Proceedings of the 17th International Conference on Manufacturing Systems – ICMaS
ISSN 1842-3183
University POLITEHNICA of Bucharest, Machine and Manufacturing Systems Department
Bucharest, Romania

FLOW RATE OF IMPELLERS WITH RADIAL DISCHARGE

Constantin TACĂ

Abstract: *A new mathematic model for the flow rate of the impellers with radial discharge was deduced due to the methods of dimensional analysis. Through experimental determinations the general calculation relation was particularized for the case of the Rushton turbine. Research results are presented by graphics, depending of impeller's parameters: speed, diameter and width of blades. Experimental data are used for establishing a general relation of impeller flow rate, taking into account the liquid density and viscosity.*

Key words: *mixing, agitation, impeller, flow rate, Rushton turbine.*

1. INTRODUCTION

The resultant motion of liquid in a cylindrical vessel with rotational mechanical impeller can be decomposed in an axial flow, a radial flow and a tangential flow, in accordance to the rectangular system with the vertical axis in direction of the vessel revolution axis (Fig. 1).

Usually, in the aim of axial and radial flow intensification, is decreased the tangential current through the use of baffles [1]. Thus, the fundamental studies considering the mixing in vessels with baffles directed to the two types of impellers: impellers with radial discharging flow pattern and impellers with axial discharging flow pattern (Fig. 2).

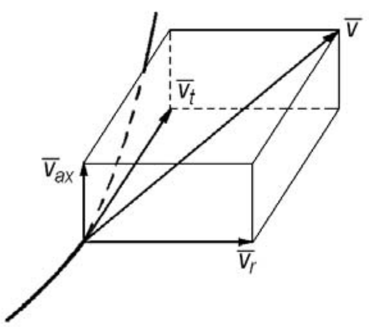

Fig. 1. The resultant speed $\overline{v}$ and his components: $\overline{v}_{ax}$ - axial speed, $\overline{v}_r$ - radial speed, $\overline{v}_t$ - tangential speed.

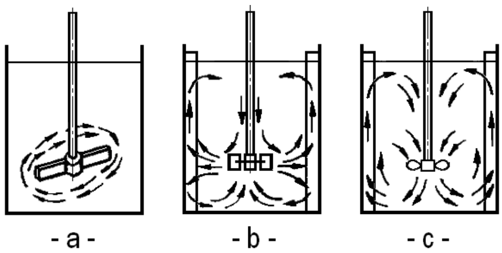

Fig. 2. Flow pattern of mechanical rotary impellers: a - tangential; b - radial; c- axial.

But apart from the circulation flow of liquid, must be tacking into account the turbulence which is created through the impeller action, this turbulence being characterized by highly cutting tensions. So, because the aims of mixing processes are very varied, the impeller must chose in such kind that to assure an optimum ratio between circulation and the turbulence [2, 3].

With this end in view, the impellers were classified depending on the size of ratio among the power number and the pumping number N_p / N_q:

$$N_p = \frac{N}{\rho \cdot n^3 \cdot d^5}, \tag{1}$$

$$N_q = \frac{q}{n \cdot d^3} \tag{2}$$

in which: q is the impeller's flow rate, n – speed of impeller, d – diameter of impeller, ρ – density of liquid and N – power input.

The impellers with a big value of ratio N_p / N_q are so called "cutting impellers", for instance the disc-turbine and the impeller with six right blades which present a radial discharging flow pattern.

The impellers with a small value of ratio N_p / N_q are so called "recirculation impellers", for instance the propeller with six blades and the six 45^0 – pitched blade turbine which present an axial discharging flow pattern.

2. THEORETICAL

Within the framework of installations from the industries of process are frequent used different types of impellers with radial discharging flow pattern. These impellers are used specially for the operations which require high degrees of turbulences, such as, the emulsions obtain, or the dispersions of gases in liquids obtain [4, 5].

Due to high performances of these impellers, they constituted the object of numerous theoretical and experimental researches. Thus, has been establish numerous relations for the calculus for the impeller power input, for the speed of discharged liquid and for the flow rate of discharged liquid, depending on the impeller's form and the impeller's geometrical parameters [6, 7].

Thus, Šterbácek and Tausk [8] showed as, generally, the radial flow of an impeller can be expressed through the product among the area surface generate of the blade's peaks of impellers on the rotation move and the radial speed of fluid:

$$q = \pi \cdot d \cdot h \cdot v\,; \tag{3}$$

in the previously relation d is the diameter of impeller, h – width of blades and v - the radial speed of fluid.

