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University POLITEHNICA of Bucharest, Machine and Manufacturing Systems Department
Bucharest, Romania

NEW ASPECTS CONCERNING THE KINEMATIC GENERATION OF THE GENERATRIX AND DIRECTRIX CURVES OF THE GEOMETRICAL SURFACES

Ioan-Gheorghe SANDU, Eugen STRAJESCU

Abstract: *The paper presents a theoretical study of the principle of the kinematics generation of the D and G curves of the geometric complex surfaces using the composition of the simple movements. In that sense, as a new original application, it is presented the cinematic generation of two largely used curves G and D of the real complex surfaces generated on machine-tools, respectively the generation of the hypocycloid at the generation of the surfaces of the flank of the cylindrical gears having a cycloid profile and the generation of the circular helix at the generation of the cylindrical helicoidally surfaces.*

Key words: *surface generation, generatrix, directrix, hypocycloid, cycloid, gears, helical surfaces.*

1. INTRODUCTION

The generation of the real surfaces on machine-tools is realized after the cinematic generation principle of the generation of the geometrical surfaces corresponding to their form.

In this way, the geometrical surfaces are generated by a generating curve *G* in it movement along the directrix curve D.

The theoretic curves *G* and *D* of a geometric surface has the form gave by the geometric form of the surface and these curves are generated by materialization, by cinematic way or by programming.

In the case of the complex surfaces [1, 3], the curves *G* and *D* have a complex geometrical form, namely different than the circle or the right line (epicycloids, hypocycloid, involutes, circular helix etc.) These can be generated by cinematic ways using the composition of the simple movements cinematically coordinated in order to obtain their theoretical form.

2. COMPOSITIONS OF SIMPLE MOVEMENTS

In the anterior original studies [2, 4], in which was treated the generation cinematic as complexes trajectories *G* and *D* of some curves with a large utility in engineering, there was advanced the notions of simple movement, composition of simple movements and ratio R_{CCIN} – ratio that assures the cinematic coordination of the simple movements being composed so that the generating element can generate the trajectory *G* or *D* with the given form.

On them bases, the paper presents new original aspects of cinematic generation of another two largely used curves *G* and *D* at the real complex surfaces' generation on machine tools.

2.1. The composition in the same plane of two rotation movements by interior rolling.

It is considered a circular base having the radius R_B with the center *O* in the origin of the co-ordinates system *XOY* (Fig. 1). A circular rolling curve having the R_R radius makes simultaneously a rotation movement around it center O_r with the angular speed ω_1 and a rotation movement of the center O_r around the center *O* with the angular speed ω_2 , so that it is rolling on the base without sliding on the base, in it interior.

It is considered a generating point *M* of the rolling curve. For an instant position of the rolling curve gave by the rolling angle Φ, the point *M* is angular positioned by the angle φ. It size can be deduced from the condition of the rolling without sliding of the rolling curve on the base, that impose the congruence of the size of the arcs *AB* and *BM*.

$$AB = BM, \tag{1}$$

where:

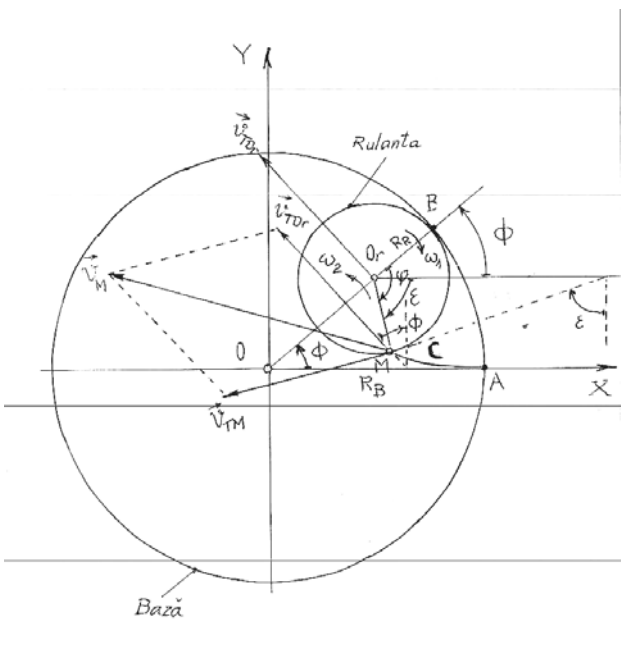

Fig. 1. The cinematics of the composition in a plane of the simple movements.

