime, fatigue, pressure vessel, materials fracture mechanic, crack.

### 1. INTRODUCTION

Whatever is the constructive form (cylindrical, spherical, ellipsoidal, conical etc.), the revolution shells with thin wall are manufactured through welding joint technological process of some components (shells, lids, flanges, pipes etc.). Preliminarily, these elements are obtained through different technological processes of plastic deformations at cold or warm, which are followed of processing operation onto machine-tool and thermal treatment for obtain an unstressed structure.

Because of these technological processes the elements of pressure vessel can result with important errors towards the right form, the most affected being the cylindrical shells and the rounded lids of vessels.

Thus, in practice, the frequently formal errors are the following types (Fig. 1): the swerve from circularity of cylindrical body and the swerve from the rectilinear form of generating line of cylindrical body (local deformations in the welding joint zones of cylindrical shells, difference in level of the brims of cylindrical shells, protuberances etc.). Must mention that the each of these formal errors types represent a stress concentrator with different values, which drives to the modification of normal state of stresses which has been foresee by calculus.

Taking into account the economical importance of the pressure vessels, which equip the industrial installations, were establish technically prescriptions as regards the manufacturing of these equipments. The technically prescriptions comprise quantitative precautions concerning the swerves from the correct geometrical form of constructions elements and of vessels on the whole their.

Usually, is considered that the effect of these form swerves about stress state from the vessel wall is covered through the use of safety coefficient in the calculus of resistance.

Unfortunately, this reasoning holds good just in the case of static sollicitation to inner pressure of vessels. But in practice the situation is much more complexes because the pressure vessels work under low cycles fatigue conditions (oligocyclic fatigue conditions), usually, under a pulsating cycle of inner pressure.

On the other hand, must taking into account the fact as in the wall of pressure vessel can exist differently types of defects (pores, cracks, zones with modified structures) which have been resulted because of manufacturing technological process.

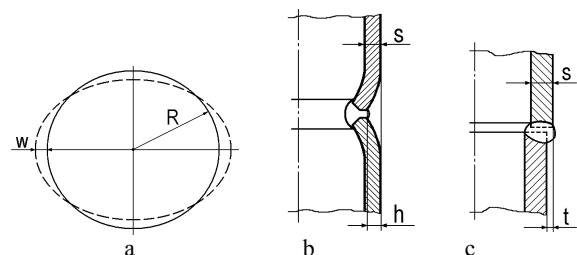
Thus, because of fatigue phenomenon of construction material, the initially cracks can expand until is reached the critical length, when is produced the material fracture and the shut-down of the equipment. As well must add the aggressiveness of processed fluids, the effects of the temperature and temperature variation, the influence of tensions and residual tensions etc. In this conditions, appear the necessity to corroborate these effects, in order to can estimate by means of an appropriate calculation procedure, the real state of stress from the wall of a pressure vessel, as well as his lifetime.

### 2. THE INFLUENCE OF FORMAL ERRORS ABOUT STATE OF STRESSES

#### 2.1. The swerves from circularity of cylindrical shell

The cylindrical shell is the constructive basic element of the pressure vessels, which so is known, is frequent affected of swerves from circularity. These swerves appear, usually, under action of shells own weight, during of the vessel body assembling operation.

In the practically activity of projection must take into account the technical prescriptions C4-2003 in which the swerves from circularity of cylindrical shell are limited to the value calculated with relation (Fig. 1a):



**Fig. 1.** Types of formal errors: a – the swerve from circularity,  $w$ , of cylindrical shell; b – local deformations,  $h$ , in the welding joint zones of two cylindrical shells; c – difference in level of the brims,  $t$ , of two cylindrical shells welded head to head.

$$w = 0.1 \cdot s + 3 \text{ mm}, \quad (1)$$

where  $s$  is the thickness of wall;  $w$  - the swerve from the right form of median surface, measured after the ray.

From theoretical viewpoint, in the case of cylindrical vessels with inner pressure, because of swerves from circularity appear supplementary flexural circumferential stresses. Thus, in the case of elliptical form of shell, the concentration coefficient of circumferential stresses, caused of supplementary stresses, can be determined with relation [1, 2]:

$$\alpha_k = 1 + \frac{6 \cdot \frac{w}{s}}{1 + 4 \cdot (1 - \mu^2) \cdot \frac{p}{E} \cdot \left(\frac{R}{s}\right)^3}, \quad (2)$$

in which  $R$  is the ray of median surface of cylindrical shell with correct geometry;  $p$  - the inner pressure;  $E$  - the module of longitudinally elasticity of shell construction material;  $\mu$  - Poisson's coefficient.

