

University POLITEHNICA of Bucharest, Machine and Manufacturing Systems Department Bucharest, Romania

MATHEMATICAL MODELLING AND COMPUTER SIMULATION OF THE TRIBOLOGICAL BEHAVIOUR OF THE HYDRAULIC DRIVING SYSTEMS SEALS

Corneliu CRISTESCU, Petrin DRUMEA, Constanța CRISTESCU, Petrică KREVEY

Abstract: The paper presents some technical aspects regarding the translation mobile seals, based on the continue liquid films, used in the hydraulic driving systems, There is presented the importance of the liquid film thickness, in order to assure the seals lubrication and a minimum leakage liquid flow for a good sealing process. There is presented the mathematical modeling of the seals based on height pressure film. Finally, there are presented some graphical results obtained by the numerical simulation of the fluid sealing process and some interesting conclusions.

Key words: tribology of seals, sealing process, lubrication process, height pressure, piston rod seals.

1. INTRODUCTION

In the hydraulic drive systems, the sealings may be *steady or mobile*, and the mobile ones, from the point of view of the relative motion between the sealing surfaces may be with rotation or translation motion, the last ones being the subject of the present article [1].

The mobile translation sealings are specific to the hydraulic cylinders, Fig. 1, where, for exemple, realize the sealing on the rod with dyameter d, being in recirpocating translation motion on the stroke, of the fluid with the viscosity η and under pressure p.

În the Fig. 1, *d* is the rod dyameter, S – the rod stroke, *v* and v_r the velocities în the both sens.

At the mobile sealings with contact, with component parts in relative motion, matter much the modality of obtaining compression needed for sealing, usually due to external or internal forces.

In the hydraulic power systems are used mainly sealings realized under the effect of internal forces, more precisely of internal pressure which is sealed by different kinds of gasket. The sealing process is influenced, beside the phenomena, from the sealing interstice, by the gasket type, material and shape and the characteristics of the sealed medium.

The sealing process by contact, related to an interstice which must be closed, may be well defined by the pressure gradient (dp/dt), size which determines the allure of the curve representing the pressure decline along the length *l* of the sealing interstice.



Fig. 1. The sealing of the hydraulic cylinders rod.

The presence of a pressure decline shows in fact that there is present a leakage too, even if this may be totally insignificant. A high pressure decline which is equivalent to a high pressure gradient and imposes a heavy compression applied to the fitting for impeding leakages.

The fluid loss caused by lack of tightness represents the quantity which at the reverse stroke does not get back into the space under pressure. This loss may be determined by calculus using elements from the hydrodynamic theory of lubrication.

2. THE PHYSICAL MODEL AND THE OPERATIONAL HYPOTHESIS

For analyzing the sealing process it is considered a mobile sealing with the rod diameter d [m] and width l [m], for the rod of a hydraulic cylinder, Fig. 2, which consists of one or more sealing rings.

In order to known the sealing process, must be determined the distribution of pressure and tangential shearing efforts for evaluating the portent force of the sealing, respectively of the coefficient of fluid friction between rod and the sealing ring, in the conditions of the leakages caused by the working pressure of the hydraulic cylinder.

By the action of the oil pressure, which appear inside the hydraulic cylinder, and its tendency to get out it is created an interstice between sleeve and rod where it is produced a sudden pressure decline from the working pressure to the atmospheric pressure from outside the sealed area. The shape of this interstice depends on the working pressure, the reliability and stiffness of the material from which it is made the sealing sleeve and may be inclined plane, parabolic or wedge form.

In what follows it will be considered *a wedge shaped interstice*, Fig. 2, where the height of the fluid film hvary linearly, from a minimum value, almost null to the exterior area of the sealing to a maximum one of tens of microns inside the sealed and pressurized area.

Because the thickness of the film is small, there is no flow on the direction of the axis y. In the same time



Fig. 2. The wedge shaped interstice.

taking into account that in the direction of the axis z there is no fluid leaking, the flow on this direction is null.

In the sealing, the leakage is made from inside to outside and for this reason it is considered that during the sealing process, the leakage appears only on the direction *x*, this meaning that the mobile sealing depends only on a plane flow.

In what follows, it will be considered that the fluid is incompressible, the inertial forces are almost null, the pressure is constant on the fluid film layer and its dynamic viscosity it is also constant.

In these hypothesis, for describing the process of fluid sealing at the mobile sealing with an wedge shaped interstice, will be used the well-known equations from the mechanics of fluids.

