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METHOD TO DETERMINEUNCERTAIN CONFIGURATIONS OF ARTICULATED RECONFIGURABLE ROBOT

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Abstract: *In this paper we present a solution to the inverse kinematics problems for articulated reconfigurable robot (ARR). The inverse kinematics is a method by which for a specified hand position the robot' variable joint angles and variable link offset distances can be found. The method presented in this paper is general in that it will apply to robots with any link lengths, offset lengths, twist angles and joint angles and in that it can be applied to robots with revolute, cylindrical or prismatic joints. The equations used are derived using power products and dialytic elimination. Because result four configurations the choice of optimal solution for articulated reconfigurable robot to make using the criterion of kinematic accuracy for robots. Finally a numerical example is used to demonstrate the feasibility of the approach.*

Key words: *articulated reconfigurable robot, inverse kinematics, mechanical hand, robot module.*

1. INTRODUCTION

A self-reconfigurable robot belongs to a class of robotic system than can change its shape and functionality without external help. Such robots are composed of many robotic modules where the different types of the modules are much less than the number of modules. These modules can have their own power supply, actuators, sensors, communication system and computational capabilities. Many configurations, such as a manipulator, a crawler, a legged robot or other robotic configurations can be built by the combination of (identical) modules.

The Articulated Reconfigurable Robot (ARR) are made of a base rotation module and k articulated arm module, which can be positioned straight or perpendicular (Fig. 1.) of this k - 1 module are identically and the last is different because of the end-effector catching system. The maximum rotation angle θ_2 of each articulated arm module compare with the previous one is 180º during the actuation. For reconfiguration the angle θ_3 between two successive modules is 90º [4, 6].

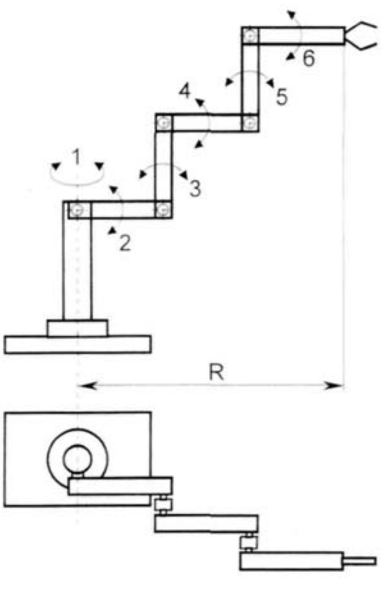

Fig. 1. Articulated Reconfigurable Robot.

According to Fig. 2 ARR can be considered Planar Robot which was added one base rotation degree of freedom (θ_1 = 360º).

One of the problems that have to be solved at the reconfiguration of this type of robot is the choice of the optimal kinematic solution. For example, in Fig. 2 are presented four configuration solutions of an ARR, made of six modules: base rotation module and five articulated arm modules (solutions *a*, *b*, *d* and *e*) which have the same positioning of the mechanical hand in cylindrical work volume. Solution techniques for the geometric design problem may be classified into two categories: exact synthesis and approximate synthesis.

Exact synthesis methods result in mechanisms and manipulators which guide a rigid body exactly through the specified precision points. Solutions in the exact synthesis exist only of the number of independent design equations obtained by the precision points is less than or equal to the number of design parameters.

If the number of design equations is less than the number of design parameters, then several of the design parameters can be regarded as free choices and their values can be selected arbitrarily so that a well-determined system is obtained.

The number of precision point that may be prescribed for a given mechanism or an industrial robot is limited by the system type and the number of design parameters that are selected to be free choices [1].

The number of precision points can be calculated using Tsai and Roth's formula [2, 3].

In approximate synthesis, using an optimization algorithm, a mechanism is found that, although not guiding a rigid body exactly through the desired phases, it optimises objective function defined using information from all desired phases.

