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ASSISTED DESIGN OF THE WORM-GEARING USING RIGIDITY VARIATION CRITERION

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Abstract: *The paper presents some aspects regarding the achievement of a worm-gearing tooth with a constant rigidity. When we want a constant rigidity at which the values for amplitude are as low as possible, we refer to performing a rigidity curve with short jumps, as flattened as possible. The theory of vibrations shows that a rigidity characteristic with jumps represents an interne source for the excitation of the gear drive. So, reducing the amplitude it is obtained a smooth meshing, because the magnitude of the vibrations and noise will become lower. It is obvious that the reduction of amplitude of rigidity curve is another mode to obtain high rigidity for the gearing tooth.*

Key words: *optimal design, worm-gearing, numerical method, amplitude, rigidity, computer simulation, meshing.*

1. AMPLITUDE OF RIGIDITY OF WORM-GEARINGTOOTH

The elasticity characteristic represents the variation of rigidity of the worm-gearing tooth depending on the rolling angle ($j \cdot \Delta\varphi$), where j is the rolling angular parameter [3].

It is cvasisinusoidal curve with the high jumps when a tooth binds or recesses (Fig. 1) and it was developed by the authors by means of the original and special software for determining the rigidity [1].

This program can be adopted for any kind of cylindrical worm-gear drives, as well as for spur-gear drives and bevel-gear drives [2].

The investigation of the elasticity characteristic is very important for the study of an elastic system, such as: gearing, linkage, machine-tool [9].

Hence, the introduction of this concept contributes to the completion of the used gearing study and it leads to increase of the gearing tooth rigidity.

The influence of the geometrical parameters on the rigidity was obtained by means of the computerized simulation [8].

It was applied to 150 worm-gear drives and we can present the following conclusions:

The influence of the geometrical parameters on the rigidity of the worm-gearing tooth was studied in [1] by means of computer simulation.

Hence, the amplitude of the rigidity reduces if:

- number of gear teeth z_2 increases (Table 1, Fig. 2);
- profile angle α reduces (Table 2, Fig. 3);
- diametral quotient q reduces (Table 3, Fig. 4 and 5);
- radius of profile curvature R increases (Table 4, Fig. 4 and 5).

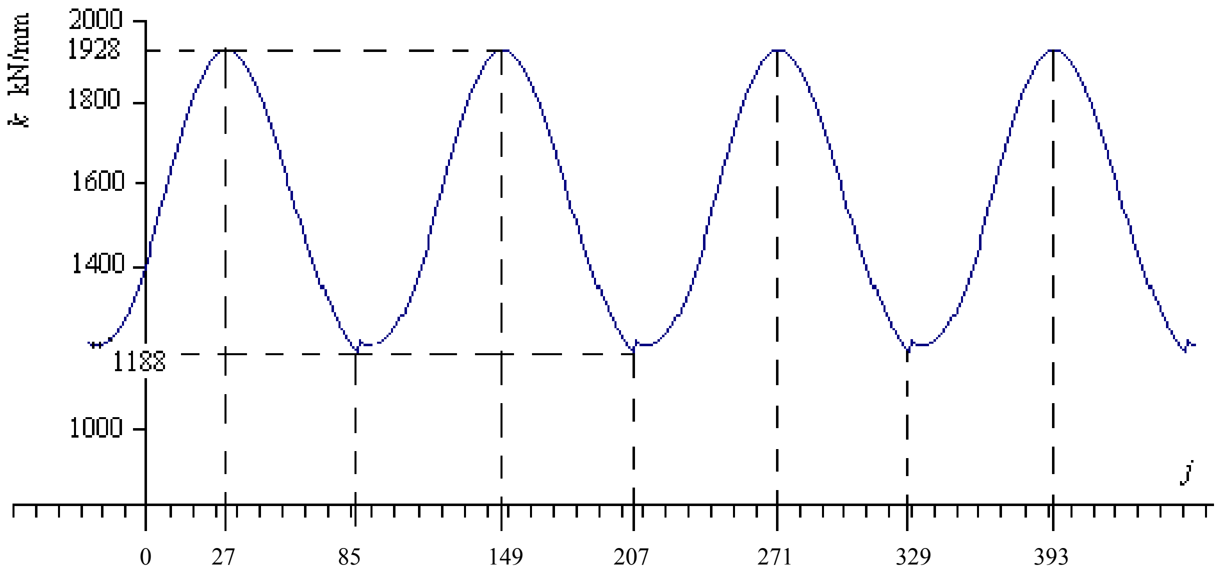

Fig. 1. Elastic characteristic of the worm-gearing tooth.

