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NUMERICAL MODELING OF WIRES AND FIBERS EXTRUSION

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Abstract: *This paper is an analysis of an extruded wire/fiber cooled by a fluid that flows in the same direction as the workpiece. The governing equations are written in dimensionless form and they are solved using the finite differences method. The obtained temperature and velocity fields are analyzed for different parameters: Peclet number and wire/fiber velocity. This analysis offers a better understanding of the extrusion process and a direct technological application by presenting the variation of the cooling tunnel optimum length as a function of the most important parameter: the Peclet number.*

Key words: *extrusion, wires, fibers, cooling, forced convection, design, industrial system.*

1. INTRODUCTION

The wires and fibers extrusion is a research domain that received great attention in the last decades [1 − 4]. The complete knowledge of the phenomena that are taking place after the wires/fibers extrusion in the cooling tunnel can help us not only to understand the influence of different process or material parameters on the process development but also to design the technological process and installation.

This research subject is of special interest not only for the wires/fibers extrusion but for hot rolling, glass fibers drawing, continuous casting, etc. The general problem is concerning a cylinder with a high initial temperature coming out from the die or furnace and moving horizontally [1, 3, 4] (or vertically [2], in some cases) with a constant velocity. An aiding or opposing fluid flow is generating a forced convection process that accelerates the wire/fiber cooling process.

Steady state [4] or transient [1, 2, 3] approaches were used in order to establish and solve the conservation equations for mass, momentum and energy. The solution of the resulting system of equations is revealing the velocity and temperature fields for the cooling fluid and the workpiece. Thus, in order to fully understand the cooling process, further research on the influence of different parameters on this process was developed.

This paper is making a step forward in the analysis of wires/fibers extrusion. It considers a horizontal cylindrical wire/fiber moving with a constant velocity and a cooling fluid moving in the same direction. Steady-state situation was considered and the finite differences method was used to solve the governing equations in dimensionless form. But, having in view the large difference between the radial and axial dimensions of the set-up, a different approach was used by defining the axial computational coordinate, ξ. After the velocity and temperature fields were found, we proceeded in two directions: a complete analysis of the technological process and the definition of optimum design principles for the industrial installation. The dependence of the cooling system optimum length on the Peclet number and the fluid velocity is discussed in the paper.

This paper, through an original and proper derivation, is bringing the theoretical analysis of the wire/fiber cooling process to the final practical point of determinning the cooling optimum length. It opens the road to further research that could finally establish a design formula of the industrial system.

2. MATHEMATICAL MODEL

Figure 1a presents the cylindrical workpiece of radius r_s, the workpiece that emerges from the extrusion die or furnace. It moves with the constant velocity u_s while the cooling fluid moving in the same direction determines a forced convection process with the initial velocity u_{inf}. The fluid has an initial temperature T_0. The cooling tunnel length is L_t. Neglecting the gravitational force influence, the problem becomes symmetric and only the upper half will be analyzed in a cylindrical coordinates system as presented by Fig. 1.

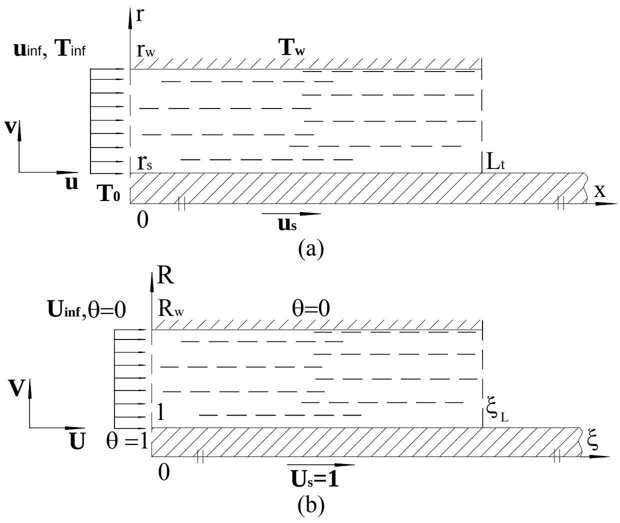


Fig. 1. Wires and fibers extrusion (a); dimensionless problem (b).

