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ALGORITHM TO PROFILE RACK-TOOL FOR PROFILES KNOWN IN DISCRETE FORM BY USING BEZIER POLYNOMIAL APPROXIMATIONS

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Abstract: *The methods used to profile rack-tool reciprocal enwrapped to a whirl of cylindrical or helical surfaces, as possibility to do the constructive design of tools to cut gears teeth, require knowing the analytical form of profiles to be generated. Practical necessities can, sometimes, impose us to solve this problem when only a limited number of points from generated profiles are known in discrete form. A solution to such a problem can be enounced if using Bezier polynomial approximation of rack-tool profile. This paper presents the rack-tool profiling problem enouncement together to a specific algorithm which allows finding rack-tool profile by using Bezier polynomial approximation together with fundamental theorems concerning profiles associated to rolling centrods enwrapping.*

Key words: *rack-tool profile, Bezier approximation, discrete points.*

1. INTRODUCTION

Many surfaces, found by using enwrapping conditions or by directly measuring their generators, belonging to ordinate whirls and associated to rolling centrods are expressed under a “discrete” form as an ensemble of points whose co-ordinates are numerical known.

There are also known specific algorithms to study the enwrapped of this kind of surfaces/profiles, algorithms elaborated based on “Minimum Distance Method” [2] or on “Tangents Method” [4].

Same time, graphical soft products, 2D or 3D, represent the surfaces/profiles in numerical form and analysis tools, specific to their discrete expression, are basically used to profile the cutting tools [5].

In this paper, an application of approximation method based on Bezier polynomial functions, to profile cutting tools (rack-tools in this case), is suggested, to generate types of profiles known as shape but expressed through a small number (3-4) of points owning to them.

To validate the method, numerical results obtained after its application are compared to those furnished by classic analytical method used to solve the same case.

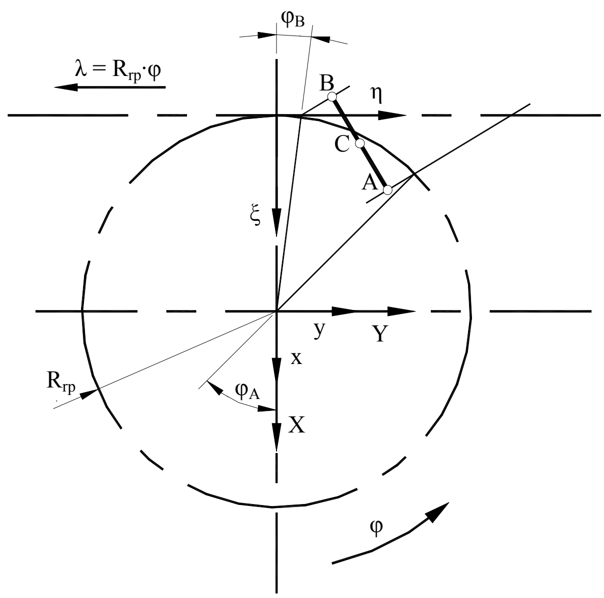

Fig. 1. Refernce systems and generating process kinematics.

2. ALGORITHM TO PROFILE THE RACK-TOOL

2.1. Reference Systems

We first consider the case when the profile to be generated, as part in a composed profile, is known as shape, in principle, but is given only through a small number of points owning to it; if considering a rectilinear profile, then three points are enough (Fig. 1).

Conform to notations from Fig. 1, the following reference systems are defined:

- *xy*, meaning a fix system;
- *XY* – a mobile system, attached to profile being generated;
- ξη – a mobile system, attached to the rack-tool.

The co-ordinates of known points (by measurement), from given profile, accepted as rectilinear, are referred to *XY* mobile system:

$$A[X_A, Y_A],\ B[X_B, Y_B],\ C[X_C, Y_C]. \tag{1}$$

2.2. Profile to Be Generated Equations

Based on (1), the profile to be generated can be expressed through a first degree Bezier function [6],

$$\Delta \left| \begin{aligned} X &= \lambda \cdot X_A + (1-\lambda) X_B ; \\ Y &= \lambda \cdot Y_A + (1-\lambda) Y_B . \end{aligned} \right. \tag{2}$$

where λ is a parameter having values between 0 and 1.