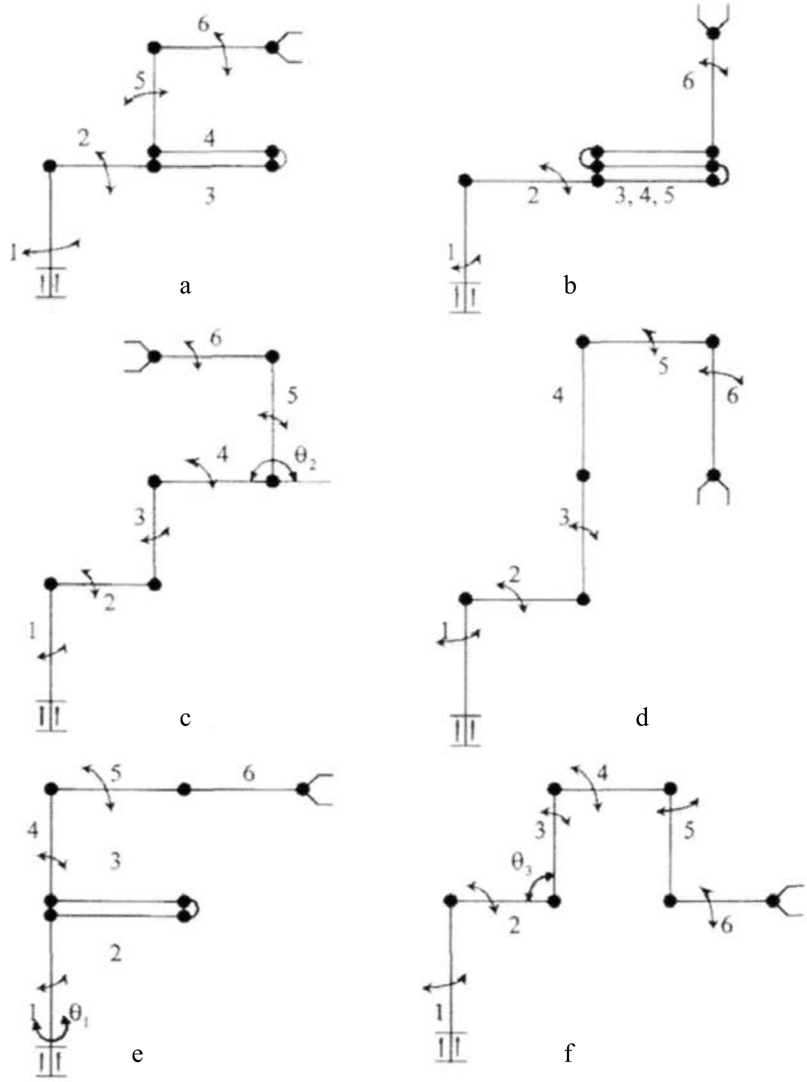

Fig. 2. Kinematics structure of ARR.

2. FORMULATION OF THE PROBLEM

The nomenclature used in this paper is the same as Tsai and Morgan (1985). The 6 R manipulator consists of seven links, the base link is number one and the hand is number seven. A fixed coordinate axis system (number one) is positioned at the first revolute pair with its z axis coincident with the axis of rotation and is mounted on the fixed link. Each link has an axis system mounted on the distal end again with the z axis coincident with the axis of rotation of the revolute pair. Each axis system is numbered the same as the link to which it is attached.

The relationship between axis systems $i + 1$ and i can be given by the following 4×4 transformation matrix:

$$A_i = \begin{bmatrix} c_i & -s_i \cdot \lambda_i & s_i \cdot \mu_i & a_i \cdot c_i \\ s_i & c_i \cdot \lambda_i & -c_i \cdot \mu_i & a_i \cdot s_i \\ 0 & \mu_i & \lambda_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (1)$$

where, $s_i = \sin\theta_i$; $c_i = \cos\theta_i$; θ_i = joint angle at joint i; $\lambda_i = \cos\alpha_i$; $\mu_i = \sin\alpha_i$; α_i = twist angle between axis of joint i and joint $i + 1$; a_i = link length of link $i + 1$; a_i = offset at joint i.