The fluid governing equations for mass, momentum and energy as well as the workpiece governing equation for the energy are considered in vorticity (ω) – stream function (ψ) formulation:

$$u = \frac{1}{r}\frac{\partial\psi}{\partial r}\,;\; v = -\frac{1}{r}\frac{\partial\psi}{\partial x}\,;\; \omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial r}\,, \tag{1}$$

where (u, v) is the fluid velocity field with its two components on the x and r axis.

The governing equations and the whole problem are transformed in dimensionless form (Fig. 1, b) using the following dimensionless variables [1]:

$$X = \frac{x}{r_s}\,; R = \frac{r}{r_s}\,; U = \frac{u}{u_s}\,; V = \frac{v}{u_s}\,;$$
$$\Psi = \frac{\psi}{u_s r_s^2}\,; \Omega = \frac{\omega r_s}{u_s}\,; \tau = \frac{t u_s}{r_s}\,; \theta = \left(\frac{T - T_\infty}{T_0 - T_\infty}\right);$$
$$Pe = \frac{u_s r_0}{\alpha_s}\,; Pr = \frac{\nu}{\alpha}\,; Re = \frac{u_s r_s}{\nu}\,, \tag{2}$$

where t is time, α_s and α are the thermal diffusivities of the workpiece and fluid, respectively, ν is the fluid kinematic viscosity, $P_e = u_s r_s/\alpha_s$, $P_r = \nu/\alpha$ and $R_e = u_s r_s/\nu$ are Peclet, Prandtl and Reynolds number, respectively. Having in view the difference between the x and r dimensions and a higher precision for the solution, we used the transformation $X = e^{\xi} - 1$ that defines the (ξ, R) computational domain. The governing equations become:

$$\frac{1}{Re^{2\xi}}\frac{\partial^2\Psi}{\partial\xi^2} + \frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\Psi}{\partial R}\right) = -\Omega + \frac{\partial\Psi}{\partial\tau}\,; \tag{3}$$

$$\frac{\partial\Omega}{\partial\tau} + \frac{1}{e^{\xi}}\frac{\partial(U\Omega)}{\partial\xi} + \frac{\partial(V\Omega)}{\partial R} =$$
$$= \frac{1}{Re}\left[\frac{1}{e^{2\xi}}\frac{\partial^2\Omega}{\partial\xi^2} + \frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial(R\Omega)}{\partial R}\right)\right]; \tag{4}$$

$$\frac{1}{e^{\xi}}\frac{\partial(U\theta)}{\partial\xi} + \frac{1}{R}\frac{\partial(RV\theta)}{\partial R} =$$
$$= \frac{1}{Re\cdot Pr}\left[\frac{1}{e^{2\xi}}\frac{\partial^2\theta}{\partial\xi^2} + \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta}{\partial R}\right)\right]; \tag{5}$$

$$\frac{1}{e^{\xi}}\frac{\partial\theta}{\partial\xi} = \frac{1}{R}\left[\frac{1}{e^{2\xi}}\frac{\partial^2\theta}{\partial\xi^2} + \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta}{\partial R}\right)\right]; \tag{6}$$

The boundary conditions imposed in the computational domain express the physical realities:

- at the *right boundary* of the domain, $(X = L)$, $\xi = \xi_L$, small variations of the variables are imposed [1]:

$$\frac{\partial\Psi}{\partial\xi} = \frac{\partial^2\Omega}{\partial\xi^2} = \frac{\partial\theta}{\partial\xi} = 0\,. \tag{7}$$

- at the *left boundary* of the domain $(X = 0)$, $\xi = 0$, the boundary conditions for the fluid and workpiece correspond to the entrance region:

$$\Psi = \frac{U_{inf}R^2}{2}\,;$$

$$\Omega = 0\,;$$

$$\theta = 0 \text{ for the fluid;}$$
$$\theta = 1 \text{ for the solid} \tag{8}$$