The value of λ_C parameter, corresponding to C point, can be calculated by using the co-ordinates of measured points, from (1):

$$\lambda_C = \frac{\sqrt{(X_C - X_A)^2 + (Y_C - Y_A)^2}}{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2}}. \quad (3)$$

This way, the polynomial function which substitutes profile equations can by fully defined by measuring only the co-ordinates of three points owning to profile.

2.3. Enveloping Condition (Rolling Angles)

Starting from the condition that the normal at the profile expressed through (2) equations

$$[X - X(\lambda)]\cdot\dot{X}_\lambda + [Y - Y(\lambda)]\cdot\dot{Y}_\lambda = 0 \quad (4)$$

intersects the rolling circle of R_{rp} radius,

$$C \left| \begin{aligned} &X = -R_{rp}\cdot\cos\varphi; \\ &Y = R_{rp}\cdot\sin\varphi. \end{aligned} \right. \quad (5)$$

the values of rolling angles (the enveloping condition corresponding to this points) can be found as

$$\begin{aligned} &\left[-R_{rp}\cdot\cos\varphi - X(\lambda)\right](X_A - X_B) + \\ &\left[-R_{rp}\cdot\sin\varphi - Y(\lambda)\right](Y_A - Y_B) \end{aligned} \quad (6)$$

2.4. Rack-Tool Profile Approximation

By knowing the values of rolling angles, corresponding to the three points characterizing the considered profile, let them be φ_A, φ_B and φ_C, the co-ordinates of three points from the rack-tool theoretical profile can be calculated, as it follows:

$$\left| \begin{aligned} &\xi_A = X_A\cdot\cos\varphi_A - Y_A\cdot\sin\varphi_A + R_{rp}; \\ &\eta_A = X_A\cdot\sin\varphi_A + Y_A\cdot\cos\varphi_A + R_{rp}\cdot\varphi_A, \end{aligned} \right. \quad (7)$$

in the case of the point corresponding to A point and similar for the other two points:

$$\left| \begin{aligned} &\xi_B = X_B\cdot\cos\varphi_B - Y_B\cdot\sin\varphi_B + R_{rp}; \\ &\eta_B = X_B\cdot\sin\varphi_B + Y_B\cdot\cos\varphi_B + R_{rp}\cdot\varphi_B, \end{aligned} \right. \quad (8)$$

respective

$$\left| \begin{aligned} &\xi_C = X_C\cdot\cos\varphi_C - Y_C\cdot\sin\varphi_C + R_{rp}; \\ &\eta_C = X_C\cdot\sin\varphi_C + Y_C\cdot\cos\varphi_C + R_{rp}\cdot\varphi_C. \end{aligned} \right. \quad (9)$$

2.5. Polynomial Approximation Functions

It is possible, now, because three points owning to rack-tool profile were found (see Fig. 2), to approximate it by using second degree Bezier polynomial functions, having the following expressions:

$$\left| \begin{aligned} &\xi = \lambda^2\cdot A_\xi + 2\lambda(1-\lambda)C_\xi + (1-\lambda)^2 B_\xi; \\ &\eta = \lambda^2\cdot A_\eta + 2\lambda(1-\lambda)C_\eta + (1-\lambda)^2 B_\eta, \end{aligned} \right. \quad (10)$$

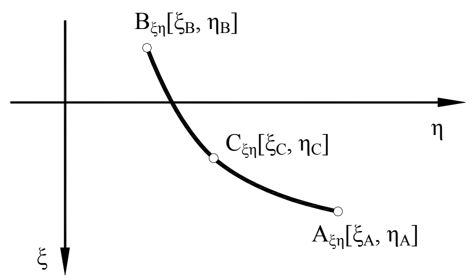

Fig. 2. Rack-tool approximated profile.

where $0 \le \lambda \le 1$.