3. THE MATHEMATICAL MODELLING OF THE MOBILE TRANSLATION SEALINGS

From *the hydrodynamic theory* of lubrication, [2, 3, 4, 5], for the *physical model* from Fig. 2, were the leakage appears into only direction *x*, result the relations from below. On the base of the equations of motion and of the cinematic conditions on edge, it may be calculated *the speeds distribution* in the fluid film on the direction, *x* is follows:

$$v_x = -\frac{1}{2\eta} \cdot \frac{\partial p}{\partial x} y \cdot (h - y) + \frac{v}{h} \cdot (h - y)$$
(1)

On the base of the *speeds distribution* may be determined the fluid flows on x direction q_{1x} , through a surface with the height *h*, width equal with the unit and oriented perpendicularly on the flow direction (on the axis x) and the rod speed *v*, as follows.

The flows leaking on the directions x, for an unitary length, is calculated as follows:

$$q_{1x} = \int_{0}^{h} v_{x} dy.1$$
 (2)

After integration, is obtained *the fluid flows leaking* on the axis x, as follows:

$$q_{1x} = -\frac{h^3}{12\eta} \cdot \frac{\partial p}{\partial x} + \frac{v.h}{2}$$
(3)

In this way, it is obtained the equation of continuity which written for the fluid column of height h and having as basis a rectangle with the dimensions dx and, respectively, 1, will be the following:

$$\frac{\partial q_{1x}}{\partial x} = 0.$$
 (4)

Replacing the flows from the relations (3), in the continuity equation (4), it results a Reynolds equation for a only direction of flow x with h = h(x).

After integration, it is obtained

$$h^{3} \cdot \frac{\mathrm{d}p}{\mathrm{d}x} = 6 \cdot \eta \cdot v \cdot h + C .$$
 (5)

Because the pressure varies from the maximum value inside the pressurized area to a value close to the atmospheric pressure outside the sealing, then the constant of integration *C* it is determined from the condition that at entering the pressurized area, when $x = x_1$, pressure has the maximum value, $p = p_{max}$, and for : $x = x_1$ and $h = h_1$, when the pressure gradient is null. It results *the pressure gradient*:

or

$$\frac{\mathrm{d}p}{\mathrm{d}x} = 6 \cdot \eta \cdot v \cdot \frac{h - h_1}{h^3}.$$

 $h^3 \cdot \frac{\mathrm{d}p}{\mathrm{d}r} = +6 \cdot \eta \cdot v \cdot (h - h_1),$

If it is known the expression of the variation of height h of the pressurized fluid interstice, the equation may be integrated and will be obtain the *pressure distribution*:

$$p(x) = \int_{0}^{x} (6 \cdot \eta \cdot v \cdot \frac{h - h_{1}}{h^{3}}) dx.$$
 (7)

(6)

For example, if the interstice has the shape of a triangular wedge with the angle at top α , as it is shows in the Fig. 2, the variation of the fluid height *h*, is:

$$h = x \cdot \tan \alpha \cong x \cdot \alpha . \tag{8}$$

Finally, it may be written *the pressure distribution* on the gasket width, it is given by the relation from below:

$$p(x) = \frac{1}{\alpha^2} \int_0^x (6 \cdot \eta \cdot v \cdot \frac{x - x_1}{x^3}) dx .$$
 (9)

The portent force produced by the fluid pressurized under gasket, on the length g of the sealing, are obtained by integration pressures on the surface (g.dx), with the relation:

$$Fp = \int_{0}^{x} p(x) \cdot g.dx \tag{10}$$

The length *g* of the sealing can be calculated, at the level of the diameter of the cylinder rod *d*, with the relation: $g = \pi d$.



Fig. 6. Pressure variation.



0 0.002 0.004 0.006 0.008 0.01 0.012

10

Fig. 9. The variation of the tangential friction force.



Fig. 10. The variation of the coefficient of fluid friction.

On the other hand, it is known that, on the base of the *Newton Law*, may be determined *the viscous shear tensions* at the contact of the fluid with the surface of the cylinder rod (y = 0), with the relations:

$$\left(\tau_{yx}\right)_{y=0} = \left\lfloor \eta \frac{\partial v_x}{\partial y} \right\rfloor_{y=0} = -\frac{1}{2} \cdot \frac{\partial p}{\partial x} \cdot h + \frac{\eta \cdot v}{h} .$$
(11)

The *tangential effort* or the *shearing tension* at the level of the fluid contact with the friction surfaces on the axis *x*, are obtained from the known relation:

$$\tau_{yx} = -\frac{1}{2} \cdot \frac{\partial p}{\partial x} \cdot h + \frac{\eta \cdot v}{h} = -\frac{1}{2} \cdot \frac{dp}{dx} \cdot h - \frac{\eta \cdot v}{h} .$$
(12)