- at the *solid/fluid interface*, $R = 1$, no slip and no penetration conditions are imposed for the velocity field while the continuity of temperature and heat flux are imposed for the temperature field :

$$\Psi = 0\,;$$

$$\Omega_{1,j} = \frac{1}{R_{surf}} - \frac{2\lfloor\Psi_{2,j} - \Psi_{1,j} - R_{surf}\cdot\Delta R\rfloor}{R_{surf}\cdot(\Delta R)^2}\,, \quad [1];$$

$$\theta_{N,j} = \frac{\frac{1}{\Delta R_s}}{\frac{1}{\Delta R_s} + \frac{K}{\Delta R_f}}\theta_{N-1,j} + \frac{\frac{K}{\Delta R_f}}{\frac{1}{\Delta R_s} + \frac{K}{\Delta R_f}}\theta_{N+1,j}\,, \tag{9}$$

where: the first subscript is the line of the grid and the second subscript is the column number; ΔR_f is the space on the radial network in the fluid domain: $R_w - 1 = (N-1)\Delta R_f$, while ΔR_s is the space on the radial network in the workpiece domain: $1 = (N-1)\Delta R_s$; K is the ratio of the fluid and solid thermal conductivities.

- at the tunnel wall (*the upper boundary*), $R = R_w$, the no slip and no penetration conditions are imposed for the velocity field while a constant temperature case is considered for the temperature field:

$$\Psi = \frac{U_{inf}R_w^2}{2}\,;$$

$$\Omega_{N,j} = \frac{2\lfloor\Psi_{N-1,j} - \Psi_{N,j}\rfloor}{R_w\cdot(\Delta R_f)^2}\,, \quad [1];$$

$$\theta = 0\,. \tag{10}$$

- at the *symmetry axis,* $R = 0$, the boundary condition for temperature express the symmetry approach:

$$\frac{\partial\theta}{\partial R} = 0\,. \tag{11}$$

Solving the governing equations (3 – 6) with the boundary conditions (7 – 11) allows the analysis of the velocity and temperature fields and determines the optimum design for the technological installation.

3. NUMERICAL METHOD

The finite differences method was used to solve the governing equations [5]. The finite differences were centered in the computation domain. The exception was made by the boundary points where forward or backward finite differences were used. An iterative process solved the velocity field calculating the vorticity and the stream function fields and, further, the temperature field was found. Crank-Nicolson method was used for Ψ and θ solutions while "alternating direction implicit" method was used for Ω.

Having in view the cooling tunnel length, L_t, and the workpiece radius, r_s, a grid with 50 points on both x and r (50 points for the fluid and another 50 points for the fluid) direction was used. A transient term was used for solving Ψ and Ω with a time step $\Delta\tau = 10^{-7}$.

4. RESULTS AND DISCUSSIONS

Intense research was performed trying to determine the parameters that have a significant influence on the cooling process and on the industrial installation design. The physical properties considered in this study are presented by Table 1 while the parameters analyzed were: the tunnel radius, R_w, the tunnel wall temperature, T_w, the fluid/solid thermal conductivities ratio, K, the fluid velocity, U_{inf}, etc.

Fig. 2 presents the isotherms (Fig. 2a) and the streamlines (Fig. 2b) for the following parameters: P_e = 100, R_e = 20, R_w = 6, U_{inf} = 2. From Fig. 2a, we can notice that a high Peclet number, a small wire/fiber thermal diffusivity, leads to a slow cooling of the workpiece. A comparison can be made studying the isotherms (Fig. 3a) and the streamlines (Fig. 3b) obtained by changing only Peclet number, P_e = 0.6. The influence of this parameter on the temperature field underlines the dependence of the cooling tunnel length on Peclet number.