The identification of A_ξ, C_ξ, B_ξ, A_η, C_η, B_η coefficients, from the equations of polynomial function which substitutes the rack-tool profile leads to finding its final form and, from here, to obtain the co-ordinates of points from rack-tool profile, reciprocal enwrapped to rectilinear profile to be generated, given through measured points co-ordinates.

In Table 1 there is synthetically presented the algorithm suggested to be used to find Bezier polynomial function to substitute rack-tool profile for generating a rectilinear profile, known under discrete form.

3. NUMERICAL APPLICATION

A numerical application, by using upper suggested method, is solved in comparison to the analytical method of finding rack-tool profile to generate the same profile (see Fig. 3).

By using the same reference systems as upper defined (see 2.1), the profile to be generated is defined through relations (11, 12, 13):

$$\Sigma \left| \begin{aligned} &X = X_A - u\cdot\cos\alpha; \\ &Y = Y_A - u\cdot\sin\alpha,. \end{aligned} \right. \quad (11)$$

$$\tan\alpha = \frac{|Y_A - Y_B|}{|X_A - X_B|}. \quad (12)$$

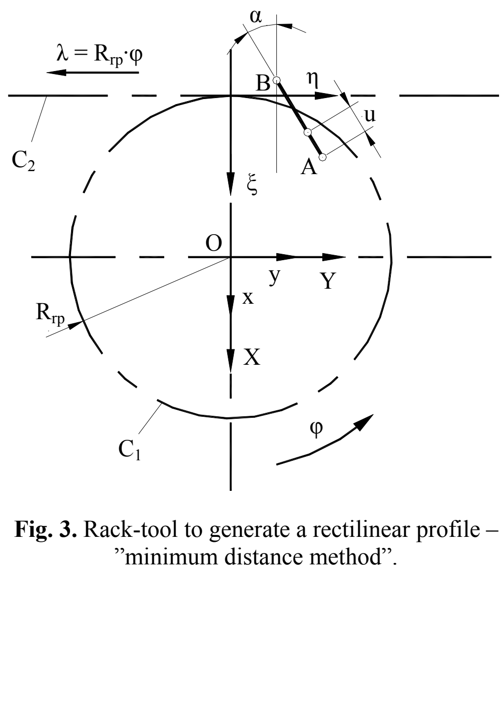

Fig. 3. Rack-tool to generate a rectilinear profile – ”minimum distance method”.

Table 1

Algorithm to profile the rack-tool for profiles known in discrete form by using Bezier polynomial approximations

Co-ordinates of measured profile	Values of λ parameter	Values of rolling angles
1	2	3
X_A Y_A	1	$(X_A - X_B)(-R_{rp} \cdot \cos\varphi_A - X_A) + (Y_A - Y_B)(R_{rp} \cdot \sin\varphi_A - Y_A) = 0$
X_C Y_C	$\lambda_C = \dfrac{\sqrt{(X_C - X_A)^2 + (Y_C - Y_A)^2}}{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2}}$	$(X_A - X_B)(-R_{rp} \cdot \cos\varphi_C - X_C) + (Y_A - Y_B)(R_{rp} \cdot \sin\varphi_C - Y_C) = 0$
X_B Y_B	0	$(X_A - X_B)(-R_{rp} \cdot \cos\varphi_B - X_B) + (Y_A - Y_B)(R_{rp} \cdot \sin\varphi_B - Y_B) = 0$