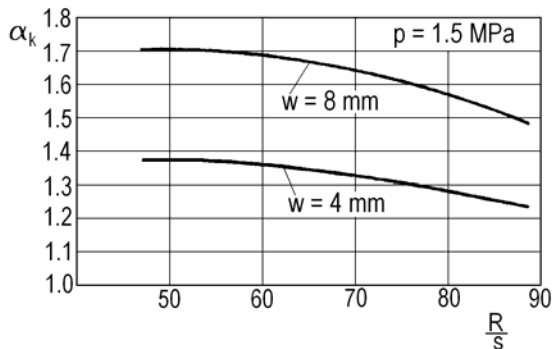
Applying the relation (2) for values of ratio  $R/s$  contained between 50 and 83.3 and for a inner pressure  $p = 1.5$  MPa, it obtained values of concentration coefficient of stresses contained between 1.35 and respective 1.27 in the case of a swerve from circularity  $w = 4$  mm and between 1.71 and respective 1.53 in the case of  $w = 8$  mm (Fig. 2). Therefore, is can said as in the case of structures more elastic, which are in the case of a bigger ratio  $R/s$ , the concentration coefficient of stresses diminishes. In the same time, through the increase of swerve from circularity from 4 mm to 8 mm, it noticed an increase of concentration coefficient of stresses with 19%.

Must be remarked as the results presented in Fig. 2 will be used afterwards for the determination of the maximum stresses in the zones of cylindrical shell with supplementary tensions caused of swerve from right form:

$$\sigma_{\max} = \sigma_2^{(p)} \cdot \alpha_k. \quad (3)$$

In the previous relation  $\sigma_2^{(p)}$  is the circumferential stress caused of the inner pressure:

$$\sigma_2^{(p)} = \frac{p \cdot R}{s}. \quad (4)$$



**Fig. 2.** The variation of stresses concentration coefficient  $\alpha_k$ , depending on the ratio  $R/s$ , for a cylindrical shell with different values of swerve from circularity  $w$ .

## 2.2. The swerves from rectilinear form of generating line of cylindrical body

These swerves from right form can appear to the assemblage of shells through circular welding seams and can meet in two variants: local deformation in the welding joint zones of cylindrical shells (Fig. 1.b) and difference in level of the brims of cylindrical shells in the welding joint zones (Fig. 1.c). In the case of local deformation, the maximum admissible values of swerves can be established with following relation:

$$h = 0.1 \cdot s + 3 \text{ mm}, \quad (5)$$

and in the case of brims difference in level

$$t = 0.1 \cdot s + 0.5 \text{ mm}. \quad (6)$$

The consequence of such swerves consist in the fact as in the welding joint zones appear supplementary flexural axial stresses, with lead to the circumferential and axial concentration coefficients of stresses, which can be calculated, respectively, with relations [2]:

$$\alpha_{k,1} = 0.5 + 3 \cdot \left(0.1 + \frac{3}{s}\right) \quad (7)$$

and

$$\alpha_{k,2} = 1 + 3 \cdot \mu \cdot \left(0.1 + \frac{3}{s}\right). \quad (8)$$

Applying the breaking energetically theory result as the expression of equivalent concentration coefficients of stresses has form:

$$\alpha_{k,ech} = \frac{1}{\sqrt{3}} \cdot \sqrt{\alpha_{k,1}^2 + 4 \cdot \alpha_{k,2}^2 - 2 \cdot \alpha_{k,1} \cdot \alpha_{k,2}}. \quad (9)$$

Applying the relation (9) for values of the cylindrical shell thickness contained between 5 and 20 mm it obtain values of equivalent concentration coefficients of stresses contained between 1.72 and respectively 1.22. Therefore is can said as the influence of these swerves from the right form types is with as much elder with how much the thickness of shell is else little.

## 3. THE LIFETIME OF PRESSURE VESSELS WITH FORMAL ERRORS

The calculus of the lifetime of pressure vessels which work in fatigue conditions is a verification calculus. This is applied in the zones with maximum values of stresses, for instance, in the zones with formal errors, which have been examined in the previous paragraph. Furthermore, the lifetime of vessel is influenced of fact as in the zones with formal errors (where the stresses are maxims) can exist differently types of defects (pores, cracks, inclusions of slag particles etc.).

Thus, because of fatigue phenomenon of pressure vessel, the rupture of material will be started in a zone with swerves from the right form in which already exist structural defects of construction material. The initially cracks can expand until is reached the critical length, when is produced the material fracture and the shut-down of the equipment. In this situation, the calculus durability

of the pressure vessels is can done by dint of method which is based on the utilization of the materials fracture mechanics concepts. This presupposes the knowing of the characteristics of the construction material, the determination of the maximum stresses, as well as of the structural defects of material. This method contains the stages which shall be described further on.