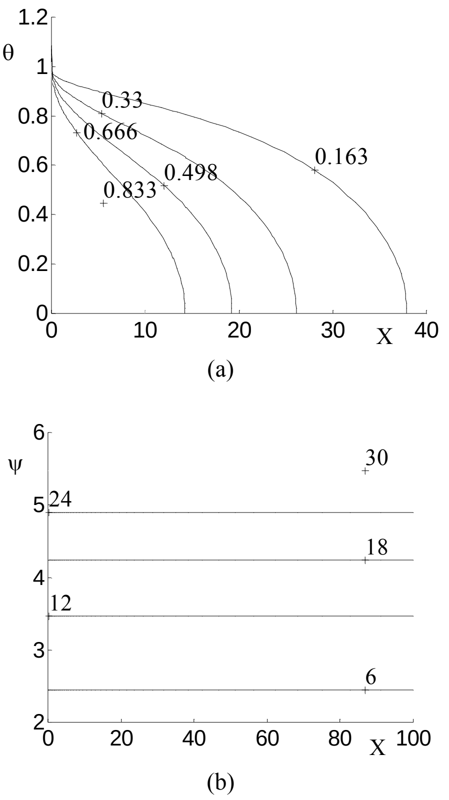


Fig. 2. Isotherms (a) and streamlines (b) for P_e = 100, R_w = 6, U_{inf} = 2.

Table 1

Physical properties

Solid/ Liquid	**Thermal diffusivity**	**Thermal conductivity**	**Kinematic viscosity**
Teflon	0.001cm^2/s	0.23W/m/K	-
Water	0.001cm^2/s	0.59W/m/K	0.01cm^2/s

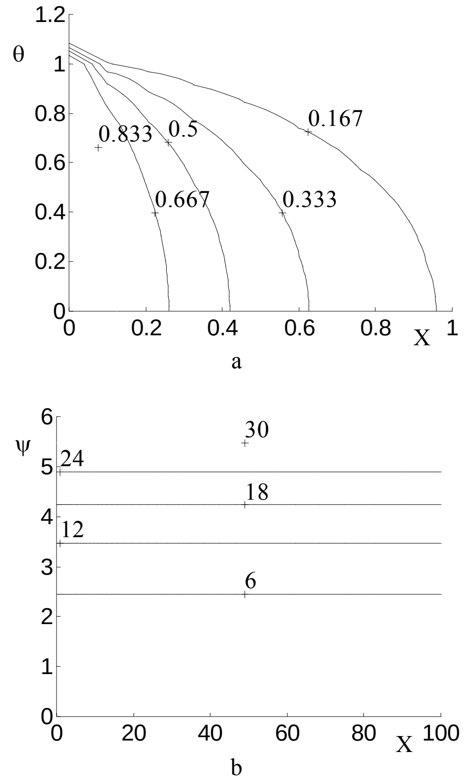


Fig. 3. Isotherms (a) and streamlines (b) for P_e = 0.6, R_w = 6, U_{inf} = 2.

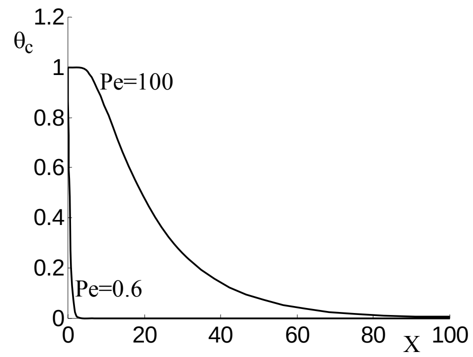


Fig. 4. Centerline temperature variation, θ_c, for two values of Peclet number: 100 and 0.6. R_w=6, U_{inf}=2.

This aspect is clearly emphasized by Fig. 4 that presents the centerline wire/fiber temperature variation for two values of Peclet number: 100 and 0.6. A higher Peclet number (in other words, a smaller workpiece thermal diffusivity and/or a higher wire velocity and/or a bigger wire radius) implies a slower cooling process. Consequently, the value of the cooling tunnel length of the industrial system must be higher.

Defining L_{opt} as the cooling tunnel optimum length at the end of which the temperature θ has a value of 0.01, Fig. 5 presents the L_{opt} variation as a function of Peclet number. Evidently, L_{opt} increases as Peclet number, P_e, increases (the wire/fiber velocity and/or radius increases and/or the workpiece thermal diffusivity decreases).