Table 1

Influence of number of gear teeth z_2 on amplitude

z_2	Maximum Rigidity [kN/mm]	Minimum Rigidity [kN/mm]	Amplitude of Rigidity [kN/mm]
53	2267.385	1215.140	1052.245
80	1727.633	1132.201	595.432
90	1581.896	1079.696	502.200
114	1586.144	1195.729	390.415
169	1055.990	853.826	202.164

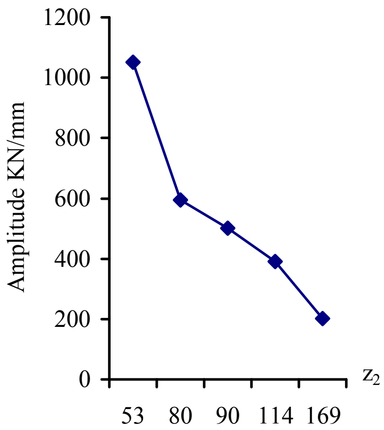

Fig. 2. Amplitude of rigidity depending on z_2.

Table 2

Influence of profile angle on amplitude

α[°]	Amplitude of rigidity [kN/mm] for R=3·m_x and q=7	Amplitude of rigidity [kN/mm] for R=4·m_x and q=7
10	439.768	300.690
15	487.297	365.662
20	537.490	449.511
25	590.143	525.614
30	632.843	593.135

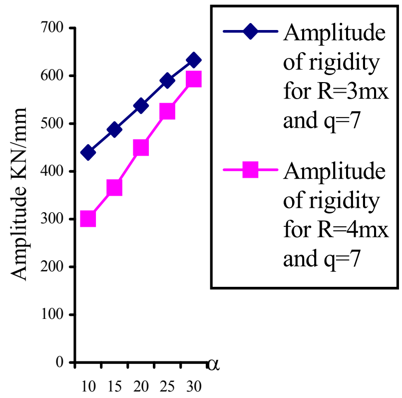

Fig. 3. Amplitude of rigidity depending on profile angle.

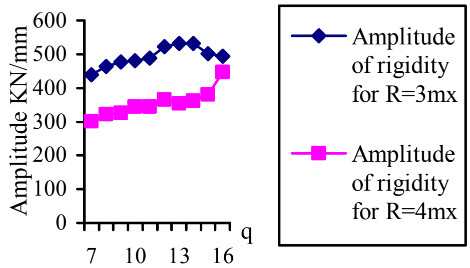

Fig. 4. Amplitude depending on q, R, $\alpha = 10°$.

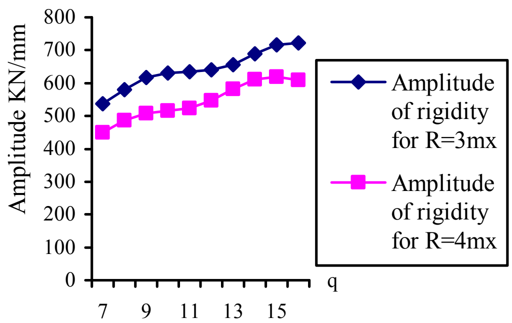

Fig. 5. Amplitude depending on q, R, $\alpha = 20°$.

Table 3

Influence of diametral quotient and radius of profile curvature on amplitude

q	m_x [mm]	Amplitude of rigidity [kN/mm] for $R = 3{\cdot}m_x$, $\alpha = 10°$	Amplitude of rigidity [kN/mm] for $R = 4{\cdot}m_x$, $\alpha = 10°$	Amplitude of rigidity [kN/mm] for $R = 3{\cdot}m_x$, $\alpha = 20°$	Amplitude of rigidity [kN/mm] for $R = 4{\cdot}m_x$, $\alpha = 20°$
7	5.20	439.768	300.690	537.490	449.511
8	5.16	463.877	320.888	580.426	486.240
9	5.12	476.271	326.327	617.484	508.882
10	5.08	480.393	343.617	631.263	515.258
11	5.04	488.338	345.394	635.026	523.156
12	5	521.871	365.301	640.248	546.103
13	4.96	532.208	354.449	656.227	581.251
14	4.92	530.959	361.054	689.048	611.675
15	4.88	500.997	381.046	716.582	618.388
16	4.84	493.212	446.091	722.285	609.272