### 3.1. The state stress determination in the zone which is verified to the fatigue

In the case of present article must be determined the values of maximum stresses which appear in the zones of cylindrical shell with swerves from the right form, under the action of maximum inner pressure of solicitation cycle.

In accordance with the relation (3), the determination of maximum tension is done with well-known relations from shell's theory. As well are necessary the values of concentration coefficients of stresses, which depends on the types of swerves from the right form.

### 3.2. The critical length of crack determination

Usually, the cracks appear in the thermal influenced zone of welding seams, on the inner surface of cylindrical body. In the case of present article are studied the following types of cracks: the internal circular crack, the internal elliptical crack and the surface semi-elliptical crack (Fig. 3). Knowing the type and the sizes of crack it can passed to the determination of critical length of fissure, using the expression [3], [4]:

$$a_{cr} = \frac{1}{\pi \cdot f} \left( \frac{K_C}{\sigma_{cr}} \right)^2, \quad (10)$$

where  $K_C$  is the tenacity to fracture of material;  $f$  – shape factor of crack;  $\sigma_{cr}$  – critical stress of construction material ( $\sigma_{cr} \leq \sigma_c$ , that is the elastic domain of solicitation).

In this work has been studied the influence of maximum stress about crack's critical length, in the case of a carbon steel (R 510, STAS 2883/2-91), taking into account the three types of cracks already mentioned. Results of research, presented in Fig. 4, pointed out the fact as the most dangerous type of structure defect is the surface semi-elliptical crack, which is characterized through the shape factor:

$$f = \frac{1,2}{\Phi^2}, \quad (11)$$

in which  $\Phi$  is the elliptically integral of the second order

$$\Phi = \int_0^{\frac{\pi}{2}} \sqrt{1 - \frac{a^2 - b^2}{a^2} \sin^2 \varphi} \cdot d\varphi. \quad (12)$$

For sizes of the crack which fulfill the conditions  $a/b = 4$  and  $\varphi = \pi/2$ ,  $\Phi = 1.15$ . Because the surface semi-elliptical crack is the most dangerous type of crack, that is the critical length of this crack has the most little values to same critical stress, means as the results obtained in this case are covering for the others type of cracks (the internal circular crack and the internal elliptical crack).

Seeing that in practice it can meet differently concrete situations of work, has been calculated the critical length of semi-elliptical crack for different values of ratio  $a/b$ , just as is can see in Fig. 4.

### 3.3. The admissible length of crack determination

Knowing the critical length of crack, whereat can to appear the material rupture, it can be determined the admissible length of crack, through the applying of a safety coefficient  $C_S = 3 \div 4$ , for the prevention of fragile rupture:

$$a_{ad} = \frac{a_{cr}}{C_S}. \quad (13)$$

### 3.4 The number of cycles until rupture determination

The number of cycles until rupture can be calculated with the relation (14) obtained through the integration of Paris equation in which:  $a_i$  and  $a_{cr}$  are the initial length and, respectively, the final length of fissures;  $C$ ,  $n$  – constants

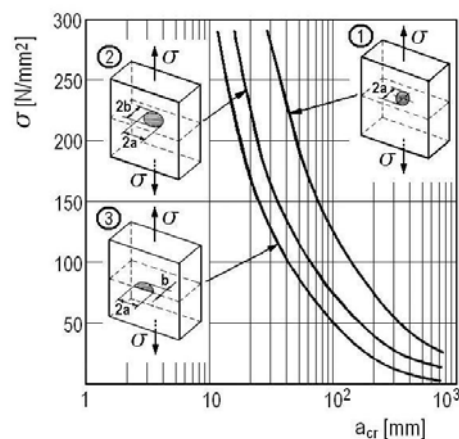


Fig. 3. The dependence between the critical length of crack and the stress whereat is set off the fracture, for carbon steel: 1– internal circular crack; 2– internal elliptical crack; 3– surface semi-elliptical crack.