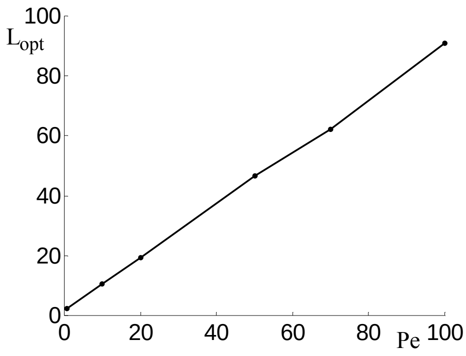


Fig. 5. The variation of the cooling tunnel optimum length, L_{opt}, as a function of Peclet number, P_e; $R_w = 6$, $U_{inf} = 2$.

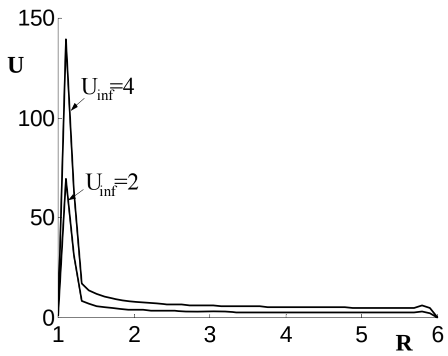
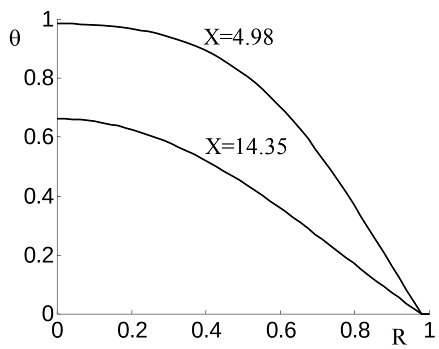
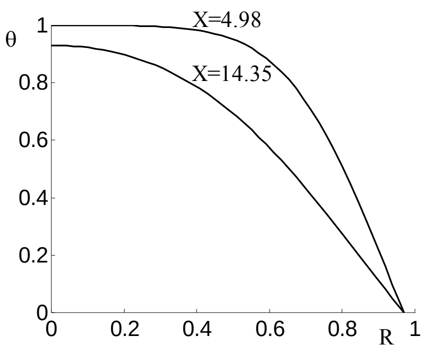


Fig. 6. Horizontal velocity variation, U, in a cross-section, for two values of fluid velocity, $U_{inf} = 2$ and 4; $P_e = 100$, $R_w = 6$.



(a)



(b)

Fig. 7. Temperature variation in two workpiece cross-sections: $X = 4.98$ and $X = 14.35$, for $U_{inf} = 2$ (a) and $U_{inf} = 4$ (b); $P_e = 100$, $R_w = 6$.

The influence of the initial fluid velocity, U_{inf}, on the optimum length of the cooling system, L_{opt}, was also studied using the developed model. If the influence of the initial fluid velocity on the velocity field can be analyzed using Fig. 6, its influence on the workpiece temperature field profile is presented by Fig. 7 for two of its values: $U_{inf} = 2$ (Fig. 7a) and $U_{inf} = 4$ (Fig. 7b). Even if its influence can be noticed for the center workpiece temperature evolution, at two positions on the X axis: 4.98 and 10.35, in the end, the variation of the cooling system optimum length on the initial fluid velocity is not significant and Peclet number remains the most influential parameter.

5. CONCLUSIONS

This work presents the numerical model of wires/fibers extrusion. The stationary case is considered for a cooling fluid flowing in the same direction as the workpiece in a dimensionless vorticity-potential formulation;

The governing equations are solved using the finite differences method through an iterative process in an original analytical and numerical derivation;

The optimum length of the cooling tunnel is defined and analyzed as a function of different material and process parameters. The Peclet number has the most significant influence on the optimum length, L_{opt};

This analysis offers solutions for the optimum design of the wires/fibers extrusion system and can be used further to define a design formula for the cooling system optimum length.

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