The end effector of a 6 link serial manipulator thus can be found by the matrix multiplication.

$$A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6 = A_{hand}, \quad (2)$$

where,

$$A_{hand} = \begin{bmatrix} L_x & M_x & N_x & P_x \\ L_y & M_y & N_y & P_y \\ L_x & M_z & N_z & P_z \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (3)$$

The vectors L, M and N describe the rotation of the coordinate system at the hand relative to the base coordinate system. The P vector is a vector from the origin of the base coordinate system to the origin of the hand coordinate system.

The forward kinematics is a somewhat easy problem. For 6 R serial robot ARR, given α_i, a_i, d_i, and specified joint variables θ_i, the position of the hand can be found be matrix multiplication (2). The inverse kinematics is much more complicated. Here, the quantities α_i, a_i, λ_i, μ_i, $i = 1,\ldots, 6$, appearing in the matrices on the left hand side of equation (2) are all known. The unknown quantities are θ_i, $i = 1,\ldots, 6$, and the above matrix equation must be solved for them.

The matrix equation (2) is equivalent to 12 scalar equations of these only 6 equations are independent, because the sub-matrix comprised of the first 3 rows and columns is orthogonal.

We perform operations on these multivariate equations and eliminate all but one variable, thus reducing to the problem to the solutions of a simple equation in one variable.

This paper uses the solution method developed by Raghavan and Roth (1993) for solving the inverse kinematics of 6R series chain manipulators of general geometry [5].

Raghavan and Roth propose three methods for solving systems of polynomial equations. Dialytic Elimination, Polynomial Continuation and Grobner bases. In our experience the Dialytic Elimination procedure is very useful for small problems with up to 6 variables. It gives the analyst, algebraic and geometric insight into the problem by permitting studies of the solution space as a function of a linkage's structural parameters. When combined with the eigen value formulation, the Dialytic Elimination procedure often yields computationally-fast solutions algorithms.

There are six steps in using the Dialytic Elimination method to solve a set of nonlinear equations:

1. Rewrite equations with one variable suppressed.
2. Define the remaining power products as new linear homogenous unknowns.
3. Obtain new linear equations so as to have as many linearly independent homogenous equations as linear unknowns.
4. Set the determinant of the coefficient matrix of the set of equations formed from steps 2 and 3, to zero and obtain a polynomial in the suppressed variable.
5. Determine the roots of the characteristic polynomial or the eigen values of the matrix.
6. Substitute (one of the roots or eigen values) for the suppressed variable and solve the linear system for the remaining unknowns. Repeat this for each value of the suppressed variable.

Taken into consideration the coordinates $x_A = 0$; $y_A = 900$ mm; $z_A = 420$ mm, of the mechanical hand for the robot from Fig. 2, there have been obtained after applying the procedure mentioned above, a number of 11 possible variants (Fig.3) of configuration.

The problem which it is posed: which is the optimum possibility of reconfiguration? The choice of this one can be made taking into consideration some criteria; we mention the followings:

- imposed kinematic precision criteria;
- maximum stiffness criteria;
- mechanical hand accessibility in the working point criteria.

For example, a robot's joint parameters (stiffness and damping) are essential for establishing an accurate dynamic model for a modular robot. Identifying the physical parameters of a modular robot is much more complicated than for a general industrial robot with few joints. Yet parameter identification is a prerequisite step in accurate dynamic modelling and effective control of modular robots.

A modular robot is composed of many modules connected via joints and the dynamic characteristics of the elastic joints have an important influence on the behaviour of the whole robot. This is because there are more joints in a modular robot than in a general robot. The modules are rigid, while the joint connections are elastic and weak. Robot vibrations will not only induce errors in the robot's operational accuracy and repeatability, but also speed up the wear of its parts. If resonance occurs, the robot can not be controlled to perform motions over a large range unless the vibration is controlled.

3. KINEMATIC PRECISION CRITERIA

An optimised articulated reconfigurable robot (ARA) is proposed in this paper. It has potential application in the biomedical field such as camera placement and light positioning for surgeon assistance.

For choosing the optimum variant from the 11 presented configurations, the kinematic precision criteria have been used.