$AB = R_B\, \Phi$ and $BM = R_R\, \varphi$.

By consequence,

$$R_B\, \Phi = R_R\, \varphi, \tag{2}$$

from where it results:

$$\varphi = (\frac{R_B}{R_R})\, \Phi. \tag{3}$$

From the equality (2) it is to deduce by differentiating around the time dT

$$R_B \frac{\mathrm{d}\Phi}{\mathrm{d}T} = R_R \frac{\mathrm{d}\varphi}{\mathrm{d}T} \tag{4}$$

so it is

$$R_B\, \omega_2 = R_R\ \omega_1, \tag{5}$$

or

$$R_{CCIN} = \frac{\omega_2}{\omega_1} = \frac{R_R}{R_B}, \tag{6}$$

that impose to the two rotation movements to be composed the cinematic coordination of the rolling without slide of the rolling curve on the base (Fig. 1).

In the generating point M appear two speed vectors $\vec{v}_{TM}$, as a tangent speed in the rotation movement of the rolling curve algebraic composed with the rotation movement of the center O_r and $\vec{v}_{TOr}$, as a tangent speed in the center O_r in its rotation movement in face to the center O. By consequence the resulting speed in the point M will be the vector $\vec{v}_M$, given by the vectorial relationship:

$$\vec{v}_M = \vec{v}_{TM} + \vec{v}_{TO_r} \tag{7}$$

The size of the component speeds are similar. So:

$$V_{TM} = R_R\,(\omega_1 - \omega_2) = R_R\,\omega_1 - R_R\,\omega_2. \tag{8}$$

Considering the equality (5), it results:

$$V_{TM} = (R_B - R_R)\ \omega_2 \tag{9}$$

On another side, it is to observe that the size of the speed vector is given too by the relation (9).

If we project the vectorial relation (7) on the co-ordinate axes we obtain the sizes of the speed components on these axes, that have the next relations:

$$\begin{aligned} v_{MX} &= -(R_B - R_R)\omega_2 \sin\varepsilon - (R_B - R_R)\omega_2 \sin\Phi \\ v_{MX} &= -(R_B - R_R)\omega_2 \cos\varepsilon - (R_B - R_R)\omega_2 \cos\Phi \end{aligned} . \tag{10}$$

From the Fig. 1 we can observe that

$$\varepsilon = \varphi - \Phi. \tag{11}$$

Replacing the relation (3) in the relation (11), we obtain:

$$\varepsilon = \left(\frac{R_B - R_R}{R_R}\right)\Phi \tag{12}$$

If we consider the (12) relationship in the relations (4) and (10), we obtain the final expressions of the sizes of the speed $\vec{v}_M$ components on the coordinates axes:

$$\begin{aligned} v_{MX} &= -(R_B - R_R)\omega_2 \sin\left(\frac{R_B - R_R}{R_R}\right)\Phi - \\ &- (R_B - R_R)\omega_2 \sin\Phi \\ v_{MX} &= -(R_B - R_R)\omega_2 \cos\left(\frac{R_B - R_R}{R_R}\right)\Phi - \\ &- (R_B - R_R)\omega_2 \cos\Phi \end{aligned} \tag{12}$$

The differential coordinated dX and dY of the generating point M are differential space traversed respectively with the speeds v_{MX} and v_{MY}, in a differential time dT:

$$\begin{cases} dX = v_{MX} dT \\ dY = v_{MY} dT \end{cases} \tag{14}$$

where:

$$dT = \frac{d\Phi}{dT} \tag{15}$$

If the relationships (13) and (15) are replaced in the relationship (14), we obtain the expressions of the instant coordinates X and Y of the generating point.

