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THE INFLUENCE OF MEASURED POINTS POSITION ON TOOLS PROFILING WHEN USING BEZIER POLYNOMIAL APPROXIMATIONS

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Abstract: *Practical necessities can, sometimes, impose us to solve the problem of profiling a rack-tool reciprocal enwrapped to a whirl of cylindrical or helical surfaces, as possibility to do the constructive design of tools to cut gears teeth, when only a limited number of points from generated profiles are known in discrete form. A solution to such a problem can be enounced if using Bezier polynomial approximation of rack-tool profile. This paper presents the rack-tool profiling problem enouncement, when a circular profile has to be generated, together to a specific algorithm which allows finding rack-tool profile, by using Bezier polynomial approximation; the influence of measured points position onto problem solution precision is also analyzed by using a dedicated soft.*

Key words: *rack-tool profile, Bezier approximation, discrete points, circular profile.*

1. INTRODUCTION

Many profiles which are frequently used in industry to realize the constructive shape of teeth from tools to cut out (disc cutters), of triangular slots, of slots with parallel flanks, are composed profiles, usually including arcs of circle [1, 3, 4]; they form tools teeth active surfaces (chip bearing face, flank face) generators or fillets between other types of elementary profiles (Fig. 1).

Generator tools profiling – in the given case the rack-tool – by considering on these elementary profiles a small number of points (3 or 4, effectively measured) and by substituting tools profiles through Bezier polynomial functions [6], could represent a simple and efficient method to realize such generator tools.

The same profiling methodology can be also used to profile tools associated to circular centrods-pinion cutter or rotating cutter.

2. PROFILING ALGORITHM

There are considered (also Fig. 2) the following ensemble of centrods and reference systems [2]:

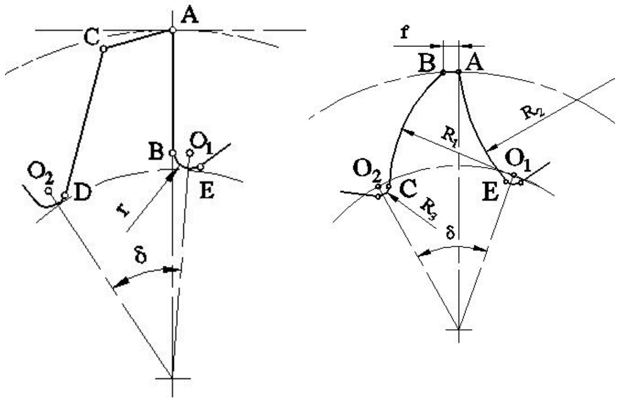

Fig. 1. Examples of composed profiles including circle arcs.

- *xOy*, meaning a fix system;
- *XOY* – mobile system, attached to C_1 centrod;
- ξη – mobile system, attached to C_2 centrod (rack-tool rolling line).

On the curved profile to be generated (but known from the beginning as circular), the co-ordinates of at least three points must be effectively measured: $A(X_A, Y_A)$, $B(X_B, Y_B)$ and $C(X_C, Y_C)$.

The co-ordinates of profile circle center, $O_C(X_{OC}, Y_{OC})$ can be found (identified) by analytically solving the system

$$\begin{cases} \overline{O_C A} = \overline{O_C B}; \\ \overline{O_C A} = \overline{O_C C}. \end{cases} \quad (1)$$

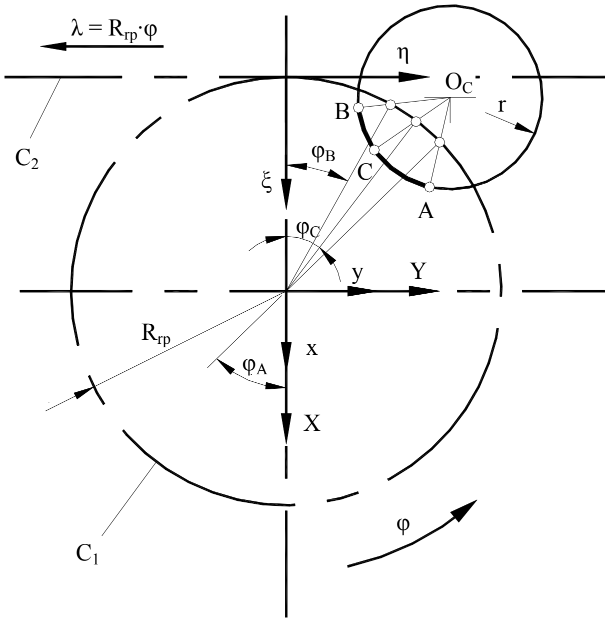

Fig. 2. Reference systems and generating process kinematics.