Table 4

Influence of radius of profile curvature, which depends on axial module, on rigidity

z_2	m_x [mm]	$R = 3{\cdot}m_x$, $q = 7$, $\alpha = 20°$				$R = 4{\cdot}m_x$, $q = 7$, $\alpha = 20°$			
		Max Rigidity	Min. Rigidity	Med. Rigidity	Amplitude of Rigidity	Max Rigidity	Min. Rigidity	Med. Rigidity	Amplitude of Rigidity
53	10.5	1533.2	470.56	1001.8	1062.6	1608.6	721.91	1165.2	886.70
114	5.2	817.64	280.15	548.89	537.49	861.19	411.68	636.43	449.51

2. OPTIMIZATION USING RIGIDITY VARIATION CRITERION

Studding the influence of geometrical parameters on amplitude of rigidity of the worm-gearing tooth [4, 6], we performed a design algorithm (Fig. 6) which ensures a constant rigidity for the gear drive.

Generally, the methodology regarding the achievement of amplitude as low as possible is:

- increasing number of gear teeth z_2;
- adoption of minimum diametral quotient q;
- increasing radius of profile curvature R;
- reduction of worm profile angle α.

2.1. Case of the chosen number of the gear teeth z_2

In the design phase, number of gear teeth is adopted as great as possible. There are two situation:

a) radius of profile curvature has an independent value;

In this case, we established that if z_2 increases three times, then amplitude increases 5.2 times [3].

Consequently, number of gear teeth is the most influent parameter. But, its influence reduces while increasing z_2 is lower.

b) radius of profile curvature depends on axial module m_x ($R = a \cdot m_x$, where a is constructive parameter).

Kipping the same overall size, if number of gear teeth z_2 increases, then: axial module reduces, radius of profile curvature reduces, finally resulting lower rigidity (Table 4). Radius of profile curvature is the second parameter which must be taken into consideration in order to reduce amplitude of rigidity. This parameter must be increased.

As may be seen in the Table 3, the greatest influence of radius of profile curvature is in the case of the lowest value for profile angle α = 10°.

From the developed study [5], we can draw the conclusion that diametral quotient must be reduced much as much possible in order to reduce the amplitude of rigidity. This parameter has a low enough influence on amplitude, but it represents the second parameter, after z_2, which is adopted in the design phase.

Studding the influence of profile angle on the reduction of amplitude, we established that the value of this parameter must be as low as possible. On the basis of those established, we present the hierarchy of geometrical parameters, as follows:

a) radius of profile curvature has an independent value:

1) number of gear teeth z_2;
2) radius of profile curvature R;
3) profile angle α;
4) diametral quotient q.

b) radius of profile curvature depends on axial module m_x:

1) radius of profile curvature R;
2) number of gear teeth z_2;
3) profile angle α;
4) diametral quotient q.

2.2. Case of the requested number of gear teeth z_2

For a requested number of gear teeth z_2, referring to results of the analyzed cases up to now, the hierarchy of geometrical parameters is:

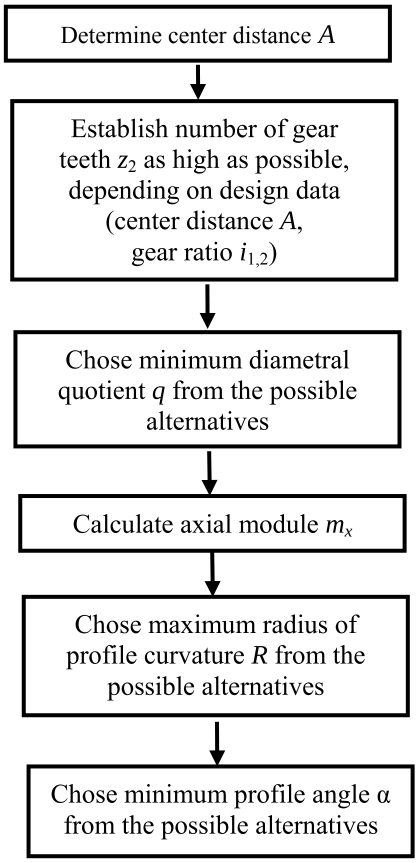

Fig. 6. Design algorithm.