In the same time, analogously with the pumps centrifugal machines, they proposed for the calculus of impeller's flow rate with radial discharging flow pattern the relation:

$$q = \pi^2 \cdot d^2 \cdot n \cdot h \tag{4}$$

in which n is the speed of impeller.

After Nagata [9], the radial flow rate of an impeller with arms is expressed through following dependency:

$$q \sim D^{1.5} \cdot d^{0.5} \cdot n \cdot h \cdot z^{0.7}, \tag{5}$$

in which z is the impeller's number of blades.

With all diversity of the range of types known in the practice, yet, is can appreciated as most frequently used impeller from these category is the disc-turbine with six blades, which is meet in the specialty literature below the names of "the Rushton turbine" or "the opened turbine"(Fig. 3). Researching the behaviors of this type of impeller, Bowen [10] arrived at the conclusion as his radial flow rate has the expression:

$$q = 6.2 \cdot D^{-0.3} \cdot d^{2.3} \cdot n \cdot h\,. \tag{6}$$

According as he arises from one presented, usually, the radial flow rate of an impeller is presented as be dependent on speed, as well as of a series of geometrical parameters: the diameter of impeller, the diameter of vessel, the width of impeller's blades etc.

But do not is considered the effect of working liquid. However, he is obviously the influence of his density and viscosity about the hydrodynamic regime produced in reactor. For this reason, in the frame of this work the author he proposed, in first the stage, to establish a general calculus relation for the radial flow rate of an impeller, which holds the account of the working liquid properties.

Because of the complexity of hydrodynamic process which is in progress in vessels with mechanical mixing devices, the deduction of the mathematical model has been realized with help of the conventional method of dimensional analysis, completed then through experimental determinations.

Thus, faithful to the Buckingham method, which is based on the produces theorem, anything complete relation

$$f(A_1, A_2, A_3 \ldots A_p) = 0 \tag{7}$$

between the physical sizes $A_1, A_2, A_3 \ldots A_p$ which cause a phenomenon, can be writhed in the form:

$$\varphi(\pi_1, \pi_2, \pi_3 \ldots \pi_{p-m}) = 0 \tag{8}$$

or

$$\pi_1 = k(\pi_2)^a (\pi_3)^b \ldots (\pi_{p-m})^r, \tag{9}$$

where $\pi_1, \pi_2, \pi_3 \ldots \pi_{p-m}$ are dimensionless produces, free between they, realized with sizes A_1, A_2, A_3... A_p, and m is the degree of dimensional matrix.

Tacking into account the results published in the specialty literature is can asserted as the parameters which can influence the flow rate of the impellers with radial discharging flow pattern are: n –speed of impeller, d - diameter of impeller; h – impeller's width of blades, - μ - dynamic viscosity of working liquid; ρ – density of working liquid; g – gravitational acceleration.

Among these parameters don't included the number of blade of impeller, because the researches were limited about impellers with six blades. So, taking count of parameters which can influence the studied phenomenon is can formed the following dimension matrix:

$$\begin{array}{c|ccccccc} & q & d & h & n & \mu & \rho & g \\ \hline L & 3 & 1 & 1 & 0 & -1 & -3 & 1 \\ M & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ T & -1 & 0 & 0 & -1 & -1 & 0 & -2 \end{array}. \tag{10}$$

The degree of matrix is $m = 3$, because the determinant of the three order formed of the elements of the columns 5, 6 and 7 is non-zero. As initial sizes are selected μ, ρ and g, and on their basis drew up the four one dimensionless produces (p – m = 7 – 3 = 4):

$$\begin{aligned} \pi_1 &= \mu^{a1} \cdot \rho^{b_1} \cdot g^{c_1} \cdot q \\ \pi_2 &= \mu^{a2} \cdot \rho^{b2} \cdot g^{c2} \cdot d \\ \pi_3 &= \mu^{a3} \cdot \rho^{b_3} \cdot g^{c3} \cdot h \\ \pi_4 &= \mu^{a4} \cdot \rho^{b4} \cdot g^{c4} \cdot n \end{aligned} \tag{11}$$