The circular profile to be generated can be approximated by using a second degree Bezier polynomial function [8],

$$\begin{vmatrix} X = \lambda^2 \cdot A_X + 2\lambda(1-\lambda)C_X + (1-\lambda)^2 B_X\,; \\ Y = \lambda^2 \cdot A_Y + 2\lambda(1-\lambda)C_Y + (1-\lambda)^2 B_Y\,. \end{vmatrix} \tag{2}$$

The coefficients of polynomial functions from (2) can be identified, by using the co-ordinates of measured points, as it follows:

$$\lambda = 0 \rightarrow\; B_X = X_A\,;\; B_Y = Y_A\,; \tag{3}$$

$$\lambda = 1 \rightarrow\; A_X = X_B\,;\; A_Y = Y_B\,; \tag{4}$$

$$\lambda = \lambda_C = \frac{\varphi_1}{\varphi_1 + \varphi_2} \rightarrow$$

$$C_X = \frac{X_C - \lambda_C^2 \cdot X_B - (1-\lambda_C)^2 X_A}{2\lambda_C(1-\lambda_C)}; \tag{5}$$

$$C_Y = \frac{Y_C - \lambda_C^2 \cdot Y_B - (1-\lambda_C)^2 Y_A}{2\lambda_C(1-\lambda_C)}.$$

The magnitudes of φ_1 and φ_2 angles can be defined in connection with points co-ordinates, previously measured on the profile:

$$\varphi_1 = \arcsin\left(\frac{\overline{AC}}{2 \cdot R}\right), \tag{6}$$

$$\varphi_2 = \arcsin\left(\frac{\overline{CB}}{2 \cdot R}\right), \tag{7}$$

while $$\left|\overline{AC}\right| = \sqrt{(X_A - X_C)^2 + (Y_A - Y_C)^2}\,, \tag{8}$$

$$\left|\overline{CB}\right| = \sqrt{(X_C - X_B)^2 + (Y_C - Y_B)^2} \tag{9}$$

and $$R = \sqrt{(X_O - X_C)^2 + (Y_O - Y_C)^2}\,, \tag{10}$$

where R means the radius of the circle including the measured points (Fig. 3).

Once identified the Bezier polynomial approximation function owning the measured points, rack-tool profile reciprocal enwrapped to given profile can also be found, if the following parameters of the circle arc are considered (Fig. 3):

$$\theta_A = \arcsin\left(\frac{Y_A}{R}\right);$$

$$\theta_B = \arcsin\left(\frac{Y_B}{R}\right); \tag{11}$$

and $$\theta_C = \arcsin\left(\frac{Y_C}{R}\right).$$

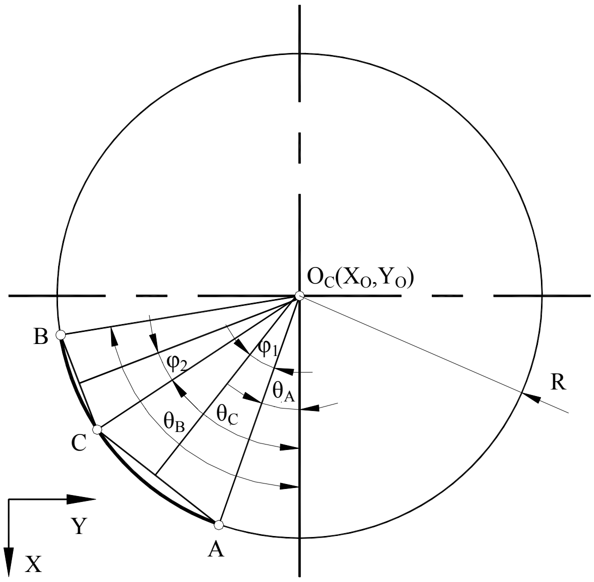

Fig. 3. Circular profile parameters.