By the accomplishing of the integrals, we obtain the coordinates X and Y) of the point M, that represent the parametric equations of the cinematic curve C, described by the generating point M as a consequence of the composition of the two rotation movements.

$$\begin{cases} X = (R_B - R_R)\cos\Phi + R_R \cos\left(\frac{R_B - R_R}{R_R}\right)\Phi + C_x \\ Y = (R_B - R_R)\sin\Phi - R_R \cos\left(\frac{R_B - R_R}{R_R}\right)\Phi + C_y \end{cases} \tag{17}$$

where the integration constants C_x and C_y are determinate knowing that at $\Phi = 0$, $X = R_B$ and $Y = 0$, resulting $C_X = 0$ and $C_Y = 0$.

So, the parametric equations of the cinematic curve C are:

$$\begin{cases} X = (R_B - R_R)\cos\Phi + R_R \cos\left(\frac{R_B - R_R}{R_R}\right)\Phi \\ Y = (R_B - R_R)\sin\Phi - R_R \cos\left(\frac{R_B - R_R}{R_R}\right)\Phi \end{cases}, \tag{18}$$

that represent the parametric equations of the cycloid.

In conclusion, by composing of two movements of plane rotation (in the same plane), a generating point M of a circular rolling curve described as a cinematic trajectory C a hypocycloid with the condition to respect the R_{CCIN} given by the relationship (6) that impose to the two movements the cinematic coordination of the rolling without sliding of the rolling curve inside the circular base.

Practically, at the generation of the real surfaces, the cinematic curve C as hypocycloids generated as a generating curve G of the complex surface.

An example of the applicability of this case of composition of simple movements is the realization of the generatrix G at the generation of the cylindrical wheel gears' flanks with cycloidal profile (Fig. 2).

The serration with a cycloidal profile is used in the fine mechanics, in the watch field.. The flank profile of the tooth, that represents the generatrix G of the flank's surface is composed by two cycloidal curves jointed in the P point, on the base circle that is in the same time the rolling circle of the wheel gear (the radius $R_p = R_B$) .

The head of the tooth has the epicycloidal profile E_p, obtained by the exterior rolling of the rolling curve with the center O'_r and the radius R_R', on the rolling circle of the wheel gear with the center in O and the radius R_p, and the foot of the tooth have a hypocycloidal profile H_p, by the interior rolling of the rolling curve with the center O''_r and the radius R''_R on the same rolling circle of the gear.

For a same tooth, the profile of the head in not conditioned by the tooth profile and inverse, so the rolling curves' radius can be different ($R'_R \neq R''_R$). In the same time, everyone of these profiles determines the conjugated profiles of the teeth of the wheel gears with it put into gear.

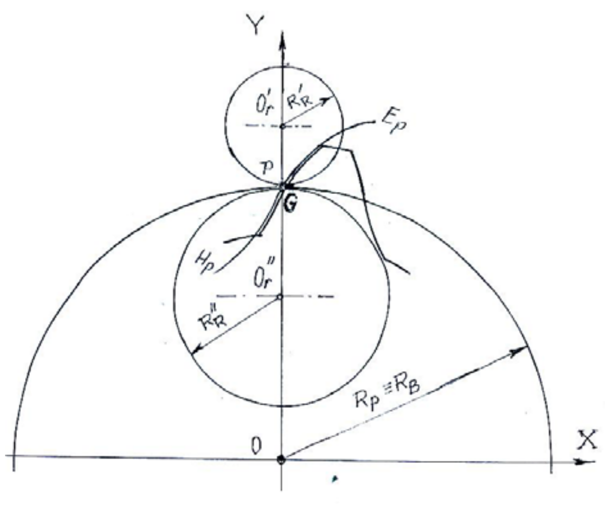

Fig. 2. The G curve of the cycloidal profile.