The kinematic error of an industrial robot is define as the linear and angular error of the position of the mechanical hand in rapport with the programmed position.

The linear kinematic error of an industrial robot can be determined as follows. It is known the programmed position of the mechanical hand in the space (the coordinates' system) defined by the coordinates' system $O_n x_n y_n z_n$ (Fig. 4). The real position of the mechanical hand is determined in the system $O'_n x'_n y'_n z'_n$.

The vector $\overline{\Delta r} = \overline{O_n \cdot O_n'}$ determines the linear error of an industrial robot.

As $\bar{r} = \bar{r}(q_i)$ is the function of the generalised coordinate q_i, then:

$$\Delta \bar{r} = \sum_{i=1}^{n} \frac{\partial \overline{r_i}}{\partial q_i} \cdot \Delta q_i \,, \tag{4}$$

in which Δq_i is the error of the generalised coordinate q_i.

In the matrix shape the coordinates of the point O_n can be written as:

$$[x_n] = M_1 \cdot M_2 \cdot M_3 \cdot \ldots \cdot M_{n-1} \cdot [x_{n-1}] \tag{5}$$

in which $[x_{n-1}]$ is the column of the coordinates of the point O_n in the system $O_{n-1}\, x_{n-1}\, y_{n-1}\, z_{n-1}$ attached to the n - 1 element of the industrial robot; $[x_n]$ is the column of the coordinates of the point O_n and the reference fixed system $Ox_0y_0z_0$. M_i is the 4[th] grade matrix for passing from the system $O_i\, x_i\, y_i\, z_i$ to the system $O_{i-1}\, x_{i-1}\, y_{i-1}\, z_{i-1}$ (i = 1, …, n).

Taking the differential from equation (5) it will be obtained the expression of the linear kinematic error as matrix:

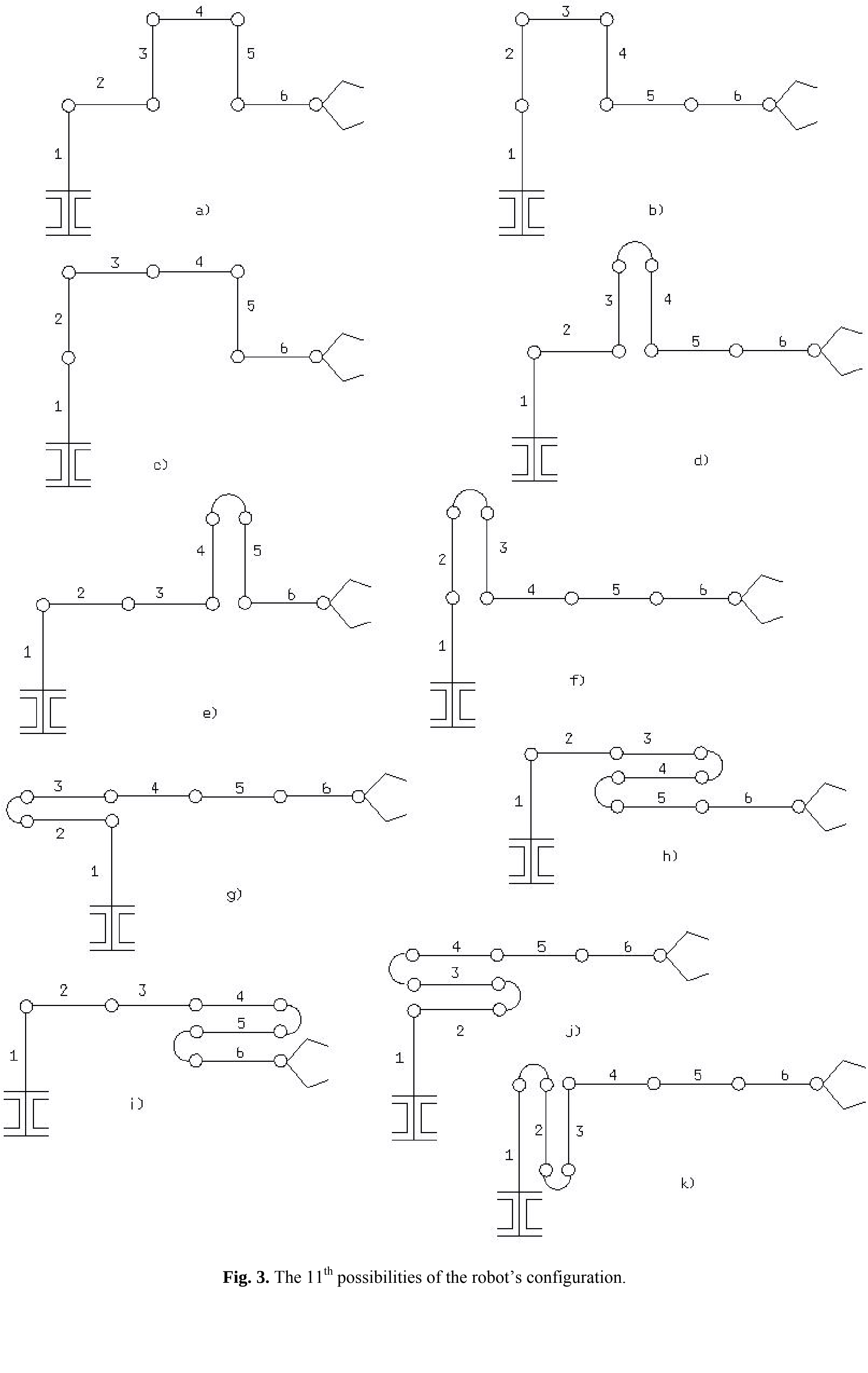

Fig. 3. The 11th possibilities of the robot's configuration.

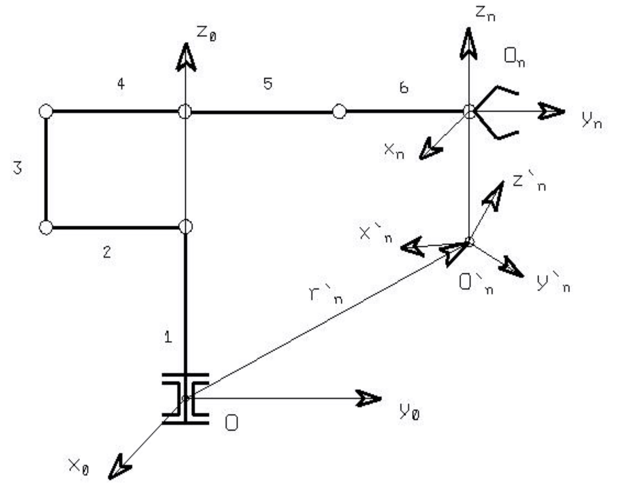

Fig. 4. The programmed position of the mechanical hand.

$$\Delta x_n = \Delta M_i \prod_{i=2}^{n-1} M_i \cdot [x_{i-1}] + M_1 \cdot \Delta M_2 \prod_{i=3}^{n-1} M_i \cdot [x_{i-1}] +$$

$$+ M_1 \cdot M_2 \cdot \Delta M_3 \prod_{i=4}^{i-1} M_i \cdot [x_{n-1}] + \ldots + M_1 \cdot M_2 \cdot M_3 \ldots \cdot \quad (6)$$

$$\cdot \Delta M_s \prod_{i=s+1}^{n-1} [x_{n-1}]$$

The modulus of the linear error is:

$$\Delta r_n = \sqrt{(\Delta x_n)^2 + (\Delta y_n)^2 + (\Delta z_n)^2}\,, \qquad (7)$$

in which $\Delta x_n, \Delta y_n, \Delta z_n$, are the positioning errors on the axis *x, y, z* of the fixed coordinates' system which depend on the generalised coordinates errors.