Further on, for the determination of exponents of dimensionless produces is necessary to obtain firstly the dimensional equations:

$$\begin{aligned} L^0 M^0 T^0 &= L^{-a_1} M^{a_1} T^{-a_1} L^{-3b_1} M^{b_1} L^{c_1} T^{-2c_1} L^3 T^{-1} \\ L^0 M^0 T^0 &= L^{-a_2} M^{a_2} T^{-a_2} L^{-3b_2} M^{b_2} L^{c_2} T^{-2c_2} L \\ L^0 M^0 T^0 &= L^{-a_3} M^{a_3} T^{-a_3} L^{-3b_3} M^{b_3} L^{c_3} T^{-2c_3} L \\ L^0 M^0 T^0 &= L^{-a_4} M^{a_4} T^{-a_4} L^{-3b_4} M^{b_4} L^{c_4} T^{-2c_4} T^{-1} \end{aligned} \tag{12}$$

and the finally forms of dimensionless produces are:

$$\begin{aligned} \pi_1 &= \mu^{-\frac{5}{3}} \cdot \rho^{\frac{5}{3}} \cdot g^{\frac{1}{3}} \cdot q;\ \pi_2 = \mu^{-\frac{2}{3}} \cdot \rho^{\frac{2}{3}} \cdot g^{\frac{1}{3}} \cdot d \\ \pi_3 &= \mu^{-\frac{2}{3}} \cdot \rho^{\frac{2}{3}} \cdot g^{\frac{1}{3}} \cdot h;\ \pi_4 = \mu^{\frac{1}{3}} \cdot \rho^{-\frac{1}{3}} \cdot g^{-\frac{2}{3}} \cdot n \end{aligned} \tag{13}$$

Taking into account the expressions (13), the relation (9) becomes:

$$\frac{q\cdot\rho^{\frac{5}{3}}g^{\frac{1}{3}}}{\mu^{\frac{5}{3}}}=k\cdot\left(\frac{\rho^{\frac{2}{3}}g^{\frac{1}{3}}d}{\mu^{\frac{2}{3}}}\right)^{a}\left(\frac{\rho^{\frac{2}{3}}g^{\frac{1}{3}}h}{\mu^{\frac{2}{3}}}\right)^{b}\left(\frac{\mu^{\frac{1}{3}}n}{\rho^{\frac{1}{3}}g^{\frac{2}{3}}}\right)^{c}, \quad (14)$$

whence results, finally, the relation of calculus for the radial flow rate of impellers:

$$q=k\cdot d^{a}\cdot h^{b}\cdot n^{c}\cdot\mu^{e}\cdot\rho^{f}\cdot g^{i}, \quad (15)$$

where

$$e=\frac{-2a-2b+c+5}{3};\quad f=\frac{2a+2b-c-5}{3};\quad i=\frac{a+b-2c-1}{3}. \quad (16)$$

3. EXPERIMENTAL PROCEDURE

The relation (15) represents the general expression of flow rate for the impellers with radial discharging flow pattern, which is the same for all type of impellers.

The particular forms of this relation depend on values of coefficient k and of exponents a, b, c, d, e and g, which will be experimentally determinate for the researched type of impeller, respectively the disc - turbine (the Rushton turbine) (Fig. 3). For the touch this aim has been used the experimental set-up presented in Fig. 4.

Seeing that in the case of a radial flow pattern the absorption of the liquid is done in the axial direction, so to the upper part and to the lower part of vessel, the impeller has been located between one two sections (2 and 10) of a draught tube. The liquid pumped of impeller in radial direction raise one in the vessel 10 and, finally, pour out above the edge of vessel 10 and is collected in the gutter 3.

By means of the pipes 7 and 9 is done the supply of impeller, as much to the upper part and to the downside. For the avoiding formation of central vortex were foresee the radial baffles 5, as much for the draught tubes 2 and 10, and for the vessel 8.

The flow rate of liquid pumped of impeller is determined measuring the volume of liquid evacuee through the pipe 4 in the clock unit.

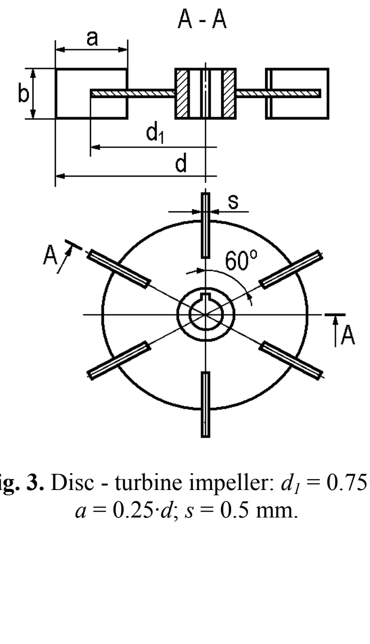

Fig. 3. Disc - turbine impeller: $d_1 = 0.75\cdot d$; $a = 0.25\cdot d$; $s = 0.5$ mm.