Thus, the value of λ_C parameter can be recalculated by using the relation

$$\lambda_C = \frac{\theta_C - \theta_A}{\theta_B - \theta_A}. \tag{12}$$

In Tables 1 and 2, there are presented the algorithms to identify Bezier approximation polynomial functions of 2nd and 3rd degree, representing the approximations of rack-tool profile reciprocal enwrapped to curved (circular) profile, given through measured points co-ordinates.

3. NUMERICAL APPLICATIONS

Based on upper presented algorithm, numerical applications are presented, for a circular profile known by measuring the co-ordinates of three points from the profile to be generated:

$A(-40.681;\, 5.176)$; $B(-40.681;\, -5.176)$; $C(-40;\, 0)$.

In fact, the mentioned points are on a circle of radius r = 20 mm and having the center $O_C(-60;\, 0)$. Rolling radius, R_{rp} was chosen of 41 mm.

In Table 2, there are presented numerical results to compare the rack-tool profiles found by each one of the two methods (theoretical profile and profile approximated by Bezier polynomial function), together to the magnitude of the error [7], defined as minimum distance between the two profiles.

Bezier polynomial approximation function degree can be chosen 2, 3 or higher; from motives concerning simplicity of calculus, in this case the approximation was made by a function of 2nd degree.

To analyze the influence of measured points co-ordinates onto precision of approximated generator tool profile, the position of intermediary point, C, initially at the middle between A and B, was successively modified to C' $(-40.012;\, 0.697)$ respective C'' $(-40.048;\, 1.395)$. The results in these cases are shown in Tables 3 and 4.

Table 1

Algorithm to profile the rack-tool for profiles known in discrete form by using Bezier polynomial approximations

Co-ordinates of measured profile	Values of λ parameter	Values of rolling angles
1	2	3
X_A Y_A	1	$\varphi_A = \arcsin\dfrac{X_A \sin\theta_A + Y_A \cos\theta_A}{R_{rp}} + \theta_A$
X_C Y_C	$\lambda_C = \dfrac{\varphi_1}{\varphi_1 + \varphi_2}$	$\varphi_B = \arcsin\dfrac{X_B \sin\theta_B + Y_B \cos\theta_B}{R_{rp}} + \theta_B$
X_B Y_B	0	$\varphi_C = \arcsin\dfrac{X_C \sin\theta_C + Y_C \cos\theta_C}{R_{rp}} + \theta_C$

The points from tool profile	Values of λ parameter	Bezier approximation polynomial function coefficients
4	5	6
$\xi_A = X_A \cdot \cos\varphi_A - Y_A \cdot \sin\varphi_A + R_{rp}$; $\eta_A = X_A \cdot \sin\varphi_A + Y_A \cdot \cos\varphi_A + R_{rp} \cdot \varphi_A$.	0	$\xi_A = B_\xi$ $\eta_A = B_\eta$
$\xi_C = X_C \cdot \cos\varphi_C - Y_C \cdot \sin\varphi_C + R_{rp}$; $\eta_C = X_C \cdot \sin\varphi_C + Y_C \cdot \cos\varphi_C + R_{rp} \cdot \varphi_C$.	λ_C	$C_\xi = \dfrac{\xi_C - \lambda_C^2 \cdot \xi_B - (1-\lambda_C)^2 \cdot \xi_A}{2\lambda_C(1-\lambda_C)}$; $C_\eta = \dfrac{\eta_C - \lambda_C^2 \cdot \eta_B - (1-\lambda_C)^2 \cdot \eta_A}{2\lambda_C(1-\lambda_C)}$.
$\xi_B = X_B \cdot \cos\varphi_B - Y_B \cdot \sin\varphi_B + R_{rp}$; $\eta_B = X_B \cdot \sin\varphi_B + Y_B \cdot \cos\varphi_B + R_{rp} \cdot \varphi_B$.	1	$\xi_B = A_\xi$ $\eta_B = A_\eta$