In the case in which only the medium error is needed in a finite number of points the medium positioning error can be used:

$$\Delta r_n = \sum_{i=1}^{n} \left(\frac{\Delta r_i}{n} \right) = \sum_{i=1}^{n} \left(\frac{\sqrt{(\Delta x_i)^2 + (\Delta y_i)^2 + (\Delta z_i)^2}}{n} \right). \qquad (8)$$

The angular error of an industrial robot is characterised by the angle with which the mechanical hand can be rotated as the axis of the system $O'_n\, x'_n\, y'_n\, z'_n$ should remain parallel with the axis of the system $O_n x_n y_n z_n$.

As a vector the angular error has the expression:

$$\overline{\Delta\psi} = \Delta\psi \cdot \overline{p}\,, \qquad (9)$$

in which, $\overline{p}$ is the unitary vector of the angular error of the mechanical hand. If the angular error is considered as a small value, then:

$$\overline{\Delta\psi} = \sum_{i=1}^{n} \Delta q_i \cdot \overline{e_i}\,, \qquad (10)$$

in which $\overline{e}$ is the unitary vector from the rotational kinematic joint of an industrial robot.

Table 1

The values of the kinematic errors

Constructive variant of the industrial robot	Kinematic error [mm]
ARR (*a*)	0.41
ARR (*b*)	0.28
ARR (*c*)	0.73
ARR (*d*)	0.92

From the equation (10) it results:

$$[\Delta\psi] = \sum_{i=1}^{n} \Delta q_i \cdot L_{0i} \cdot [e_i], \qquad (11)$$

in which, $[\Delta\psi]$ is the column of the angular error coordinates in the system $O_0x_0y_0z_0$; L_{0i} is the 3rd grade matrix for passing from the system $O_i\ x_i\ y_i\ z_i$ to the system $O_0x_0y_0z_0$ which is determined with the formula:

$$L_{0i} = L_1 \cdot L_2 \cdot L_3 \cdot \ldots \cdot L_{i-1} \cdot L_i\,, \qquad (12)$$

where L_i is the 3rd grade matrix for passing from the system $O_i x_i y_i z_i$ to the system $O_{i-1} x_{i-1} y_{i-1} z_{i-1}$;

[e$_i$] is the column of the $\overline{e_i}$ vector's coordinates in the system $O_i x_i y_i z_i$.

For illustrating the application of the optimisation criteria proposed in the paper, there have been taken into consideration for the calculus the values $a_{2\text{-}6}$=300 mm of the lengths of the articulated modules and a_1=420 mm the length of the rotation module of the base.

Using calculation software Optirob PC the values of the kinematic errors presented in the Table 1 have been obtained, for the first four variants from Fig. 3. It can be noticed that from the 4th variants analysed of configuration, the variant ARR-*b* has the highest performance.

4. CONCLUSIONS

Designers of industrial robots are often caught in a dilemma whether to design a new mechanical system for moving rigid body through specified locations or to use a generic off-the-self multi-axis robot to perform the task.

In most of the uses the off-the-set robots are preferred, as they are ready to be used and do not require a prototype phase and a test phase as in the case of a newly designed mechanism.

However off-the-self robots are very expensive and considerably large in size. In addition, for respective tasks, which is usually the case, the use of a multi-axis robots is highly unjustified as several of the axis remain under utilised because of the redundancy in degrees of freedom. On the other hand, if the designer decides to design and built a new system the design algorithms either do not exist or are very complicated.

Therefore, there is a need to develop design methodologies for spatial task orientated robotic system.

This paper examines the design of articulated reconfigurable robot (ARR).

ARR presents the following advantages:

1. Their construction needs a minimum number of module (one base rotational module and k identical module type articulated arm).
2. Give the possibility to obtain with the same number of module different robot structure, with different number of freedom degree.
3. By reconfiguration of the workspace can be easily modified.
4. Changing the structure can increase the stiffness of the robot.
5. Has high degree of adaptability: by reconfiguration or adding new arm articulated modules can adapt to any workspace.
6. Keep the specific advantages of the module construction of the robot.

Solutions based on the dialytic elimination procedure is selected using the kinematic accuracy criteria of industrial robot.

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