The points from tool profile	Values of λ parameter	Bezier approximation polynomial function coefficients
4	5	6
$\xi_A = X_A \cdot \cos\varphi_A - Y_A \cdot \sin\varphi_A + R_{rp}$; $\eta_A = X_A \cdot \sin\varphi_A + Y_A \cdot \cos\varphi_A + R_{rp} \cdot \varphi_A$.	0	$\xi_A = B_\xi$ $\eta_A = B_\eta$
$\xi_C = X_C \cdot \cos\varphi_C - Y_C \cdot \sin\varphi_C + R_{rp}$; $\eta_C = X_C \cdot \sin\varphi_C + Y_C \cdot \cos\varphi_C + R_{rp} \cdot \varphi_C$.	λ_C	$C_\xi = \dfrac{\xi_C - \lambda_C{}^2 \cdot \xi_B - (1-\lambda_C)^2 \cdot \xi_A}{2\lambda_C(1-\lambda_C)}$; $C_\eta = \dfrac{\eta_C - \lambda_C{}^2 \cdot \eta_B - (1-\lambda_C)^2 \cdot \eta_A}{2\lambda_C(1-\lambda_C)}$.
$\xi_B = X_B \cdot \cos\varphi_B - Y_B \cdot \sin\varphi_B + R_{rp}$; $\eta_B = X_B \cdot \sin\varphi_B + Y_B \cdot \cos\varphi_B + R_{rp} \cdot \varphi_B$.	1	$\xi_B = A_\xi$ $\eta_B = A_\eta$

Maximum value of u parameter from (11) is

$$u_{max} = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2}\,. \qquad (13)$$

During its motion referred to ξη system, the profile Σ describes the family of profiles:

$$(\Sigma)_\varphi \begin{vmatrix} \xi = (X_A - u \cdot cos\,\alpha) cos\,\varphi - \\ \quad - (Y_A - u \cdot sin\,\alpha) sin\,\varphi + R_{rp}; \\ \eta = (X_A - u \cdot cos\,\alpha) sin\,\varphi + \\ \quad + (Y_A - u \cdot sin\,\alpha) cos\,\varphi + R_{rp} \cdot \varphi, \end{vmatrix} \qquad (14)$$

to whom the enveloping condition (according to "minimum distance method" [8]) is attached:

$$\xi \cdot \xi_u{}' + (\eta - R_{rp} \cdot \varphi) \cdot \eta_u{}' = 0, \qquad (15)$$

where

$$\begin{aligned} \xi_u{}' &= - cos\,\alpha \cdot cos\,\varphi + sin\,\alpha \cdot sin\,\varphi = - cos(\alpha + \varphi); \\ \eta_u{}' &= - cos\,\alpha \cdot sin\,\varphi - sin\,\alpha \cdot cos\,\varphi = - sin(\alpha + \varphi). \end{aligned} \qquad (16)$$

The ensemble of equations (14) and (15) are giving the profile of the rack-tool to generate by enwrapping the whirl of rectilinear profiles $\overline{AB}$.

In Table 2, there are presented numerical results to compare the rack-tool profiles found by each one of the two methods (theoretical profile and profile approximated by Bezier polynomial function), together to the magnitude of the error, defined as minimum distance between the two profiles.

Table 2

Rack-tool profile – numerical results

Crt No.	Theoretical Profile		Approximated Profile		Error [mm]
	ξ [mm]	η [mm]	ξ [mm]	η [mm]	
1	4.3290	8.7254	4.3398	8.7380	0.0166
2	4.2688	8.6552	4.2782	8.6664	0.0146
3	4.2086	8.5853	4.2166	8.5953	0.0128
4	4.1482	8.5157	4.1550	8.5245	0.0111
5	4.0877	8.4464	4.0934	8.4541	0.0096
6	4.0271	8.3774	4.0317	8.3841	0.0081
7	3.9664	8.3087	3.9700	8.3145	0.0068
8	3.9056	8.2404	3.9083	8.2453	0.0056
9	3.8446	8.1724	3.8465	8.1764	0.0044
10	3.7836	8.1047	3.7848	8.1080	0.0035
…	…	…	…	…	…
51	1.1593	5.5827	1.1663	5.5872	0.0083
52	1.0965	5.5311	1.1034	5.5356	0.0082
53	1.0337	5.4800	1.0405	5.4844	0.0081
54	0.9710	5.4293	0.9775	5.4335	0.0077
55	0.9083	5.3791	0.9145	5.3831	0.0074
56	0.8457	5.3293	0.8515	5.3331	0.0069
57	0.7832	5.2799	0.7884	5.2834	0.0063
58	0.7208	5.2311	0.7254	5.2342	0.0055
59	0.6584	5.1826	0.6623	5.1853	0.0047
60	0.5961	5.1347	0.5991	5.1368	0.0037