Taking in consideration the pressure gradient expression, it obtaining the next equation:

$$\tau_{yx} = -\frac{1}{2} \cdot 6\eta \cdot v \frac{h - h_1}{h^3} h - \frac{\eta \cdot v}{h} = -\eta \cdot v \left(\frac{4h - 3h_1}{h^2}\right).$$
(13)

The friction force, or fluid friction on the gasket width, will be.

$$F_{f} = \int_{0}^{x} \tau_{yx} \cdot g \cdot dx = \int_{0}^{x} \left[-\eta \cdot v \cdot \left(\frac{4h - 3h_{1}}{h^{2}}\right) \cdot g \cdot dx \text{ (14)} \right]$$
$$F_{f} = -\eta \cdot v \cdot g \cdot \int_{0}^{x} \frac{4h - 3h_{1}}{h^{2}} \cdot dx \text{ (15)}$$

The friction coefficient will be:

$$\mu = \frac{T_f}{F_p}, \qquad (16)$$

$$= \frac{-\eta \cdot v \cdot g \cdot \int_{0}^{x} \frac{4h - 3h_1}{h^2} \cdot dx}{\int_{0}^{x} (1 - 1)^2} = \frac{-\int_{0}^{x} \left(\frac{4h - 3h_1}{h^2}\right) dx}{\int_{0}^{x} (1 - 1)^2}. (17)$$

$$\mu = \frac{0}{6 \cdot \eta \cdot v \cdot g \cdot \int_{0}^{x} \left(\frac{h-h_{1}}{h^{3}}\right) \cdot dx} = \frac{0}{6} \int_{0}^{x} \left(\frac{h-h_{1}}{h^{3}}\right) dx} .(17)$$

Based on this mathematical model, it was elaborated a systemically and informational model in order to simulating the trybological phenomena of the fluid sealing.

4. THE SIMULATION EXPERIMENTS OF THE MOBILE TRANSLATION SEALING PROCESS

For computer simulation of the sealing process, was used the environment graphics programming MATLAB with Simulink. Using the library block standard simulation environment, and some blocks specifically designed, was developed a program on computer for simulation the tribologic behaviour of the mobile translation sealing subjected to high pressure. To run on computer the presented simulation model, were used as test data the constructive-functional characteristics of a mobile sealing of one hydraulic cylinder with a rod diameter with d = 100 mm, the width l = 10 mm and rod speed over the sealing is v = 2.5 m / s.

The obtained graphics results by computer simulation, regarding the variation on the main parameters of mobile translation sealing, are presented in the Figs. 3 - 10.

5. CONCLUSIONS

The article presents some technical and operational aspects of mobile translation sealing systems, used in the construction of hydraulic cylinders. Following analysis of hydrodynamic phenomena occurring at sealing with gaskets elastic and the alternative movement, it highlights some aspects of the process of sealing and are presented the conditions for achieving fluid sealing.

To determine the distribution of pressure and tangential efforts at the contact of the friction areas, in order to evaluate the sealing portant force and the coefficient of friction between the rod and fluid sealing ring, it propose a physical model based on a *wedge shaped interstice*, who developed a mathematical model.

After systemic and informatics modeling of the mobile translation sealing and computer simulation, have been obtained graphic variation of parameters that characterize the process of fluid sealing between the gasket and rod of the hydraulic cylinder.

The quantitative and qualitative variations of the parameters of interest, especially the coefficient of fluid friction, are in accordance with the data from the literature, being a logical and dimensional validation for the mathematical models and simulation programs.

REFERENCES

- Cristea, V., Creța, G., Ivan, D.D. (1973). *Etaşări* (Sealings), Edit. Tehnică, Bucharest.
- [2] Pavelescu, D., Muşat, M., Tudor, A. (1971). *Tribologie*. (Tribology), Edit. Didactică şi Pedagogică, Bucharest.
- [3] Oroveanu, Th. (1967). Mecanica fluidelor viscoase (Mechanics of viscous fluids), Edit. Academiei Române, Bucharest.
- [4] Catrina, Gh. (2002). Introducere în tribologie (Introduction in tribology), Edit. Universitaria Craiova, Romania.
- [5] Catrina, Gh. (2007). Organe de maşini, Vol. I, Transmisii mecanice (Machine components. Mechanical transmissions), Edit. Universitaria Craiova, Romania.

Authors:

PhD. Eng., Corneliu CRISTESCU, INOE 2000-IHP Bucharest, Senior Resarcher, General Hydraulic Department,

- E-mail: cristescu.ihp@fluidas.ro
- PhD. Eng., Petrin DRUMEA, Institute manager

Eng., Petrică KREVEY

Eng. Constanța CRISTESCU, Scientific Researchers