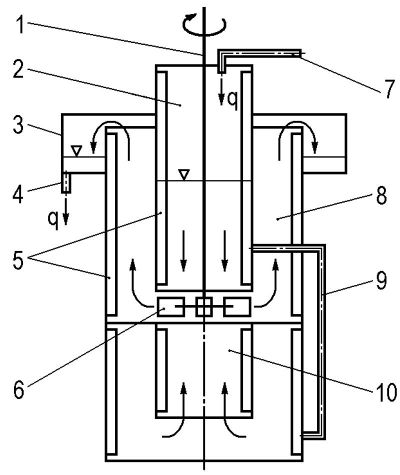

Fig. 4. The experimental set-up: 1- impeller's shaft; 2- upper draught tube; 3- gutter; 4- drain pipe; 5- baffles; 6- impeller; 7- supplying pipe; 8- vessel; 9- liaison pipe; 10- lower draught tube.

In the frame of experimental determinations has been used five constructive variants of impeller presented in the Fig. 3, with the width of blades h = 15, 20 and 25 mm and with diameters $d = D / 4$, $D / 3$ and $D / 2$, where the diameter of the vessel D = 0.250 m.

4. RESULTS AND DISCUSSIONS

The purpose of experimental researches has been the identification of influence for all parameters from relation (15) about the radial flow rate of impeller. Thus, were varied the speed of impeller, the impeller's diameter and the width of blades, but in the same time the others parameters were maintained constantly. The values determinate on this path were represented the chart in logarithmic coordinates just as is showed in the Figs. 5 – 7.

The graphical processing of results from these diagrams put in evidence the following proportionality relations:

$$q \sim n;\quad q \sim d^{3};\quad q \sim h. \quad (17)$$

In this way has been founded the exponents a, b and c from the relation (10): $a = 3$; $b = 1$; $c = 1$; the others exponents can be deduced with the help of equations (16): $e = -0.666$; $f = 0.666$; $i = 0.333$.

Through the assemblage of results is obtained following relation:

$$q\sim n\cdot d^{3}\cdot h\cdot g^{0.333}\cdot\left(\frac{\rho}{\mu}\right)^{0.666}. \quad (18)$$

Confronting the experimental results with the values obtained through the application of relation (18) he arrived at the conclusion as the average value of coefficient k is 7.5 10^{-4}. Thus, the final form of this equation is:

$$q=7.5\cdot10^{-4}\cdot n\cdot d^{3}\cdot h\cdot g^{0.333}\cdot\left(\frac{\rho}{\mu}\right)^{0.666}. \quad (19)$$

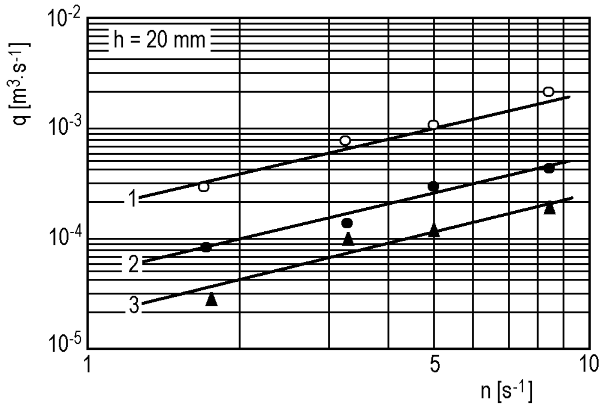

Fig. 5. Influence of impeller's speed about radial flow rate of disc - turbine impeller:
1 - d = D/2; 2 - d = D/3; 3 – d = D/4.