2.2. The composition of a rotation movement with a translation rectilinear movement made in an axial direction, normal on the rotation plane.

We consider a rotation right cylinder, with the radius R and a circular base in the plane YOZ of the coordinate system. To this cylinder we impart a rotation movement around the OX axe with the frequency n (rot/min). In the same time, an initial generating point M_o of the cylinder makes a rectilinear translation movement on the axial direction of the OX axis, with the speed $\vec{v}_A$, and, after a certain moment, come up in M in its resulting movement, but relative to the cylinder. In order to obtain a relative movement¸ we consider the cylinder fixed by its rotation in an opposite sense with the frequency $n^* = -n$, on the rotation angle Φ (Fig. 3).

By consequence, the relative speed's movement of the generating point M is the vector $\vec{v}_M$, given by the vectorial relationship:

$$\vec{v}_M = \vec{v}_T + \vec{v}_A, \qquad (19)$$

where $\vec{v}_T^{\,*}$ is the vector of the tangent speed from the rotation movement with the $|n^*|$ frequency, having the size:

$$\left|v_T^*\right| = 2\pi R n \ \text{[rot/min]}. \qquad (20)$$

The right linear translation movement is an uniform movement made by the generating point M along the generatrix G of the cylinder, on cyclical cinematic distances with the size p. A cinematic cycle is a rotation of the cylinder and is made in the time T_{CIC} having the size:

$$T_{CIC} = 1/n \ \text{[min]}. \qquad (21)$$

In this case, the size of the axial speed $\vec{v}_A$ from the right linear uniform movement is given by the relation:

$$v_A = \frac{p_E}{T_{CIC}} = p_E n \ \text{[m/min]}. \qquad (22)$$

In this way it results:

$$R_{CCIN} = \frac{v_A}{n} = p_E \ \text{[mm]} \qquad (23)$$

That imposes to the two corposants movements the cinematic coordination in the purpose that the generating point M cross cyclicaly along the generatrix G of the cylinder constant distances p_E.

If the vectorial relation (19) is projected on the coordinated system's axes $OXYZ$, we obtain the sizes of the components of the $\vec{v}_M$ speed on these axes, having the next expressions:

$$\begin{cases} v_{MX} = v_A = p_E \cdot n \\ v_{MY} = -\left|v_{TY}^*\right| = -\left|v_T^*\right| \sin\Phi = -2\pi R n \sin\Phi \\ v_{MZ} = -\left|v_{TZ}^*\right| = -\left|v_T^*\right| \cos\Phi = 2\pi R n \cos\Phi \end{cases} \qquad (24)$$

The differential coordinates dX, dY, dZ of the generating point M are differential spaces crossed respectively with the speeds v_{MX}, v_{MY} and v_{MZ}, in a differential time dT:

$$dX = v_{MX} \cdot dT; \ \ dY = v_{MY} \cdot dT; \ \ dZ = v_{MZ} \cdot dT \ (25)$$

where:

$$dT = d\Phi / 2\pi \cdot n \ . \qquad (26)$$

We replace now the relations (24) and (26) in the relation (25) and we obtain the expressions of the instantaneous components X, Y and Z of the generating point M.

$$X=\int v_{MX}\,dT=\frac{1}{2\pi n}\int v_{MX}\,d\Phi=\frac{p_E}{2\pi}\int d\Phi$$
$$Y=\int v_{MY}\,dT=\frac{1}{2\pi n}\int v_{MY}\,d\Phi=R\int-\sin\Phi d\Phi\,. \quad 27)$$
$$Z=\int v_{MZ}\,dT=\frac{1}{2\pi n}\int v_{MZ}\,d\Phi=R\int\cos\Phi d\Phi$$

Effectuating the integrals we obtain the coordinates X, Y and Z of the point M that represent the parametric equations of the cinematic curve C, described by the generating point M as consequence of the composition of the two movements:

$$\begin{cases} X=\dfrac{p_E}{2\pi}\Phi+C_X \\ Y=R\cos\Phi+C_Y\,, \\ Z=R\sin\Phi+C_Z \end{cases} \qquad (28)$$

where integration constants C_X, C_Y and C_Z are determinate knowing that at $\Phi = 0$, $X = 0$, $Y = R$ and $Z = 0$, resulting $C_X = 0$ $C_Y = 0$ and $C_Z = 0$.