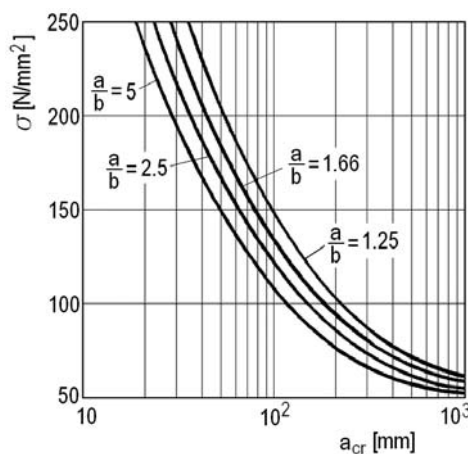


Fig. 4. The dependence between the critical length of the crack and the stress whereat is set off the material fracture, in the case of surface semi-elliptical crack with different values of ratio  $a/b$ .

material;  $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$  or  $\Delta\sigma = \sigma_{\max} - 0$  for a pulsating cycle.

$$N_r = \frac{a_{cr}^{1-\frac{n}{2}} - a_i^{1-\frac{n}{2}}}{C \cdot \left(1 - \frac{n}{2}\right) \cdot (\Delta\sigma)^n \cdot \pi^{n/2}}. \quad (14)$$

### 3.5. The admissible number of work cycles determination

The admissible number of work cycles can be calculated through the applying of a safety coefficient to durability  $C_N = 10$  [5]:

$$N_a = \frac{N_r}{C_N}. \quad (15)$$

### 3.6. The lifetime determination

Knowing the admissible number of work cycles, as well as the frequency of cycles,  $z$ , afterwards, can be calculated the lifetime  $D$ , expressed in years, of a mechanical structure:

$$D = N_a \cdot \frac{z}{8760}. \quad (16)$$

## 4. APPLICATION

The body of a pressure vessel is built from two cylindrical shells, which are jointed through a circular welding seam. The diameter of median surface of cylindrical body with rightly geometry is 1 m, and the thickness  $s = 6$  mm. After the construction of vessel, in the thermal influenced zone of welding joint have been discovered surface semi-elliptical cracks (Fig. 3), with length between 1 and 3 mm, which are characterized through the ratio  $a/b = 4$ . Also, has been observed a swerve from the circularity of vessel body  $w = 4$  mm (Fig. 1.a). Knowing as in exploitation the vessel will be solicited to the fatigue after a pulsating cycle with the duration of 8 hours, because of the inner pressure variation between 0 and 3 MPa, it can determined the number of admissible work cycles of vessel.

The construction material of vessel is steel which has the following characteristics:  $\sigma_r^{20^\circ} = 510$  MPa;  $\sigma_c^{20^\circ} = 350$  MPa; the coefficient of linear dilatation  $\alpha = 1.2 \cdot 10^{-5}$  [m/m $^\circ$ C]; the module of longitudinal elasticity  $E = 2.12 \cdot 10^5$  MPa; the Poisson's coefficient  $\mu = 0.3$ ; the tenacity to fracture  $K_C = 55$  MPa $\cdot$ m $^{0.5}$ ;  $n = 3$ ;  $C = 3.5 \cdot 10^{-12}$ . For the application of method which is based on the utilization of the materials fracture mechanics concepts firstly is necessary to be determined the value of maximum stress installed in the zone of cylindrical shell with swerves from the circularity. In accordance with relation (3), the determination of maximum stress is done with the relation of circumferential stress known from the shell's theory (relation (4)). As well is necessary the value of concentration coefficient of stress, which is calculated with the ration (2). After the carried out the calculus has been obtained the value of maximum stress  $\sigma_{\max} = 282.5$  N/mm $^2$ .

Afterwards, with help of relation (10) or using the dates presented in Fig. 4, is determined the value of crack critical length  $a_{cr} \approx 13$  mm.

Knowing the critical length of crack it can pass then to the settlement of number of cycles until rupture, with help of relation (14).

Thus, in the conditions of present application, result as the number of cycles until rupture is  $N_r = 103961$  and the admissible number of work cycles  $N_a = 10396$ . In the case in which the duration of work cycle is of 8 hours, finally result as the safe lifetime of pressure vessel with a swerve from the right form, under fatigue condition is of 9.5 years (relation (16)).

## 5. CONCLUSIONS

In the present paper is tackled the problem of lifetime determination for the mechanical structures, which present differently types of construction formal errors. Thus, is showed the fact as under influence of some swerve from the right form of pressure vessels appear concentration of stresses with values what can touch 15-20% from the value of the membrane stress, or more. Even if these states of stresses are admissible in the case of a static solicitation to inner pressure, they can become dangerous under low cycles fatigue conditions (oligocyclic fatigue conditions).

On the other hand, has been taken into account the fact as in the wall of pressure vessel exist differently types of defects (pores, cracks, inclusions of slag particles etc.) resulted because of manufacturing technological process.

In the fatigue conditions, the cracks can expand until is reached the critical length, when is produced the material fracture and the shut-down of the equipment.

Thus, in this paper is presented a calculation method for the lifetime of pressure vessels which takes count of all these aspects.

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