Table 2

Rack-tool profile – numerical results (1)

Crt No.	Theoretical Profile		Approximated Profile		Error [mm]
	ξ [mm]	η [mm]	ξ [mm]	η [mm]	
1	-0.0093	5.1887	-0.0090	5.1887	0.0003
2	0.0557	5.0266	0.0572	5.0158	0.0109
3	0.1187	4.8634	0.1211	4.8428	0.0207
4	0.1797	4.6991	0.1827	4.6698	0.0295
5	0.2386	4.5338	0.2422	4.4969	0.0371
6	0.2953	4.3674	0.2993	4.3239	0.0437
7	0.3500	4.2001	0.3543	4.1510	0.0493
8	0.4025	4.0319	0.4070	3.9780	0.0541
9	0.4529	3.8627	0.4574	3.8051	0.0578
10	0.5011	3.6928	0.5056	3.6321	0.0609
11	0.5471	3.5221	0.5516	3.4591	0.0632
12	0.5910	3.3505	0.5953	3.2862	0.0644
13	0.6327	3.1783	0.6368	3.1132	0.0652
14	0.6721	3.0054	0.6760	2.9403	0.0652
…	…	…	…	…	…
51	0.5471	-3.5220	0.5516	-3.4591	0.0631
52	0.5011	-3.6928	0.5056	-3.6321	0.0609
53	0.4529	-3.8627	0.4574	-3.8051	0.0578
54	0.4025	-4.0319	0.4070	-3.9780	0.0541
55	0.3500	-4.2001	0.3543	-4.1510	0.0493
56	0.2953	-4.3674	0.2993	-4.3239	0.0437
57	0.2386	-4.5338	0.2422	-4.4969	0.0371
58	0.1797	-4.6991	0.1827	-4.6698	0.0295
59	0.1187	-4.8634	0.1211	-4.8428	0.0207
60	0.0557	-5.0266	0.0572	-5.0158	0.0109
61	-0.0093	-5.1887	-0.0090	-5.1887	0.0003

Table 3

Rack-tool profile – numerical results (2)

Crt No.	Theoretical Profile		Approximated Profile		Error [mm]
	ξ [mm]	η [mm]	ξ [mm]	η [mm]	
1	-0.0093	5.1887	-0.0090	5.1887	0.0003
2	0.0557	5.0266	0.0572	5.0172	0.0095
3	0.1187	4.8634	0.1211	4.8457	0.0179
4	0.1797	4.6991	0.1827	4.6741	0.0252
5	0.2386	4.5338	0.2421	4.5025	0.0315
6	0.2953	4.3674	0.2993	4.3308	0.0368
7	0.3500	4.2001	0.3542	4.1590	0.0413
8	0.4025	4.0319	0.4069	3.9872	0.0449
9	0.4529	3.8627	0.4573	3.8154	0.0475
10	0.5011	3.6928	0.5055	3.6435	0.0495
11	0.5471	3.5221	0.5515	3.4716	0.0507
12	0.5910	3.3505	0.5952	3.2996	0.0511
13	0.6327	3.1783	0.6367	3.1276	0.0509
14	0.6721	3.0054	0.6759	2.9555	0.0500
…	…	…	…	…	…
51	0.5471	-3.5220	0.5515	-3.4467	0.0754
52	0.5011	-3.6928	0.5055	-3.6207	0.0722
53	0.4529	-3.8627	0.4573	-3.7947	0.0681
54	0.4025	-4.0319	0.4069	-3.9688	0.0633
55	0.3500	-4.2001	0.3542	-4.1429	0.0574
56	0.2953	-4.3674	0.2993	-4.3171	0.0505
57	0.2386	-4.5338	0.2421	-4.4913	0.0426
58	0.1797	-4.6991	0.1827	-4.6656	0.0336
59	0.1187	-4.8634	0.1211	-4.8399	0.0236
60	0.0557	-5.0266	0.0572	-5.0143	0.0124
61	-0.0093	-5.1887	-0.0090	-5.1887	0.0003