By consequence, the parametric equations of the cinematic curve C are:

$$X=p_0\Phi;Y=R\cos\Phi;Z=R\sin\Phi\,. \ (28)$$

that represent the known parametric equations of the circular helix and in which

$$p_0=p_E/2\pi \qquad (29)$$

is the parameter and p_E is the step of the circular helix.

So, by composing a rotation movement with a right linear translation movement made on an axial direction, normal on the rotation plane, a generating point M generates as a cinematic curve C a circular helix, with the condition to respect R_{CCIN} given by the relation (23) that impose to the two movements the cinematic correlation with the view to the generating point M generate the helical circular trajectory with the pitch p_E and with the radius R given.

Practically, at the generation of the complex real surfaces the curve C as circular helix is generated as directrix D of the surface.

The most extended application of this case of simple movements composition is represented the generation of the directrix D at the generation of the main extended complex surfaces in techniques, the helical cylindrical surfaces with all kind of profiles.

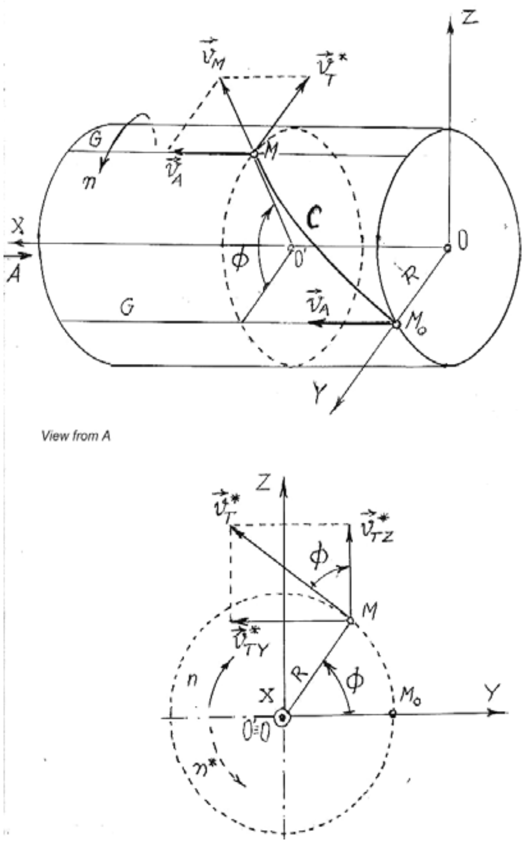

Fig. 3. The cinematics of the spatial composition of the simple movements.

3. CONCLUSIONS

The aspects shown in this paper represent original contributions concerning the cinematic generation of the hypocycloid and of the circular helix, curves used as generatrix G and directrix D at the generation of real complex surfaces of the type cycloidal denture from the watches industry, respectively cylindrical helical surfaces.

The treatment of these aspects by original and unique mathematical demonstrations contributes to the improvement of the cinematic generation theory by simple movement composition, of the plane and space curves used in practice as real complex curves G and D.

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Authors:

PhD. Eng., Ioan-Gheorghe SANDU, Professor, University “Politehnica” of Bucharest, Machines and Production Systems Department,
E-mail: ioan_gheorghe_sandu@yahoo.com;
PhD. Eng., Eugen STRĂJESCU, Professor, University "Politehnica" of Bucharest, Machines and Production Systems Department,
E-mail: eugen_strajescu@yahoo.com