1) radius of profile curvature R;
2) profile angle α;
3) diametral quotient q.

2.3. Design algorithm

In conclusion, to obtain a rigidity characteristic with amplitude as low as possible for the worm-gearing tooth with circular profile (Fig. 7), we recomand using the following algorithm:

Fig. 7 presents the axial section of the worm with constant pitch, having a circular arch profile with center O_1 for the right flank and O_2 for the left flank [7], where:

$$\begin{aligned} &u = 1.25 \cdot m / \cos\alpha; \\ &p = m/2; \\ &b = \pi \cdot m/4 - 1.25 \cdot \tan\alpha; \\ &R = \sqrt{a^2 + u^2}. \end{aligned} \quad (1)$$

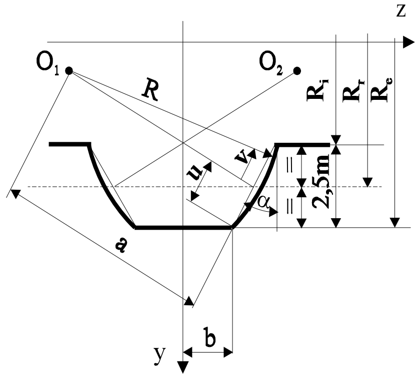

Fig. 7. Worm flank geometry.

3. CALCULUS EXEMPLE FOR OPTIMAL DESIGN USING RIGIDITY VARIATION CRITERION

The known design data are:
- center distance $A = 315$ mm;
- gear ratio $i_{1,2} = 114/1$.

So, results $z_2 = 114$.

We follow the algorithm established in this paper, at section **2.3**:

Chose diametral quotient as low as possible, $q = 7$;

Calculate axial module:

$$m_x = 2\,A\,/\,(q + z_2),$$
$$m_x = 2\cdot315\,/\,(7 + 114) = 5.2066 \text{ mm.} \quad (2)$$

We adopt standardized axial module, $m_x = 5$ mm.

If $A = 315$ mm and $z_2 = 114$, then diametral quotient becomes $q = 12$. Anticipating, this change of diametral quotient will attract increasing amplitude of the rigidity (Table 3) from 439.768 kN/mm ($q = 7$, $R = 3m_x$) to 521.871 kN/mm ($q = 12$, $R = 3m_x$), the deference being 82.103 kN/mm.

If $R = 4m_x$, increasing amplitude will be lower, from 300.690 kN/mm to 365.301 kN/mm (Table 3). In this case the deference is 64.61 kN/mm.

Another possibility would be, after the adopting of axial module $m_x = 5$ mm and diametral quotient $q = 7$, the increasing number of gear teeth z_2 ($A = 315$ mm), but the gear ratio deviation must be less than 3% ($\Delta i_{1,2} < 3\%$).

So, if:

$m_x = 5$ mm, $q = 7$, $z_2 = 119 \Longrightarrow \Delta i_{1,2} = 4.3\%$ (doesn't agree);

$m_x = 5$ mm, $q = 8$, $z_2 = 118 \Longrightarrow \Delta i_{1,2} = 3.5\%$;

$m_x = 5$ mm, $q = 9$, $z_2 = 117 \Longrightarrow \Delta i_{1,2} = 2.63\%$.

The last solution would allow to obtain higher reduction of the amplitude than in the case of $q = 12$ and $z_2 = 114$, because z_2 increases and q reduces.

Radius of profile curvature must be as large as possible, especially the value for axial module is low: $R = 4m_x = 4\cdot5.2066 = 20.82$ mm.

We recommend that in the case of the low value for axial module, to adopt a larger radius of profile curvature, limited by the technological procedures of the circular profile manufacturing.

The value for profile angle must be as low as possible, $\alpha = 10°$.

4. CONCLUSIONS

Based on the computerized simulation of the meshing and the influence of geometrical parameters on the rigidity of worm-gearing tooth, a new approach has been developed for design of the worm gear drives.

The proposed methodology improves the performances of worm-gearing, what is very important for the accuracy of the machine-tool or robot linkages.

The basic idea of the approach to obtain high rigidity is to take into account the rigidity variation criterion in the design phase.

The study presents the main steps of the proposed algorithm and a calculus example.

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