Table 4

Rack-tool profile – numerical results (3)

Crt No.	Theoretical Profile ξ [mm]	Theoretical Profile η [mm]	Approximated Profile ξ [mm]	Approximated Profile η [mm]	Error [mm]
1	-0.0093	5.1887	-0.0090	5.1887	0.0003
2	0.0557	5.0266	0.0572	5.0187	0.0080
3	0.1187	4.8634	0.1210	4.8486	0.0150
4	0.1797	4.6991	0.1827	4.6784	0.0209
5	0.2386	4.5338	0.2420	4.5081	0.0259
6	0.2953	4.3674	0.2992	4.3377	0.0300
7	0.3500	4.2001	0.3541	4.1672	0.0332
8	0.4025	4.0319	0.4068	3.9965	0.0357
9	0.4529	3.8627	0.4572	3.8258	0.0371
10	0.5011	3.6928	0.5054	3.6550	0.0380
11	0.5471	3.5221	0.5513	3.4841	0.0382
12	0.5910	3.3505	0.5950	3.3131	0.0376
13	0.6327	3.1783	0.6365	3.1420	0.0365
14	0.6721	3.0054	0.6757	2.9708	0.0348
…	…	…	…	…	…
51	0.5471	-3.5220	0.5513	-3.4342	0.0879
52	0.5011	-3.6928	0.5054	-3.6092	0.0837
53	0.4529	-3.8627	0.4572	-3.7843	0.0785
54	0.4025	-4.0319	0.4068	-3.9595	0.0725
55	0.3500	-4.2001	0.3541	-4.1348	0.0654
56	0.2953	-4.3674	0.2992	-4.3102	0.0573
57	0.2386	-4.5338	0.2420	-4.4857	0.0482
58	0.1797	-4.6991	0.1827	-4.6613	0.0379
59	0.1187	-4.8634	0.1210	-4.8370	0.0265
60	0.0557	-5.0266	0.0572	-5.0128	0.0139
61	-0.0093	-5.1887	-0.0090	-5.1887	0.0003

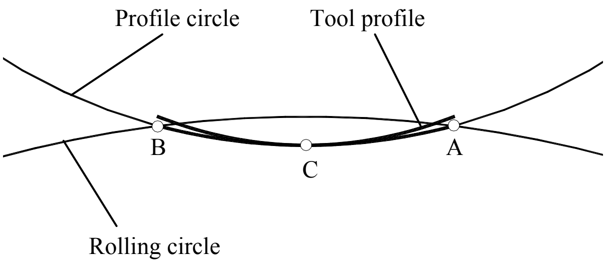

Fig. 4. Rack-tool approximated profile.

In Fig. 4 the profile to be generated, *AB*, is represented together to rack-tool profiles (the theoretical one and the approximated one); because the distance between last two profiles is insignificant referred to drawing scale, they are overlapped.

4. CONCLUSIONS

The method of approximating by using Bezier polynomial functions is easy to apply and rigorous enough to be used in profiling rack-tools, if profiles to be generated by using them aren't teeth flanks of gears transmitting important torques, at high rotation speeds. It can be used in the case of curved (circular) profiles as well as in the case of rectilinear profiles.

The method is very expedient, the only things needed to be known being the co-ordinates of three points, measured directly along the profile to be generated; obviously, the measurement precision directly affects the accuracy of results.

If analyzing the effect of measured points co-ordinates onto accuracy of approximated profile (relative to the theoretical one), a comparison between error columns from Tables 2, 3 and 4 must be made. Thus, if *C* point is in the middle of *AB* profile, the maximum error is about 65 μm; if *C* is progressively eloigning from this position, then the maximum error increases to about 80 μm (Tabel 3) respective 90 μm (Tabel 4).

Obviously, when measuring the co-ordinates of points from a certain profile, it is impossible to find exactly its middle, but we must notice that best results can be obtained if considering a point as close as possible to profile middle.

Increasing the magnitude of polynomial substitution function leads to amelioration of tool profile precision, under condition of accepting more and more complicated calculus to be done.

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