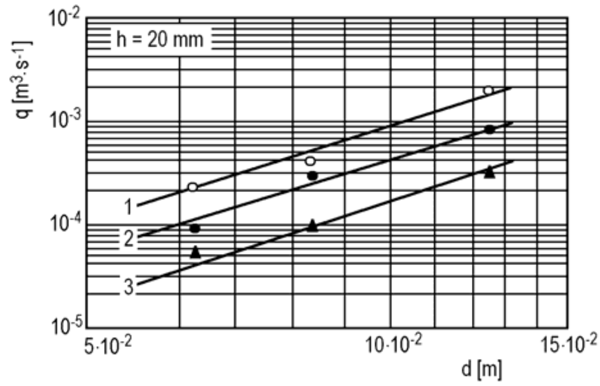

Fig. 6. Influence of impeller's diameter about radial flow rate of disc - turbine impeller: 1 - n = 500 rot/min;
2 - n = 300 rot/min; 3 – n = 100 rot/min.

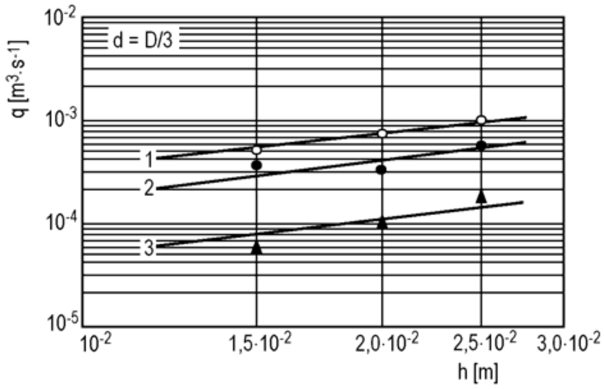

Fig. 7. Influence of blade's width about radial flow rate of disc - turbine impeller: 1 - n = 500 rot/min;
2 - n = 300 rot/min; 3 – n = 100 rot/min.

5. CONCLUSIONS

On the strength of the methods of dimensional analysis he deduced a new mathematical general model for the flow rate of impellers with radial discharge [relation (15)].

With help of experimental determinations this general relations were personalized for the case turbine - disk impeller [relation (19)].

Must did the specification as the relation proposed is valuable just if are respected the conditions of geometrical similitude, as well as the scale-up rules, in the case her applications to industrial equipments.

REFERENCES

[1] Jinescu, V.V. (1989). *Utilaj tehnologic pentru industrii de proces* (Tehnological equipment for the processing industries), Vol. 4, Edit. Tehnică, ISBN 973-31-0117-6, Bucureşti.

[2] Tacă, C., Păunescu, M. (2000). *Teoria şi practica sistemelor de amestecare* (The theory and the practice of mixing sistems), Edit. Matrix Rom, ISBN 973-685-113-3, Bucureşti.

[3] Tacă, C., Păunescu, M. (2000). *Suspension of solid particles in spherical stirred vessels*, Chemical Engineering Science, Vol. 55, No. 15, pp. 2989-2993, ISSN 0009-2509.

[4] Yoon, H.S., Hill, D.F. (2005). *Reynolds number scaling of flow in a Rushton turbine stirred tank. Part I—Mean flow, circular jet and tip vortex scaling*, Chemical Engineering Science, Vol. 60, No. 12, pp. 3169-3183, ISSN 0009-2509.

[5] Guillard, F., Tragardh, C. (2003). *Mixing in industrial Rushton turbine-agitated reactors under aerated conditions*, Chemical Engineering and Processing, Vol. 42, No. 5, pp. 373-386, ISSN 0255-2701.

[6] Tacă, C. (2000). *Influenţa formei amestecătorului asupra procesului de realizare a suspensiilor* (Influence of impeller form about suspension process), Revista de Chimie, Vol. 51, No. 10, pp. 787-792, ISSN 0034-7752.

[7] Medek, J., Fořt, I. (1979). *Pumping effect of impellers with flat inclined blades*, Coll. Czech. Chem. Commun. Vol. 44, No. 6, pp. 3077-3089, ISSN 0010-0765.

[8] Štěrbáček, Z., Tausk, P. (1969). *Amestecarea* (Mixing), Edit. Tehnică, Bucureşti.

[9] Nagata, S. (1975). *Mixing. Principles and Applications*, John Wiley and sons, ISBN 0-470-62863-4, New York.

[10] Bowen, R.L. (1986). *Effect of impeller design on the flow pattern and mixing in stirtred tanks.* Chem. Eng., Vol. 24, No. 6, pp. 55-63, ISSN 1385-8947.

Author:

PhD. Eng., Constantin TACĂ, Professor, University "Politehnica" of Bucharest, Department of Mechanical and Mechatronical Engineering,
E-mail: `dantaca2002@yahoo.com`