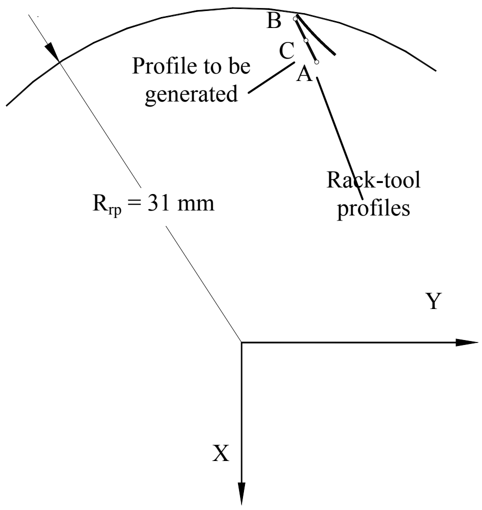

Fig. 4. Rack-tool profile – numerical application.

The results from Table 2 were obtained by considering the following three points on measured profile (accepted from the beginning as being rectilinear):

- *A*(–26; 7); B(–30; 5) and C(–28; 6);

- the rolling radius of worked piece, R_{rp}, was chosen of 31 mm.

The approximation polynomial function has, in this case, the expression:

$$\begin{cases} \xi = 0.599 \cdot \lambda^2 + 4.986 \cdot \lambda(1-\lambda) + 4.339 \cdot (1-\lambda)^2; \\ \eta = 5.137 \cdot \lambda^2 + 13.17 \cdot \lambda(1-\lambda) + 8.738 \cdot (1-\lambda)^2, \end{cases}$$

or, after calculus

$$\begin{cases} \xi = -0.048 \cdot \lambda^2 - 3.692 \cdot \lambda + 4.339; \\ \eta = 0.705 \cdot \lambda^2 - 4.306 \cdot \lambda + 8.738. \end{cases}$$

By using an original soft [7], dedicated to measure the distance between two profiles given through points, the profile of rack-tool approximated (as upper suggested) by using a Bezier second degree polynomial function was compared to the theoretical profile of the tool which could generate the same profile. The maximum distance found (meaning, in this case, the maximum error induced by the approximation that we made) is about 17 μm.

In Fig. 4, the profile to be generated, AB, is represented together to rack-tool profiles (the theoretical one and the approximated one); because the distance between last two profiles is insignificant referred to drawing scale, they are overlapped.

Notes:

- In considered numerical application, for simplicity point C was considered to be measured at the middle of AB segment; in fact, it can be anywhere between A and B; the only thing that really maters is that points A and B must be measured as closest as possible to segment to be generated extremities.

- The method is relative difficult to apply in the cases of profiles parallel to OY axis.

4. CONCLUSIONS

The method of approximating by using Bezier polynomial functions is easy to apply and rigorous enough to be used in profiling rack-tools, if profiles to be generated by using them aren't teeth flanks of gears transmitting important torques, at high rotation speeds.

The method is very expedient, the only things needed to be known being the co-ordinates of three points, measured directly along the profile to be generated; obviously, the measurement precision directly affects the accuracy of results.

Once the expressions of substitution polynomial function (10) are found, there is the possibility of extrapolating it over the limits given through points *A* and *B* (Fig. 2).

Increasing the magnitude of polynomial substitution function leads to amelioration of tool profile precision, under condition of accepting more and more complicated calculus to be done.

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